CHAPTER 1
Introduction to Structural Steel Design

Advantages of structural steel:

1. **High Strength**

The high strength per steel unit of weight means that the weight of structures will be small.

2. **Uniformity**

The properties of steel do not change over the time as do those for reinforced concrete structures.

3. **Elasticity**

Steel behaves closer to design assumptions than most materials because it follows Hook's law up to fairly high stress.

4. **Performance**
Steel frames that are properly maintained will last indefinitely.

5. **Ductility**

1.1 STRUCTURAL DESIGN

The structural design of buildings, whether of structural steel or reinforced concrete, requires:

1. The determination of the overall proportions and dimensions of the supporting framework and,
2. The selection of the cross sections of individual members.

Generally, in design of any structure:

- Several alternative designs should be prepared and their costs compared.
- To do so requires the structural analysis of the building frames and the computation of forces and bending moments in the individual members.
- For each framing plan investigated, the individual components must be designed.
- Armed with this information, the structural designer can then select the appropriate cross section.

- Before any analysis, however, a decision must be made on the primary building material to be used; it will usually be reinforced concrete, structural steel, or both. Ideally, alternative designs should be prepared with each.

1.2 LOADS

The on forces that act a structure are called loads. They belong to one of two broad categories:

1. **Dead load, and Live load.**

Dead loads are those that are permanent, including

a. The weight of the structure itself, which is sometimes called the **self-weight**
b. Dead loads in a building include the weight of nonstructural components such as floor coverings, partitions, and suspended ceilings (with light fixtures, mechanical equipment, and plumbing).

All of the loads mentioned thus far are forces resulting from gravity and are referred to as **gravity loads**.

Live loads, which can also be gravity loads, are those that are not as permanent as dead loads. They may or may not be acting on the structure at any given time, and the location may not be fixed. Examples of live loads include:

1. **Furniture,**
2. **Equipment, and**
3. **Occupants of buildings.**

Live load may be classified into three main types:

1. **Static Load** when a live load is applied slowly and is not removed and reapplied an excessive number of times.
2. **Impact load** If the load is applied suddenly, as would be the case when the structure supports a moving crane, the effects of impact must be accounted for.
3. **Fatigue load** If the load is applied and removed many times over the life of the structure, fatigue stress becomes a problem, and its effects must be accounted for.

Wind exerts a pressure or suction on the exterior surfaces of a building. Because of the relative complexity of determining wind loads, however, wind is usually considered a separate category of loading.

Lateral loads are most detrimental to tall structures, wind loads are usually not as important for low buildings, but uplift on light roof systems can be critical.

Earthquake loads are another special category and need to be considered only in those geographic locations where there is a reasonable probability of occurrence.
Simpler methods are sometimes used in which the effects of the earthquake are simulated by a system of horizontal loads, similar to those resulting from wind pressure, acting at each floor level of the building.

Snow is another live load that is treated as a separate category. Adding to the uncertainty of this load is the complication of drift, which can cause much of the load to accumulate over a relatively small area.

Other types of live load are often treated as separate categories, such as **hydrostatic pressure** and **soil pressure**, but the cases we have enumerated are the ones ordinarily encountered in the design of structural steel building frames and their members.

1.3 BUILDING CODES

Buildings must be designed and constructed according to the provisions of a building code. **Building code** is a legal document containing requirements related to such things as structural safety, fire safety, plumbing, ventilation, and accessibility to the physically disabled. Abuilding code has the force of law and is administered by a governmental entity such as a city, a county, or, for some large metropolitan areas, a consolidated government.
Building codes do not give design procedures, but they do specify the design requirements and constraints that must be satisfied.

1.4 DESIGN SPECIFICATIONS

In contrast to building codes, design specifications give more specific guidance for the design of structural members and their connections. They present the guidelines and criteria that enable a structural engineer to achieve the objectives mandated by a building code.

Design specifications represent what is considered to be good engineering practice based on the latest research. They are periodically revised and updated by the issuance of supplements or completely new editions.

The specifications of most interest to the structural steel designer are those published by the following organizations.

1. *American Institute of Steel Construction* (**AISC**): This specification provides for the design of structural steel buildings and their connections.
2. *American Association of State Highway and Transportation Officials* (**AASHTO**): This specification covers the design of highway bridges and related structures. It provides for all structural materials normally used in bridges, including steel, reinforced concrete, and timber.
3. *American Railway Engineering and Maintenance-of-Way Association* (**AREMA**): The AREMA Manual for Railway Engineering covers the design of railway bridges and related structures. This organization was formerly known as the American Railway Engineering Association (**AREA**).
4. *American Iron and Steel Institute* (**AISI**): This specification deals with cold-formed steel.

1.5 STRUCTURAL STEEL

Steel, an alloy of primarily iron and carbon, with much less carbon than cast iron.

The characteristics of steel that are of the most interest to structural engineers can be examined by plotting the results of a tensile test. If a test specimen is subjected to an axial load *P*, as shown in Figure 1, the stress and strain can be computed as follows:

$$f = \frac{P}{A} \text{ and } \varepsilon = \frac{\Delta L}{L}$$

where

f - axial tensile stress

A- cross-sectional area

ε -axial strain

L -length of specimen

ΔL- change in length

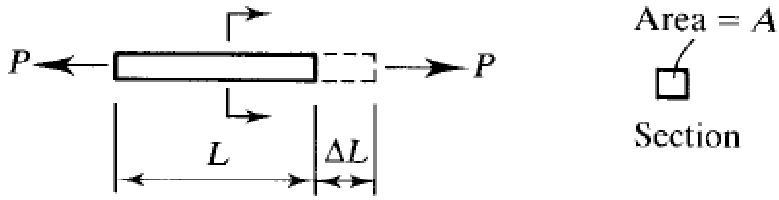


Figure 1

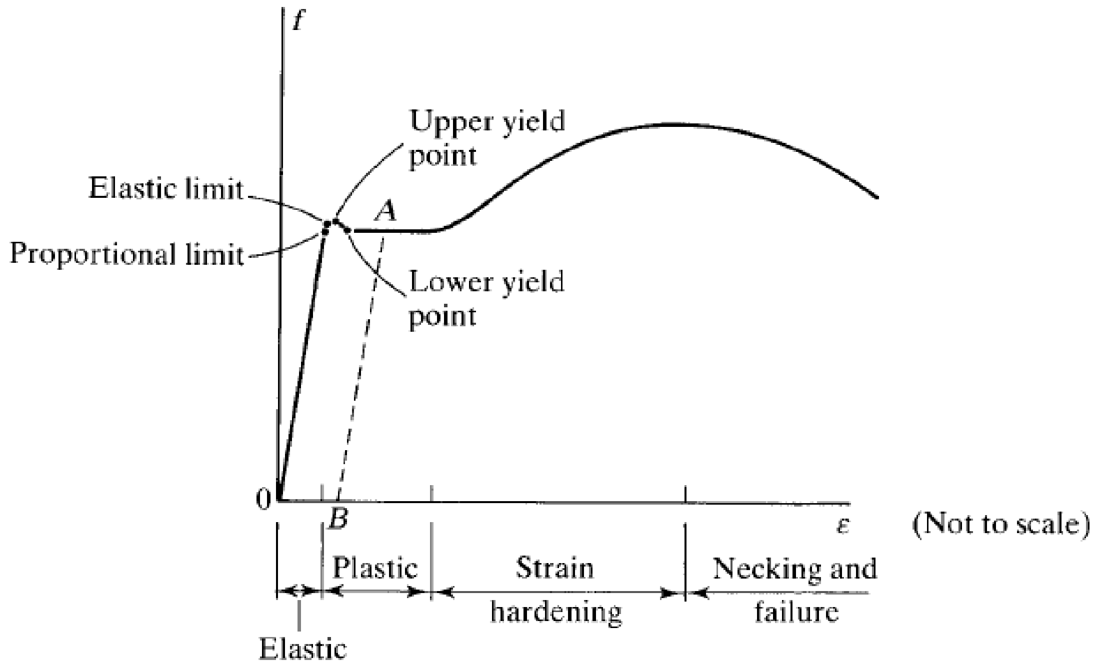


Figure 2

If the load is increased in increments from zero to the point of fracture, and stress and strain are computed at each step, a stress–strain curve such as the one shown in Figure 2 can be plotted. This curve is typical of a class of steel known as ductile, or mild, steel. The **proportional limit** relationship between stress and strain is linear, the material is said to follow Hooke's law.

yield point the peak value at the proportional limit.

plastic range The stress remains constant, even though the strain continues to increase. It is also called the **yield plateau.**

strain hardening an additional load (and stress) is required to cause additional elongation (and strain). A maximum value of stress is reached, after which the specimen begins to "neck down" as the stress decreases with increasing strain, and fracture occurs.

1.6 American Standard Cross-Sectional Shapes

In the design process outlined earlier, one of the objectives is the selection of the appropriate cross sections for the individual members of the structure being designed. Most often, this selection will entail choosing a standard cross-sectional shape that is widely available rather than requiring the fabrication of a shape with unique dimensions and properties.

Cross sections of some of the more commonly used are shown below:

1. **W-shape, (a wide-flange shape)**

Consists of two parallel flanges separated by a single web. The orientation of these elements is such that the cross section has two axes of symmetry.

A typical designation would be "**W18 × 50**"

Where:

W: indicates the type of shape,

18: is the nominal depth parallel to the web, (in)

50: is the weight in pounds per foot of length; lb/ft.

2. **S-shape**

It is similar to the W-shape in having two parallel flanges, a single web, and two axes of symmetry. The difference is in the proportions:

- The flanges of the W are wider in relation to the web than are the flanges of the S.
- The outside and inside faces of the flanges of the W-shape are parallel, whereas the inside faces of the flanges of the S-shape slope with respect to the outside faces.

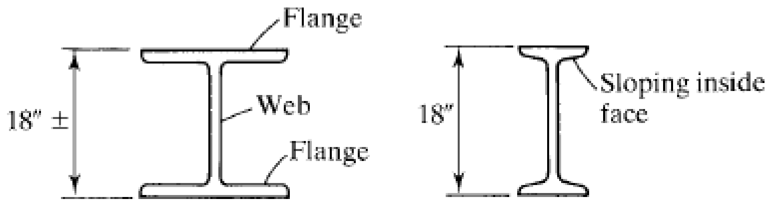
An example of the designation of an S-shape is "**S18 × 70**," with

S indicating the type of shape,

18 The depth in inches

70 The weight in pounds per foot.

These shapes are formerly called I-beams.



W-shape (W18 × 50 shown)

American Standard, S (S18 × 70 shown)

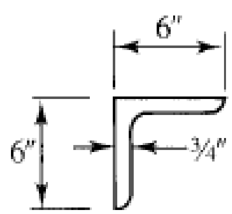
3. Angle shapes

Angle shapes are available in either equal-leg or unequal-leg versions. A typical designation would be "L6 × 6 × 3/4" or "L6 × 4 × 5/8." The three numbers are:

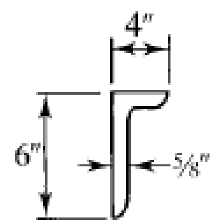
6 The lengths of each of the two legs as measured from the corner, or heel, to the toe at the other end of the leg,

3/4 The thickness, which is the same for both legs.

In the case of the unequal-leg angle (L6 × 4 × 5/8), the longer leg dimension is always given first. *The weight per foot does not provide*.



Equal-leg angle, L (L6 × 6 × ¾ shown)



Unequal-leg angle (L6 × 4 × ⅝ shown)

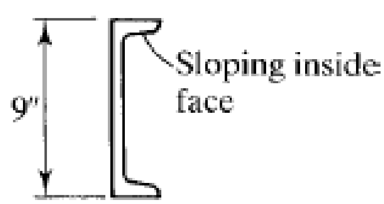
4. Channel, or C-shape,

Has two flanges and a web, with only one axis of symmetry; it carries a designation such as "C9 × 20." This notation is:

9 The total depth in inches' parallel to the web

20 The weight in pounds per linear foot.

For the channel, however, the depth is exact rather than nominal. The inside faces of the flanges are sloping, just as with the American Standard shape. Miscellaneous Channels—for example, the MC10 × 25—are similar to American Standard Channels.



American Standard Channel, C (C9 × 20 shown)

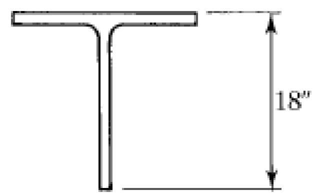
5. The Structural Tee

Produced by splitting an I-shaped member at mid depth. This shape is sometimes referred to as a split-tee. The prefix of the designation is either: WT, ST, or MT, depending on which shape is the "parent." For example,

WT18 × 105 has:

A nominal depth of 18 inches, a weight of 105 lb/ft, and cut from a W36 × 210.

Similarly, an ST10 × 33 is cut from an S20 × 66.



Structural Tee: WT, ST, or MT
(WT18 × 105 shown)

6. Miscellaneous or M-shape

The M-shape has two parallel flanges and a web, but it does not fit exactly into either the W or S categories. For example, MT5 × 4 is cut from an M10 × 8.

7. HP shape

It is used for bearing piles, has parallel flange surfaces, approximately the same width and depth, and equal flange and web thicknesses.

HP-shapes are designated in the same manner as the W-shape; for example, HP14 × 117.

8. Bars and plate

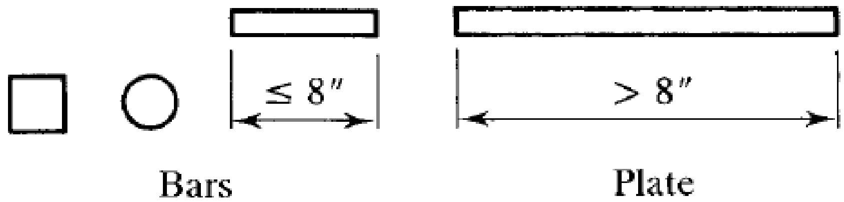
Bar can have circular, square, or rectangular cross sections.

- If the width of a rectangular shape is ≤ 8 inches, it is classified as a bar.
- If the width is > 8 inches, the shape is classified as a plate.

The usual designation abbreviation for both is:

PL (thickness in inches, the width in inches, and the length in feet and inches)

For example, PL 3/8 × 5 × 3′-2½″.



Note: Plates and bars are available in increments of 1/16 inch, it is customary to specify dimensions to the nearest 1/8 inch.

9. Hollow shapes

Can be produced either by bending plate material into the desired shape and welding the seam or by hot working to produce a seamless shape. The shapes are categorized as:

- Steel pipe,
- Round HSS,
- Square HSS
- Rectangular HSS.

The designation HSS is for "**Hollow Structural Sections**"

Steel pipe is available as:

- Standard (Pipe 5 Std.),
- Extra-strong (Pipe 5 x-strong), or
- Double-extra-strong (Pipe 5 xx-strong)

where 5 is the nominal outer diameter in inches.

For pipes whose thicknesses do not match those in the standard, extra-strong, or double-extra-strong categories, the designation is the outer diameter and wall thickness in inches, expressed to three decimal places; for example, Pipe 5.563 × 0.500.

Round HSS are designated by **HSS 8.625 × 0.250**

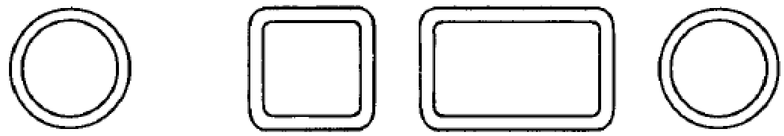
Where **8.625**: outer diameter and **0.25**: wall thickness, expressed to three decimal places.

Square and rectangular HSS are designated by **HSS 7 × 5 × 3/8**

Where

7 and **5**: nominal outside dimensions and

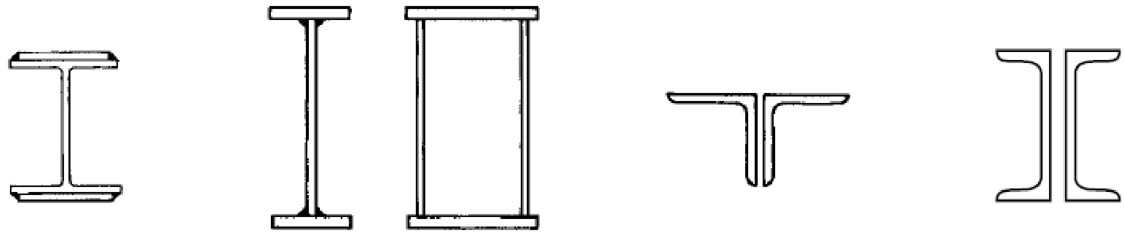
3/8: wall thickness, expressed in rational numbers.



Steel pipe Hollow Structural Sections

10.Built-up section

In most cases, one of the above standard shapes satisfy the design requirements. If the requirements are especially severe, then a built-up section may be needed. Sometimes a standard shape is augmented by additional cross-sectional elements, as when a cover plate is welded to one or both flanges of a W-shape.



W-shape with cover plates Plate girders Double angle Double channel

Chapter Two

1. Concepts in Structural Steel Design

2.1 DESIGN PHILOSOPHIES

As discussed earlier, the design of a structural member entails the selection of a cross section that will safely and economically resist the applied loads. Economy usually means minimum weight—that is, the minimum amount of steel. The fundamental requirement of structural design is that the required strength not exceed the available strength; that is,

Required strength ≤available strength

2.1.1 Allowable Strength Design (ASD),

A member is selected that has cross-sectional properties such as area and moment of inertia that are large enough to prevent the maximum applied axial force, shear, or bending moment from exceeding an allowable, or permissible, value.

This allowable value is obtained by dividing the nominal, or theoretical, strength by a factor of safety. This can be expressed as:

Required strength ≤ allowable strength

Where:

$$\text{Allowable strength} = \frac{\text{nominal strength}}{\text{safety factor}} \qquad (2.1)$$

This approach is called **Allowable Stress Design**. Strength can be an axial force strength (as in tension or compression members), a flexural strength (moment strength), or a shear strength.

If stresses are used instead of forces or moments, the relationship of **Eq. 2.1** becomes

Maximum applied stress ≤ allowable stress

The allowable stress will be in the elastic range of the material.

This approach to design is also called **elastic design or working stress design**. Working stresses are those resulting from the working loads, which are the applied loads. Working loads are also known as **service loads**.

2.1.2 Plastic Design

Is based on a consideration of failure conditions rather than working load conditions. A member is selected by using the criterion that the structure will fail at a load substantially higher than the working load. Failure in this context means either collapse or extremely large deformations. The term plastic is used because, at failure, parts of the member will be subjected to very large strains, large enough to put the member into the plastic range. When the entire cross section becomes plastic at enough locations, "plastic hinges" will form at those locations, creating a collapse mechanism. As the actual loads will be less than the failure loads by a factor of safety known as the **load factor**, members designed this way are not unsafe, despite being designed based on what happens at failure. This design procedure is roughly as follows:

1. Multiply the working loads (service loads) by the load factor to obtain the failure loads.
2. Determine the cross-sectional properties needed to resist failure under these loads. (A member with these properties is said to have sufficient strength and would be at the verge of failure when subjected to the factored loads.)
3. Select the lightest cross-sectional shape that has these properties.

Members designed by plastic theory would reach the point of failure under the factored loads but are safe under actual working loads.

2.1.3 Load and Resistance Factor Design (LRFD)

Similar to plastic design in that strength, or the failure condition, is considered. Load factors are applied to the service loads, and a member is selected that will have enough strength to resist the factored loads.

In addition, the theoretical strength of the member is reduced by the application of a resistance factor. The criterion that must be satisfied in the selection of a member is:

Factored load ≤ factored strength

In this expression,

The factored load is actually the sum of all service loads to be resisted by the member, each multiplied by its own load factor. For example, dead loads will have load factors that are different from those for live loads.

The factored strength is the theoretical strength multiplied by a resistance factor. The above equation can be rewritten:

$$\text{(Loads} \times \text{Load factors)} \leq \text{Resistance} \times \text{Resistance factor)} \qquad (2.2)$$

The factored load is a failure load greater than the total actual service load, so the load factors are usually greater than unity. However, the factored strength is a reduced, usable strength, and the resistance factor is usually less than unity. The factored loads are the loads that bring the structure or member to its limit. In terms of safety, this limit state can be fracture, yielding, or buckling, and the factored resistance is the useful strength of the member, reduced from the theoretical value by the resistance factor. The limit state can also be one of serviceability, such as a maximum acceptable deflection.

2.2 LOAD FACTORS, RESISTANCE FACTORS, AND LOAD COMBINATIONS FOR LRFD

Equation **2.2** can be written as follows:

$$\gamma_i Q_i \leq \emptyset R_n \qquad \textbf{(2.3)}$$

where,

Q_i - the load effect (a force or a moment)

γ_i - the load factor

R_n - the nominal resistance, or strength, of the component under consideration

$\emptyset$ - the resistance factor

The factored resistance $\emptyset \boldsymbol{R_n}$ is called the **Design Strength**

The summation on the left side of Equation is over the total number of load effects (including, but not limited to, dead load and live load), where each load effect can be associated with a different load factor

Equation **(2.3)** can also be written in the form

$$\boldsymbol{R_u} \leq \emptyset \boldsymbol{R_n} \qquad \textbf{(2.4)}$$

Where,

R_u = required strength = sum of factored load effects (forces or moments).

ASCE 7 (ASCE, 2010) provides load factors and load combinations based on extensive statistical studies. ASCE 7 presents the basic load combinations in the following form:

Combination 1: $1.4D$

Combination 2: $1.2D + 1.6L + 0.5\ (L_r \text{ or } S \text{ or } R)$

Combination 3: $1.2D + 1.6\ (L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$

Combination 4: $1.2D + 1.0W + L + 0.5(L_r \text{ or S or R})$

Combination 5: $1.2D + 1.0E + L + 0.2S$

Combination 6: $0.9D + 1.0W$

Combination 7: $0.9D + 1.0E$

Where:

D = dead load

L = live load due to occupancy

Lr = roof live load

S = snow load

R = rain or ice load

W = wind load

E = earthquake (seismic load)

The resistance factor ϕ for each type of resistance is given by AISC in the Specification chapter dealing with that resistance, but in most cases, one of two values will be used:

ϕ = 0.90 for limit sates involving yielding or compression buckling, and

ϕ = 0.75 for limit states involving rupture (fracture).

2.3 SAFETY FACTORS AND LOAD COMBINATIONS FOR ASD

For allowable strength design, the relationship between loads and strength can be expressed as:

$$R_a \leq \frac{R_n}{\Omega} \qquad \textbf{(2.5)}$$

Where

R_a =required strength

R_n =nominal strength (same as for LRFD)

Ω =safety factor

R_n/Ω =allowable strength

The required strength R_a is the sum of the service loads or load effects

Load combinations for ASD are also given in ASCE 7. These combinations, as presented in the AISC Steel Construction Manual (AISC 2011a), are

Combination 1: D

Combination 2: D + L

Combination 3: D + (L_r or S or R)

Combination 4: D + 0.75L + 0.75(L_r or S or R)

Combination 5: D ± (0.6W or 0.7E)

Combination 6a: D + 0.75L + 0.75(0.6W) + 0.75(Lr or S or R)

Combination 6b: D + 0.75L ± 0.75(0.7E) + 0.75S

Combinations 7 and 8: 0.6D ± (0.6W or 0.7E)

Corresponding to the two most common values of resistance factors in **LRFD** are the following values of the safety factor Ω in ASD:

- For limit states involving yielding or compression buckling, Ω=1.67.
- For limit states involving rupture, Ω= 2.00

The relationship between resistance factors and safety factors is given by:

$$\Omega = \frac{1.5}{\phi}$$

If both sides of Equation **(2.5)** are divided by area (in the case of axial load) or section modulus (in the case of bending moment), then the relationship becomes:

$$f \leq F$$

where

f = applied stress

F = allowable stress

This formulation is called *allowable stress design*.

2.4 STEEL CONSTRUCTION MANUAL

Anyone engaged in structural steel design must have access to AISC's *Steel Construction Manual*. This publication contains the AISC Specification and numerous design aids in the form of tables and graphs, as well as a "catalog" of the most widely available structural shapes.

The *Manual* is divided into 17 parts as follows:

Part 1. Dimensions and Properties.

Part 2. General Design Considerations.

Part 3. Design of Flexural Members.

Part 4. Design of Compression Members.

Part 5. Design of Tension Members.

Part 6. Design of Members Subject to Combined Loading.

Part 7. Design Considerations for Bolts.

Part 8. Design Considerations for Welds.

Part 9. Design of Connecting Elements.

Part 10. Design of Simple Shear Connections.

Part 11. Design of Partially Restrained Moment Connections.

Part 12. Design of Fully Restrained (FR) Moment Connections.

Part 13. Design of Bracing Connections and Truss Connections.

Part 14. Design of Beam Bearing Plates, Column Base Plates, Anchor Rods, and Column Splices.

Part 15. Design of Hanger Connections, Bracket Plates, and Crane–Rail Connections.

Part 16. Specifications and Codes.

Part 17. Miscellaneous Data and Mathematical Information.

Chapter 3

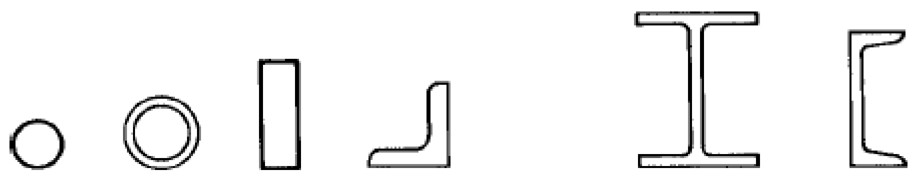
Tension Members

3.1 INTRODUCTION

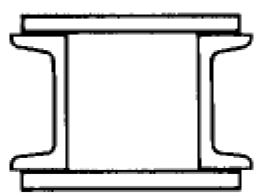
Tension members are structural elements that are subjected to **axial tensile forces**. They are used in various types of structures and include truss members, bracing for buildings and bridges, cables in suspended roof systems, and cables in suspension and cable-stayed bridges.

Note: Any cross-sectional configuration may be used, because for any given material, the only determinant of the strength of a tension member is the cross-sectional area.

Circular rods and rolled angle shapes are frequently used:



Built-up shapes, either from plates, rolled shapes, or a combination of plates and rolled shapes, are sometimes used when large loads must be resisted.



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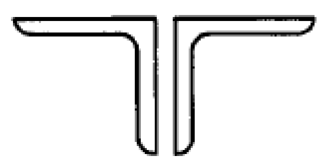

double-angle section

The stress in an axially loaded tension member is given by:

$$f = \frac{P}{A} \qquad (3.1)$$

where

P is the magnitude of the load

A is the cross-sectional area (the area normal to the load).

Note:

1. The stress given by ep. 3.1 is exact providing that the cross section is not adjacent to the point of load where the stress is not uniform.
2. If the cross sectional area of a tension member is varied along the length, the stress is a function of the section under consideration.
3. The presence of holes in a member will influence the stress at a cross section through the hole or holes.

Tension members are frequently connected at their ends with bolts, as illustrated in Figure 3.1. The tension member shown, a plate is connected to a gusset plate (لوح تقوية), which is a connection element whose purpose is to transfer the load from the member to a support or to another member. Hence, The area of the bar at ***section a–a*** is (1/2)*(8)=4 in.^{2}, but the area at ***section b–b*** is only 4 – (2)(1/2)(7/8) = 3.13 in.^{2}

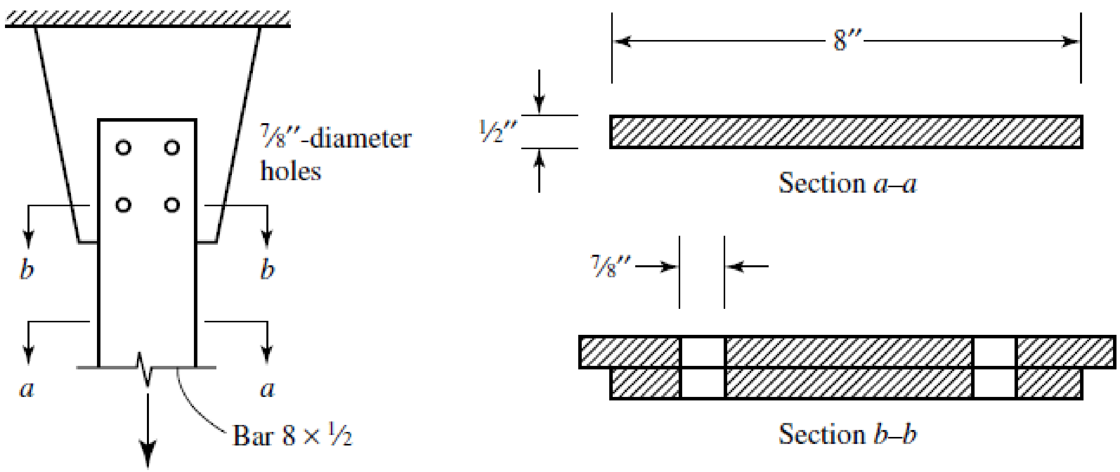

Figure 3.1

This reduced area (3.13 in.^{2}) will be more highly stressed and is referred to as the *net area*, or *net section*, and the unreduced area (4 in.^{2}) is the *gross area*.

3.2 TENSILE STRENGTH

A tension member can fail by reaching one of two limit states:

- **Excessive deformation or**
- **Fracture**.

To prevent excessive deformation, initiated by yielding, the load on the gross section must be small enough to ensure that the stress on the *gross section* is less than the yield stress F_y.

To prevent fracture, the stress on the *net section* must be less than the tensile strength F_u. In each case, the stress *P/A* must be less than a limiting stress *F* or

$$\frac{P}{A} < F$$

Thus, the load *P* must be less than *FA*, or $P < FA$

The *nominal* strength in yielding is;

$$P_n = F_y A_g$$

and the nominal strength in fracture is;

$$P_n = F_u A_e$$

Where:

A_e is the *effective* net area, which may be equal to either the net area or, in some cases, a smaller area. We discuss effective net area later.

LRFD: In load and resistance factor design, the factored tensile load is compared to the design strength. The design strength is the resistance factor times the nominal strength Eq. **(2.4)**.

$$R_u = \emptyset R_n$$

can be written for tension members as

$$P_u \leq \emptyset_t P_n$$

where P_u is the governing combination of factored loads.

The resistance factor $\emptyset_t$ is smaller for fracture than for yielding, reflecting the more serious nature of fracture.

For yielding, $\emptyset_t = 0.9$

For fracture, $\emptyset_t = 0.75$

Because there are two limit states, both of the following conditions must be satisfied:

$$P_u \leq 0.9\, F_y\, A_g$$

$$P_u \leq 0.75\, F_u\, A_e$$

The smaller of these is the design strength of the member.

ASD: In allowable strength design, the total service load is compared to the allowable load:

$$P_a = \frac{P_n}{\Omega_t}$$

Where

P_a is the required strength (applied load) and $\frac{P_n}{\Omega_t}$ is the allowable strength.

The subscript "a" indicates that the required strength is for "allowable strength design," but you can think of it as standing for "applied" load.

For yielding of the *gross section*, the safety factor $\Omega_t = 1.67$, and the allowable load is:

$$\frac{P_n}{\Omega_t} = \frac{F_y\, A_g}{1.67} = 0.6F_y\, A_g$$

(The factor 0.6 appears to be a rounded value, but recall that 1.67 is a rounded value. If $\Omega_t = \frac{5}{3}$ is used, the allowable load is exactly $0.6F_y\, A_g$)

For fracture of the net section, the safety factor is 2.00 and the allowable load is

$$P_a = \frac{P_n}{\Omega_t} = \frac{F_u\, A_e}{2} = 0.5F_u\, A_e$$

Alternatively, the service load stress can be compared to the allowable stress. This can be expressed as:

$$f_t \le F_t$$

where

f_t is the applied stress and F_t is the allowable stress

For yielding of the *gross section*,

$$f_t = \frac{P_a}{A_g} \text{ and } F_t = \frac{P_n/\Omega_t}{A_g} = \frac{0.6\, F_y A_g}{A_g} = 0.6F_y$$

For fracture of the net section,

$$f_t = \frac{P_a}{A_e} \text{ and } F_t = \frac{P_n/\Omega_t}{A_e} = \frac{0.5\, F_u A_e}{A_e} = 0.5F_u$$

You can find values of F_y and F_u for various structural steels in Table 2-3 in the *Manual.*

TABLES FOR THE GENERAL DESIGN AND SPECIFICATIONS OF MATERIALS 2–39

Table 2–3
Applicable ASTM Specifications for Various Structural Shapes

Steel Type	ASTM Designation		F_y Min. Yield Stress (ksi)	F_u Tensile Stress[a] (ksi)	Applicable Shape Series									
					W	M	S	HP	C	MC	L	HSS Rect.	HSS Round	Pipe
Carbon	A36		36	58–80[b]	▦	■	■	▦	■	■	■			
	A53 Gr. B		35	60										■
	A500	Gr. B	42	58									■	
			46	58								■		
		Gr. C	46	62									▦	
			50	62								▦		
	A501		36	58								▦	▦	
	A529[c]	Gr. 50	50	65–100	▦	▦	▦	▦	▦	▦	▦			
		Gr. 55	55	70–100	▦	▦	▦	▦	▦	▦	▦			
High-Strength Low-Alloy	A572	Gr. 42	42	60	▦	▦	▦	▦	▦	▦	▦			
		Gr. 50	50	65[d]	▦	▦	▦	■	▦	▦	▦			
		Gr. 55	55	70	▦	▦	▦	▦	▦	▦	▦			
		Gr. 60[e]	60	75	▦	▦	▦	▦	▦	▦	▦			
		Gr. 65[e]	65	80	▦	▦	▦	▦	▦	▦	▦			
	A618[f]	Gr. I & II	50[g]	70[g]								▦	▦	
		Gr. III	50	65								▦	▦	
	A913	50	50[h]	60[h]	▦	▦	▦	▦	▦	▦	▦			
		60	60	75	▦	▦	▦	▦	▦	▦	▦			
		65	65	80	▦	▦	▦	▦	▦	▦	▦			
		70	70	90	▦	▦	▦	▦	▦	▦	▦			
	A992		50–65[i]	65[i]	■	▦	▦	▦	▦	▦	▦			
Corrosion Resistant High-Strength Low-Alloy	A242		42[j]	63[j]	▦									
			46[k]	67[k]	▦			▦			▦			
			50[l]	70[l]	▦	▦	▦	▦	▦	▦	▦			
	A588		50	70	▦	▦	▦	▦	▦	▦	▦			
	A847		50	70								▦	▦	

■ = Preferred material specification.
▦ = Other applicable material specification, the availability of which should be confirmed prior to specification.
☐ = Material specification does not apply.

[a] Minimum unless a range is shown.
[b] For shapes over 426 lb/ft, only the minimum of 58 ksi applies.
[c] For shapes with a flange thickness less than or equal to 1½ in. only. To improve weldability a maximum carbon equivalent can be specified (per ASTM Supplementary Requirement S78). If desired, maximum tensile stress of 90 ksi can be specified (per ASTM Supplementary Requirement S79).
[d] If desired, maximum tensile stress of 70 ksi can be specified (per ASTM Supplementary Requirement S91).
[e] For shapes with a flange thickness less than or equal to 2 in. only.
[f] ASTM A618 can also be specified as corrosion-resistant; see ASTM A618.
[g] Minimum applies for walls nominally ¾-in. thick and under. For wall thicknesses over ¾ in., F_y = 46 ksi and F_u = 67 ksi.
[h] If desired, maximum yield stress of 65 ksi and maximum yield-to-tensile strength ratio of 0.85 can be specified (per ASTM Supplementary Requirement S75).
[i] A maximum yield-to-tensile strength ratio of 0.85 and carbon equivalent formula are included as mandatory in ASTM A992.
[j] For shapes with a flange thickness greater than 2 in. only.
[k] For shapes with a flange thickness greater than 1½ in. and less than or equal to 2 in. only.
[l] For shapes with a flange thickness less than or equal to 1½ in. only.

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All of the steels that are available for various hot-rolled shapes are indicated by **shaded areas**.

The **black areas** correspond to preferred materials, and The **gray areas** represent other steels that are available.

Under the **W** heading, we see that A992 is the preferred material for **W** shapes, but other materials are available, usually at a higher cost.

For some steels, there is more than one grade, with each grade having different values of F_y and F_u. In these cases, the grade must be specified along with the ASTM designation—for example, A572 Grade 50. For A242 steel, F_y and F_u depend on the thickness of the flange of the cross-sectional shape. This relationship is given in footnotes in the table.

For example, to determine the properties of a W33 ×221 of ASTM A242 steel, first refer to the dimensions and properties table in Part 1 of the *Manual* and determine that the flange thickness t_f is equal to 1.28 inches. This matches the thickness range indicated in footnote*1*; therefore, $Fy = 50\ ksi$ and $Fu = 70\ ksi$.

Values of F_y and F_u for plates and bars are given in the *Manual* Table 2-4, and information on structural fasteners, including bolts and rods, can be found in Table 2-5.

AISC Specification requires (to calculate the exact amount of area to be deducted المستقطعة from the gross area to account for the presence of bolt holes) the addition of 1/16 inch to the actual hole diameter.

This amounts to using an effective hole diameter 1/8 inch larger than the fastener diameter.

This means;

hole diameter = actual hole diameter + 1/16

hole diameter = actual bolt diameter + 1/8

You can find details related to standard, oversized, and slotted holes in AISC J3.2, "Size and Use of Holes" (in Chapter J, "Design of Connections")

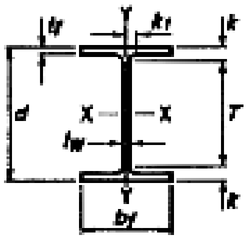


Table 1–1 (continued)
W Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		$\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	k_{det}	k_1	T	Workable Gage
	in.²	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W36×800[h]	236	42.6	42 1/2	2.38	2 3/8	1 3/16	18.0	18	4.29	4 5/16	5.24	5 5/16	2 3/8	31 3/8	7 1/2
×652[h]	192	41.1	41	1.97	2	1	17.6	17 5/8	3.54	3 9/16	4.49	4 13/16	2 3/16		
×529[h]	156	39.8	39 3/4	1.61	1 5/8	13/16	17.2	17 1/4	2.91	2 15/16	3.86	4 3/16	2		
×487[h]	143	39.3	39 3/8	1.50	1 1/2	3/4	17.1	17 1/8	2.68	2 11/16	3.63	4	1 15/16		
×441[h]	130	38.9	38 7/8	1.36	1 3/8	11/16	17.0	17	2.44	2 7/16	3.39	3 3/4	1 7/8		
×395[h]	116	38.4	38 3/8	1.22	1 1/4	5/8	16.8	16 7/8	2.20	2 3/16	3.15	3 7/16	1 13/16		
×361[h]	106	38.0	38	1.12	1 1/8	9/16	16.7	16 3/4	2.01	2	2.96	3 5/16	1 3/4		
×330	97.0	37.7	37 5/8	1.02	1	1/2	16.6	16 5/8	1.85	1 7/8	2.80	3 1/8	1 3/4		
×302	88.8	37.3	37 3/8	0.945	15/16	1/2	16.7	16 5/8	1.68	1 11/16	2.63	3	1 11/16		
×282[c]	82.9	37.1	37 1/8	0.885	7/8	7/16	16.6	16 5/8	1.57	1 9/16	2.52	2 7/8	1 5/8		
×262[c]	77.0	36.9	36 7/8	0.840	13/16	7/16	16.6	16 1/2	1.44	1 7/16	2.39	2 3/4	1 5/8		
×247[c]	72.5	36.7	36 5/8	0.800	13/16	7/16	16.5	16 1/2	1.35	1 3/8	2.30	2 5/8	1 5/8		
×231[c]	68.1	36.5	36 1/2	0.760	3/4	3/8	16.5	16 1/2	1.26	1 1/4	2.21	2 9/16	1 9/16		
W36×256	75.4	37.4	37 3/8	0.960	15/16	1/2	12.2	12 1/4	1.73	1 3/4	2.48	2 5/8	1 5/16	32 1/8	5 1/2
×232[c]	68.1	37.1	37 1/8	0.870	7/8	7/16	12.1	12 1/8	1.57	1 9/16	2.32	2 7/16	1 1/4		
×210[c]	61.8	36.7	36 3/4	0.830	13/16	7/16	12.2	12 1/8	1.36	1 3/8	2.11	2 5/16	1 1/4		
×194[c]	57.0	36.5	36 1/2	0.765	3/4	3/8	12.1	12 1/8	1.26	1 1/4	2.01	2 3/16	1 3/16		
×182[c]	53.6	36.3	36 3/8	0.725	3/4	3/8	12.1	12 1/8	1.18	1 3/16	1.93	2 1/8	1 3/16		
×170[c]	50.1	36.2	36 1/8	0.680	11/16	3/8	12.0	12	1.10	1 1/8	1.85	2	1 3/16		
×160[c]	47.0	36.0	36	0.650	5/8	5/16	12.0	12	1.02	1	1.77	1 15/16	1 1/8		
×150[c]	44.2	35.9	35 7/8	0.625	5/8	5/16	12.0	12	0.940	15/16	1.69	1 7/8	1 1/8		
×135[c,v]	39.7	35.6	35 1/2	0.600	5/8	5/16	12.0	12	0.790	13/16	1.54	1 11/16	1 1/8		
W33×387[h]	114	36.0	36	1.26	1 1/4	5/8	16.2	16 1/4	2.28	2 1/4	3.07	3 3/16	1 7/16	29 5/8	5 1/2
×354[h]	104	35.6	35 1/2	1.16	1 3/16	5/8	16.1	16 1/8	2.09	2 1/16	2.88	2 15/16	1 3/8		
×318	93.6	35.2	35 1/8	1.04	1 1/16	9/16	16.0	16	1.89	1 7/8	2.68	2 3/4	1 3/16		
×291	85.7	34.8	34 7/8	0.960	15/16	1/2	15.9	15 7/8	1.73	1 3/4	2.52	2 5/8	1 5/16		
×263	77.5	34.5	34 1/2	0.870	7/8	7/16	15.8	15 3/4	1.57	1 9/16	2.36	2 7/16	1 1/4		
×241[c]	71.0	34.2	34 1/8	0.830	13/16	7/16	15.9	15 7/8	1.40	1 3/8	2.19	2 1/4	1 1/4		
×221[c]	65.2	33.9	33 7/8	0.775	3/4	3/8	15.8	15 3/4	1.28	1 1/4	2.06	2 1/8	1 3/16		
×201[c]	59.2	33.7	33 5/8	0.715	11/16	3/8	15.7	15 3/4	1.15	1 1/8	1.94	2	1 3/16		
W33×169[c]	49.5	33.8	33 7/8	0.670	11/16	3/8	11.5	11 1/2	1.22	1 1/4	1.92	2 1/8	1 3/16	29 5/8	5 1/2
×152[c]	44.8	33.5	33 1/2	0.635	5/8	5/16	11.6	11 5/8	1.06	1 1/16	1.76	1 15/16	1 1/8		
×141[c]	41.6	33.3	33 1/4	0.605	5/8	5/16	11.5	11 1/2	0.960	15/16	1.66	1 13/16	1 1/8		
×130[c]	38.3	33.1	33 1/8	0.580	9/16	5/16	11.5	11 1/2	0.855	7/8	1.56	1 3/4	1 1/8		
×118[c,v]	34.7	32.9	32 7/8	0.550	9/16	5/16	11.5	11 1/2	0.740	3/4	1.44	1 5/8	1 1/8		

[c] Shape is slender for compression with $F_y = 50$ ksi.
[h] Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
[v] Shape does not meet the h/t_w limit for shear in Specification Section G2.1a with $F_y = 50$ ksi.

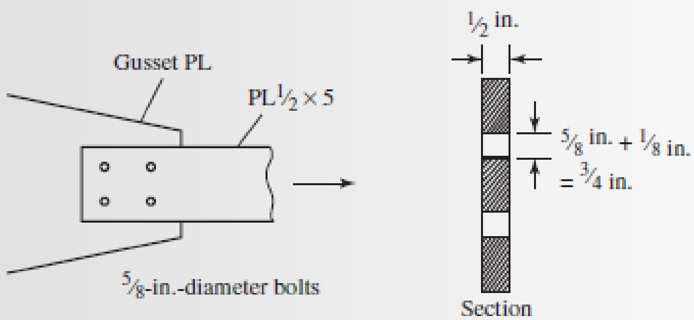
EXAMPLE 3.1

A ½ × 5 plate of A36 steel is used as a tension member. It is connected to a gusset plate with four ⅝-inch-diameter bolts as shown in Figure 3.3. Assume that the effective net area A_e equals the actual net area A_n (we cover computation of effective net area in Section 3.3).

a. What is the design strength for LRFD?

b. What is the allowable strength for ASD?

FIGURE 3.3



SOLUTION

For yielding of the gross section,

$$A_g = 5(1/2) = 2.5 \text{ in.}^2$$

and the nominal strength is

$$P_n = F_y A_g = 36(2.5) = 90.0 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 2.5 - (½)(¾) \times 2 \text{ holes}$$
$$= 2.5 - 0.75 = 1.75 \text{ in.}^2$$

$A_e = A_n = 1.75$ in.^{2} (This is true for this example, but A_e does not always equal A_n.)

The nominal strength is

$$P_n = F_u A_e = 58(1.75) = 101.5 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81.0 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(101.5) = 76.1 \text{ kips}$$

ANSWER The design strength for LRFD is the smaller value: $\phi_t P_n = 76.1$ kips.

b. The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{101.5}{2.00} = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Alternative Solution Using Allowable Stress: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(2.5) = 54.0 \text{ kips}$$

(The slight difference between this value and the one based on allowable strength is because the value of Ω in the allowable strength approach has been rounded from 5/3 to 1.67; the value based on the allowable stress is the more accurate one.)
For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(1.75) = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

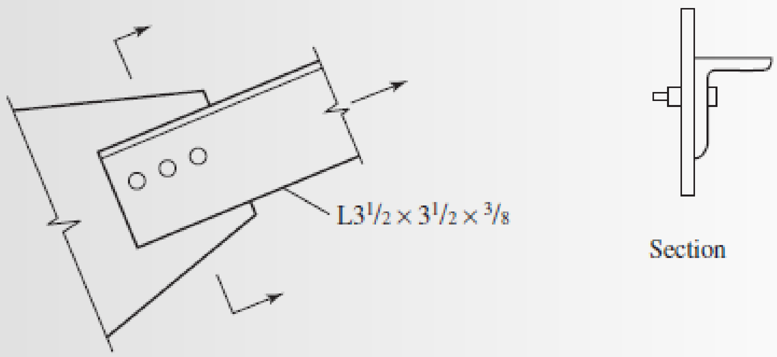
EXAMPLE 3.2

A single-angle tension member, an L3½ × 3½ × ⅜, is connected to a gusset plate with ⅞-inch-diameter bolts as shown in Figure 3.4. A36 steel is used. The service loads are 35 kips dead load and 15 kips live load. Investigate this member for compliance with the AISC Specification. Assume that the effective net area is 85% of the computed net area.

a. Use LRFD.

b. Use ASD.

FIGURE 3.4



SOLUTION First, compute the nominal strengths.

Gross section:

$$A_g = 2.50 \text{ in.}^2 \quad \text{(from Part 1 of the \textit{Manual})}$$

$$P_n = F_y A_g = 36(2.50) = 90 \text{ kips}$$

Net section:

$$A_n = 2.50 - \left(\frac{3}{8}\right)\left(\frac{7}{8}+\frac{1}{8}\right) = 2.125 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(2.125) = 1.806 \text{ in.}^2 \quad \text{(in \textit{this} example)}$$

$$P_n = F_u A_e = 58(1.806) = 104.7 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(104.7) = 78.5 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 78.5$ kips

Factored load:

When only dead load and live load are present, the only load combinations with a chance of controlling are combinations 1 and 2.

Combination 1: $1.4D = 1.4(35) = 49$ kips
Combination 2: $1.2D + 1.6L = 1.2(35) + 1.6(15) = 66$ kips

The second combination controls; $P_u = 66$ kips.

(When only dead load and live load are present, combination 2 will always control when the dead load is less than eight times the live load. In future examples, we will not check combination 1 [$1.4D$] when it obviously does not control.)

ANSWER Since $P_u < \phi_t P_n$, (66 kips < 78.5 kips), the member is satisfactory.

b. For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{104.7}{2.00} = 52.4 \text{ kips}$$

The smaller value controls; the allowable strength is 52.4 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 35 + 15 = 50 \text{ kips}$$

ANSWER Since 50 kips < 52.4 kips, the member is satisfactory.

Alternative Solution Using Allowable Stress

For the gross area, the applied stress is

$$f_t = \frac{P_a}{A_g} = \frac{50}{2.50} = 20 \text{ ksi}$$

and the allowable stress is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

For this limit state, $f_t < F_t$ (OK)

For the net section,

$$f_t = \frac{P_a}{A_e} = \frac{50}{1.806} = 27.7 \text{ ksi}$$

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi} > 27.7 \text{ ksi} \quad \text{(OK)}$$

ANSWER Since $f_t < F_t$ for both limit states, the member is satisfactory.

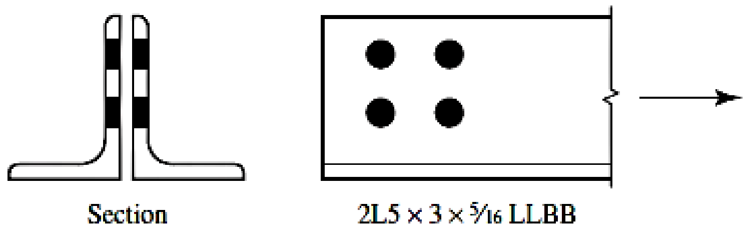
EXAMPLE 3.3

A double-angle shape is shown in Figure 3.5. The steel is A36, and the holes are for ½-inch-diameter bolts. Assume that $A_e = 0.75A_n$.

a. Determine the design tensile strength for LRFD.

b. Determine the allowable strength for ASD.

FIGURE 3.5



SOLUTION

Figure 3.5 illustrates the notation for unequal-leg double-angle shapes. The notation LLBB means "long-legs back-to-back," and SLBB indicates "short-legs back-to-back."

When a double-shape section is used, two approaches are possible: (1) consider a single shape and double everything, or (2) consider two shapes from the outset. (Properties of the double-angle shape are given in Part 1 of the *Manual*.) In this example, we consider one angle and double the result. For one angle, the nominal strength based on the gross area is

$$P_n = F_y A_g = 36(2.41) = 86.76 \text{ kips}$$

There are two holes in each angle, so the net area of one angle is

$$A_n = 2.41 - \left(\frac{5}{16}\right)\left(\frac{1}{2} + \frac{1}{8}\right) \times 2 = 2.019 \text{ in.}^2$$

The effective net area is

$$A_e = 0.75(2.019) = 1.514 \text{ in.}^2$$

The nominal strength based on the net area is

$$P_n = F_u A_e = 58(1.514) = 87.81 \text{ kips}$$

a. The design strength based on yielding of the gross area is

$$\phi_t P_n = 0.90(86.76) = 78.08 \text{ kips}$$

The design strength based on fracture of the net area is

$$\phi_t P_n = 0.75(87.81) = 65.86 \text{ kips}$$

ANSWER Because 65.86 kips < 78.08 kips, fracture of the net section controls, and the design strength for the two angles is $2 \times 65.86 = 132$ kips.

b. The allowable stress approach will be used. For the gross section,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_g = 21.6(2.41) = 52.06 \text{ kips}$$

For the net section,

$$F_t = 0.5F_u = 0.5(58) = 29 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_e = 29(1.514) = 43.91 \text{ kips}$$

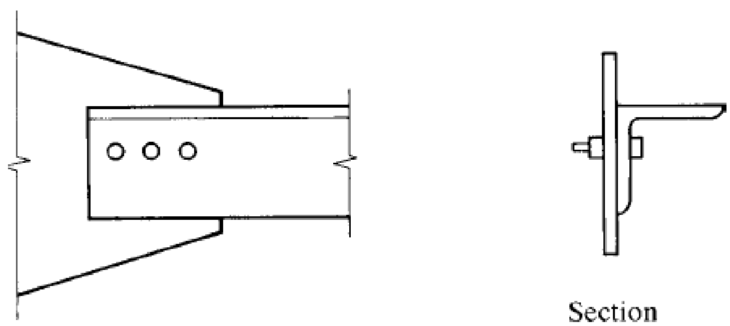
ANSWER Because 43.91 kips < 52.06 kips, fracture of the net section controls, and the allowable strength for the two angles is $2 \times 43.91 = 87.8$ kips.

3.3 Effective Area

A connection almost always weakens the member, and the measure of its influence is called the *joint efficiency*. This factor is a function of:

- The ductility of the material,
- Fastener spacing,
- Stress concentrations at holes,
- Fabrication procedure, and
- A phenomenon known as *shear lag*.

All contribute to reducing the effectiveness of the member, but shear lag is the most important. **Shear lag** occurs when some elements of the cross section are not connected, as when only one leg of an angle is bolted to a gusset plate, as shown in Figure.



The consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed. Lengthening the connected region will reduce this effect.

Research suggests that shear lag can be considered by using a reduced, or effective, net area.

Because shear lag affects both bolted and welded connections, the effective net area concept applies to both types of connections.

For bolted connections, the effective net area is

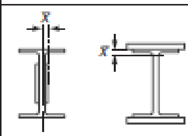
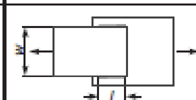
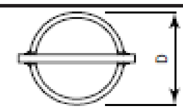
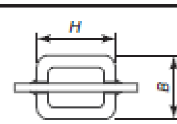
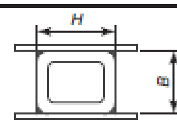
$$A_e = A_n U \qquad \textit{(AISC Equation D3-1)}$$

For welded connections, we refer to this reduced area as the effective area (rather than the effective net area), and it is given by

$$A_e = A_g U$$

where the reduction factor U is given in **AISC D3, Table D3.1**

TABLE D3.1
Shear Lag Factors for Connections to Tension Members

Case	Description of Element		Shear Lag Factor, U	Example
1	All tension members where the tension load is transmitted directly to each of cross-sectional elements by fasteners or welds. (except as in Cases 3, 4, 5 and 6)		$U = 1.0$	—
2	All tension members, except plates and HSS, where the tension load is transmitted to some but not all of the cross-sectional elements by fasteners or longitudinal welds (Alternatively, for W, M, S and HP, Case 7 may be used.)		$U = 1 - \bar{x}/l$	
3	All tension members where the tension load is transmitted by transverse welds to some but not all of the cross-sectional elements.		$U = 1.0$ and A_n = area of the directly connected elements	—
4	Plates where the tension load is transmitted by longitudinal welds only.		$l \geq 2w \ldots U = 1.0$ $2w > l \geq 1.5w \ldots U = 0.87$ $1.5w > l \geq w \ldots U = 0.75$	
5	Round HSS with a single concentric gusset plate		$l \geq 1.3D \ldots U = 1.0$ $D \leq l < 1.3D \ldots U = 1 - \bar{x}/l$ $\bar{x} = D/\pi$	
6	Rectangular HSS	with a single concentric gusset plate	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2 + 2BH}{4(B + H)}$	
		with two side gusset plates	$l \geq H \ldots U = 1 - \bar{x}/l$ $\bar{x} = \dfrac{B^2}{4(B + H)}$	
7	W, M, S or HP Shapes or Tees cut from these shapes. (If U is calculated per Case 2, the larger value is permitted to be used)	with flange connected with 3 or more fasteners per line in direction of loading	$b_f \geq 2/3d \ldots U = 0.90$ $b_f < 2/3d \ldots U = 0.85$	—
		with web connected with 4 or more fasteners in the direction of loading	$U = 0.70$	—
8	Single angles (If U is calculated per Case 2, the larger value is permitted to be used)	with 4 or more fasteners per line in direction of loading	$U = 0.80$	—
		with 2 or 3 fasteners per line in the direction of loading	$U = 0.60$	—

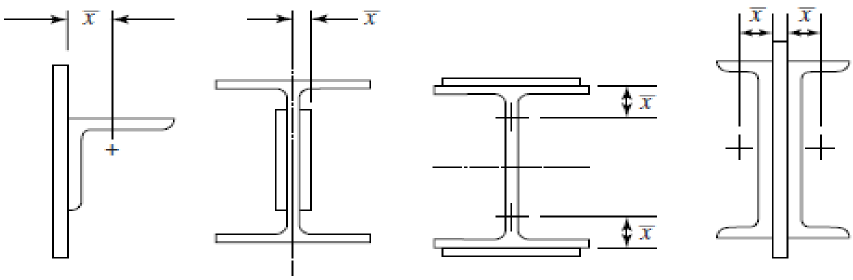
l = length of connection, in. (mm); w = plate width, in. (mm); $\bar{x}$ = connection eccentricity, in. (mm); B = overall width of rectangular HSS member, measured 90 degrees to the plane of the connection, in. (mm); H = overall height of rectangular HSS member, measured in the plane of the connection, in. (mm)

The table above gives a general equation that will cover most situations as well as alternative numerical values for specific cases.

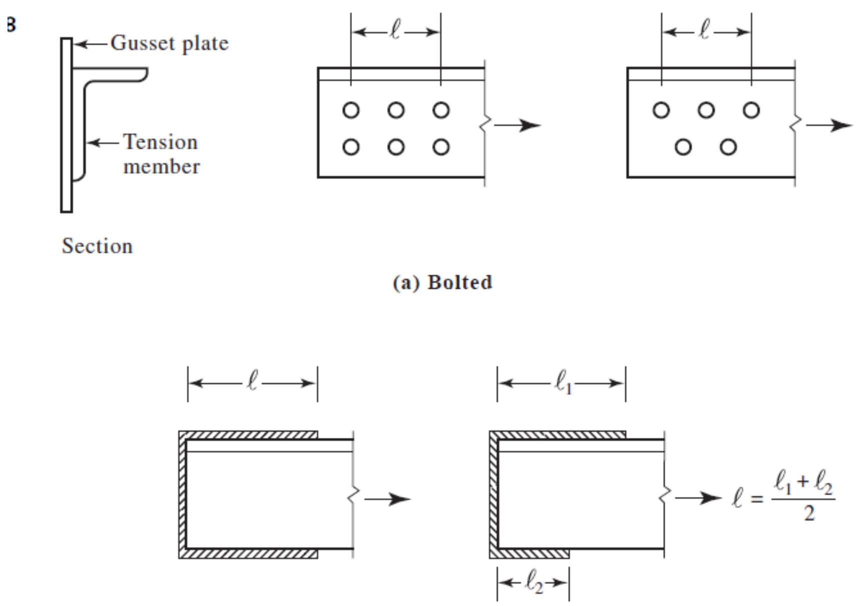
1. For any type of tension member except plates and round HSS with $\ell \geq 1.3D$

$$U = 1 - \frac{x^{-}}{\ell} \qquad (3.1)$$

where
X^{-} = distance from centroid of connected area to the plane of the connection as shown in figure below.



ℓ = length of the connection as shown in figure below.



(a) Bolted

(b) Welded

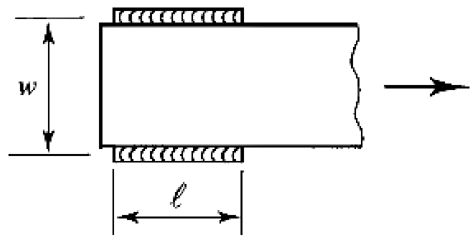
2. Plates

In general, U = 1.0 for plates, since the cross section has only one element and it is connected. There is one exception for welded plates, however. If the member is connected with longitudinal welds on each side with no transverse weld (as in Figure 3.9), the following values apply:

_ For $\ell \geq 2w\ U = 1.0$

_ For $1.5w \leq \ell < 2w, U = 0.87$

_ For $w \leq \ell < 1.5w, U = 0.75$



EXAMPLE 3.4

Determine the effective net area for the tension member shown in Figure 3.12.

SOLUTION

$$A_n = A_g - A_{\text{holes}}$$
$$= 5.77 - \frac{1}{2}\left(\frac{5}{8} + \frac{1}{8}\right)(2) = 5.02 \text{ in.}^2$$

Only one element (one leg) of the cross section is connected, so the net area must be reduced. From the properties tables in Part 1 of the *Manual,* the distance from the centroid to the outside face of the leg of an L6 × 6 × ½ is

$$\bar{x} = 1.67 \text{ in.}$$

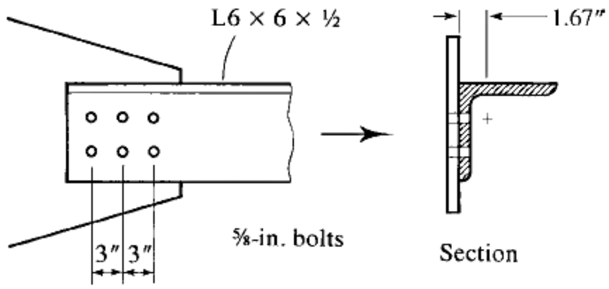
Table 1–7 (continued)
Angles
Properties

L8-L6

Shape	Axis Y-Y						Axis Z-Z			Tan α	Q_s
	I	S	r	$\bar{x}$	Z	x_p	I	S	r		$F_y = 36$ ksi
	in.^{4}	in.^{3}	in.	in.	in.^{3}	in.	in.^{4}	in.^{3}	in.		
L8×8×1 1/8	98.1	17.5	2.41	2.40	31.6	1.05	40.9	7.23	1.56	1.00	1.00
×1	89.1	15.8	2.43	2.36	28.5	0.943	36.8	6.51	1.56	1.00	1.00
×7/8	79.7	14.0	2.45	2.31	25.3	0.832	32.7	5.78	1.57	1.00	1.00
×3/4	69.9	12.2	2.46	2.26	22.0	0.720	28.5	5.04	1.57	1.00	1.00
×5/8	59.6	10.3	2.48	2.21	18.6	0.606	24.2	4.27	1.58	1.00	0.997
×9/16	54.2	9.33	2.49	2.19	16.8	0.548	22.0	3.88	1.58	1.00	0.959
×1/2	48.8	8.36	2.49	2.17	15.1	0.490	19.7	3.49	1.59	1.00	0.912
L8×6×1	38.8	8.92	1.72	1.65	16.2	0.816	21.3	4.84	1.28	0.542	1.00
×7/8	34.9	7.94	1.74	1.60	14.4	0.721	18.9	4.31	1.28	0.546	1.00
×3/4	30.8	6.92	1.75	1.56	12.5	0.624	16.5	3.78	1.29	0.550	1.00
×5/8	26.4	5.88	1.77	1.51	10.5	0.526	14.1	3.22	1.29	0.554	0.997
×9/16	24.1	5.34	1.78	1.49	9.52	0.476	12.8	2.94	1.30	0.556	0.959
×1/2	21.7	4.79	1.79	1.46	8.52	0.425	11.5	2.64	1.30	0.557	0.912
×7/16	19.3	4.23	1.80	1.44	7.50	0.374	10.2	2.35	1.31	0.559	0.850
L8×4×1	11.6	3.94	1.03	1.04	7.73	0.691	7.87	2.15	0.844	0.247	1.00
×7/8	10.5	3.51	1.04	0.997	6.77	0.612	7.01	1.93	0.846	0.252	1.00
×3/4	9.37	3.07	1.05	0.949	5.82	0.531	6.13	1.70	0.850	0.257	1.00
×5/8	8.11	2.62	1.06	0.902	4.86	0.448	5.24	1.47	0.856	0.262	0.997
×9/16	7.44	2.38	1.07	0.878	4.39	0.405	4.79	1.34	0.859	0.264	0.959
×1/2	6.75	2.15	1.08	0.854	3.91	0.363	4.32	1.22	0.863	0.266	0.912
×7/16	6.03	1.90	1.09	0.829	3.42	0.320	3.84	1.09	0.867	0.268	0.850
L7×4×3/4	9.00	3.01	1.08	1.00	5.60	0.550	5.64	1.71	0.855	0.324	1.00
×5/8	7.79	2.56	1.10	0.958	4.69	0.464	4.80	1.47	0.860	0.329	1.00
×1/2	6.48	2.10	1.11	0.910	3.77	0.376	3.95	1.21	0.866	0.334	0.965
×7/16	5.79	1.86	1.12	0.886	3.31	0.331	3.50	1.08	0.869	0.337	0.912
×3/8	5.06	1.61	1.12	0.861	2.84	0.286	3.05	0.942	0.873	0.339	0.840
L6×6×1	35.4	8.55	1.79	1.86	15.4	0.918	15.0	3.53	1.17	1.00	1.00
×7/8	31.9	7.61	1.81	1.81	13.7	0.813	13.3	3.13	1.17	1.00	1.00
×3/4	28.1	6.64	1.82	1.77	11.9	0.705	11.6	2.73	1.17	1.00	1.00
×5/8	24.1	5.64	1.84	1.72	10.1	0.594	9.83	2.32	1.17	1.00	1.00
×9/16	22.0	5.12	1.85	1.70	9.17	0.538	8.94	2.11	1.18	1.00	1.00
×1/2	19.9	4.59	1.86	1.67	8.22	0.481	8.04	1.89	1.18	1.00	1.00
×7/16	17.6	4.06	1.86	1.65	7.25	0.423	7.11	1.68	1.18	1.00	0.973
×3/8	15.4	3.51	1.87	1.62	6.26	0.365	6.17	1.45	1.19	1.00	0.912
×5/16	13.0	2.95	1.88	1.60	5.26	0.306	5.20	1.23	1.19	1.00	0.826

Note: For compactness criteria, refer to the end of Table 1–7.

FIGURE 3.12



The length of the connection is

$$\ell = 3 + 3 = 6 \text{ in.}$$

$$\therefore U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{6}\right) = 0.7217$$

$$A_e = A_n U = 5.02(0.7217) = 3.623 \text{ in.}^2$$

EXAMPLE 3.5

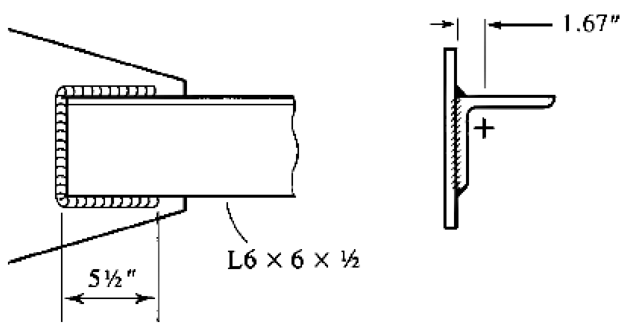
If the tension member of Example 3.4 is welded as shown in Figure 3.13, determine the effective area.

SOLUTION As in Example 3.4, only part of the cross section is connected and a reduced effective area must be used.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{5.5}\right) = 0.6964$$

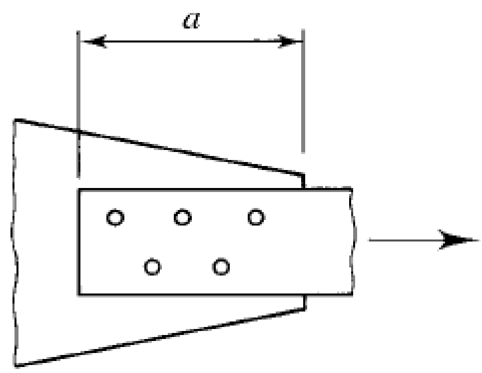
ANSWER $A_e = A_g U = 5.77(0.6964) = 4.02 \text{ in.}^2$

FIGURE 3.13



3.4 STAGGERED FASTENERS

If a tension member connection is made with bolts, the net area will be maximized if the fasteners are placed in a single line. Sometimes space limitations, such as a limit on dimension a in Figure, necessitate using more than one line. If so, the reduction in cross-sectional area is minimized if the fasteners are arranged in a staggered pattern, as shown



Cochrane (1922) proposed that when deducting the area corresponding to a staggered hole, use a reduced diameter, given by:

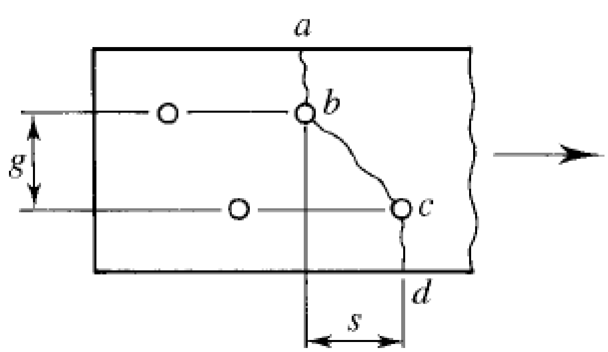
$$\boldsymbol{d'} = \boldsymbol{d} - \frac{\boldsymbol{s^2}}{\boldsymbol{4g}} \qquad \textbf{(3.2)}$$

where

d is the hole diameter,

s is the stagger, or pitch, of the bolts (spacing in the direction of the load), and

g is the gage (transverse spacing).



If the net area is treated as the product of a thickness times a net width, and the diameter from Equation is used for all holes (since *d′* = *d* when the stagger s = 0), the net width in a failure line consisting of both staggered and unstaggered holes is:

$$w_n = w_g - \sum d'$$
$$= w_g - \sum (d - S^2/4g)$$
$$= w_g - \sum d + \sum S^2/4g$$

Where;

w_n is the net width and

w_g is the gross width.

$\sum d$ is the sum of all hole diameters, and

$\sum S^2/4g$ is the sum of $S^2/4g$ for all inclined lines in the failure pattern.

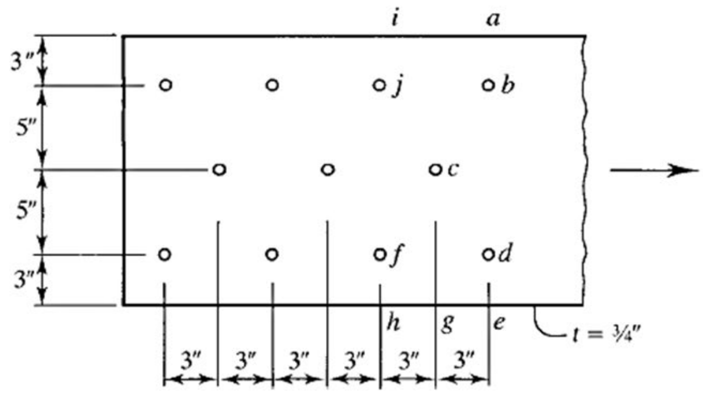
EXAMPLE 3.6

Compute the smallest net area for the plate shown in Figure 3.15. The holes are for 1-inch-diameter bolts.

SOLUTION The effective hole diameter is $1 + \frac{1}{8} = 1\frac{1}{8}$ in. For line *abde,*

$$w_n = 16 - 2(1.125) = 13.75 \text{ in.}$$

FIGURE 3.15



For line *abcde,*

$$w_n = 16 - 3(1.125) + \frac{2(3)^2}{4(5)} = 13.52 \text{ in.}$$

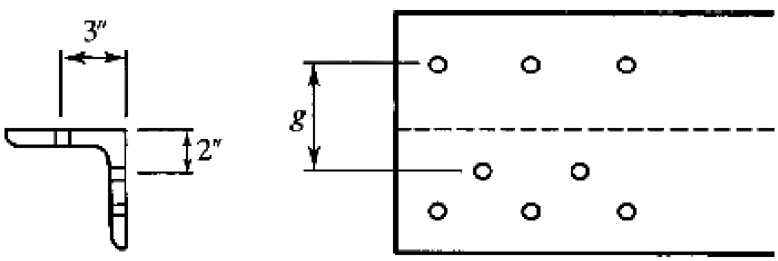
The second condition will give the smallest net area:

ANSWER $A_n = tw_n = 0.75(13.52) = 10.1 \text{ in.}^2$

Specification. If the shape is an angle, it can be visualized as a plate formed by "unfolding" the legs to more clearly identify the pitch and gage distances. AISC B4.3b specifies that any gage line crossing the heel of the angle be reduced by an amount that equals the angle thickness. Thus, the distance g in Figure 3.16, to be used in the $s^2/4g$ term, would be $3 + 2 - \frac{1}{2} = 4\frac{1}{2}$ inches.

ter 3 Tension Members

3.16



L5 × 5 × ½

EXAMPLE 3.7

An angle with staggered fasteners in each leg is shown in Figure 3.17. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

a. Determine the design strength for LRFD.
b. Determine the allowable strength for ASD.

SOLUTION From the dimensions and properties tables, the gross area is A_g = 6.80 in.^{2}. The effective hole diameter is ⅞ + ⅛ = 1 in.

For line *abdf*, the net area is

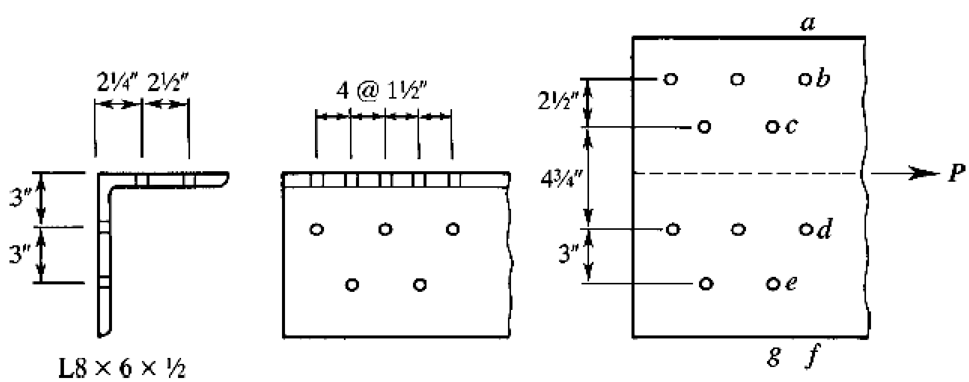
$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$
$$= 6.8\,0 - 0.5(1.0) \times 2 = 5.8\,0 \text{in.}^2$$

For line *abceg*,

$$A_n = 6.8\,0 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5(1.0) = 5.413 \text{ in.}^2$$

Because ⅒ of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only ⁹⁄₁₀ of the load. Therefore, the net area

FIGURE 3.17



of 5.413 in.^{2} should be multiplied by $\frac{10}{9}$ to obtain a net area that can be compared with those lines that resist the full load. Use $A_n = 5.413(\frac{10}{9}) = 6.014$ in.^{2} For line *abcdeg*,

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(4.75)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(3)}\right]$$

$$= 5.065 \text{ in.}^2$$

The last case controls; use

$$A_n = 5.065 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.065 \text{ in.}^2$$

The nominal strength based on fracture is

$$P_n = F_u A_e = 58(5.065) = 293.8 \text{ kips}$$

The nominal strength based on yielding is

$$P_n = F_y A_g = 36(6.80) = 244.8 \text{ kips}$$

a. The design strength based on fracture is

$$\phi_t P_n = 0.75(293.8) = 220 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(244.8) = 220 \text{ kips}$$

ANSWER Design strength = 220 kips.

b. For the limit state of fracture, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable strength is

$$F_t A_e = 29.0(5.065) = 147 \text{ kips}$$

For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

$$F_t A_g = 21.6(6.80) = 147 \text{ kips}$$

ANSWER Allowable strength = 147 kips.

EXAMPLE 3.8

Determine the smallest net area for the American Standard Channel shown in Figure 3.18. The holes are for ⅝-inch-diameter bolts.

SOLUTION

$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$

$$d = \text{bolt diameter} + \frac{1}{8} = \frac{5}{8} + \frac{1}{8} = \frac{3}{4} \text{ in.}$$

Line *abe:*

$$A_n = A_g - t_w d = 3.82 - 0.437\left(\frac{3}{4}\right) = 3.49 \text{ in.}^2$$

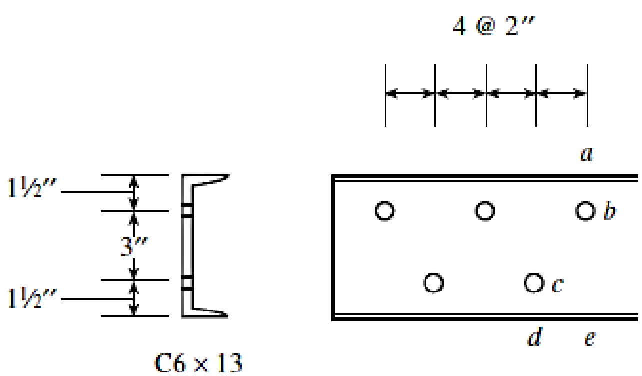
Line *abcd:*

$$A_n = A_g - t_w(d \text{ for hole at } b) - t_w(d' \text{ for hole at } c)$$

$$= 3.82 - 0.437\left(\frac{3}{4}\right) - 0.437\left[\frac{3}{4} - \frac{(2)^2}{4(3)}\right] = 3.31 \text{ in.}^2$$

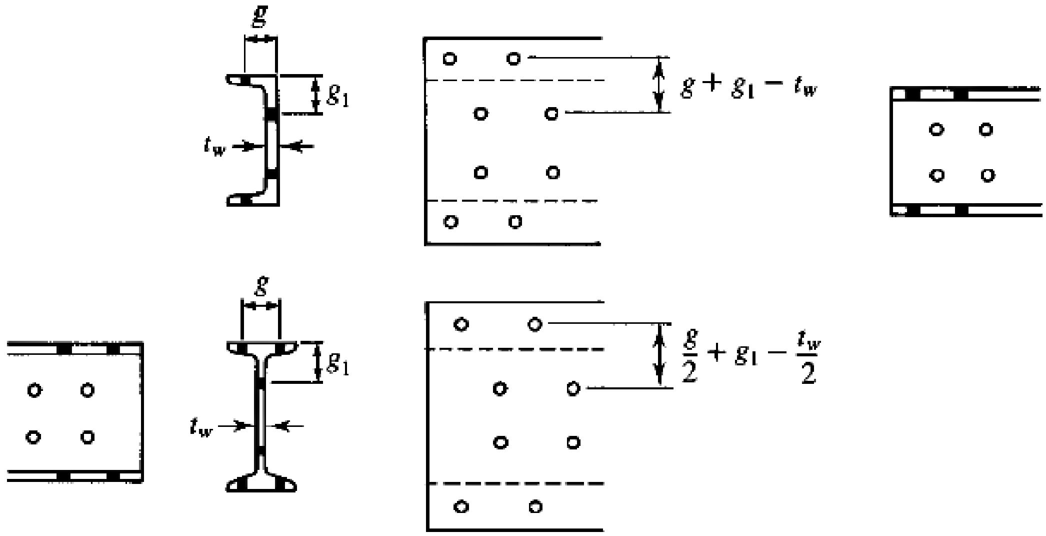
ANSWER Smallest net area = 3.31 in.^{2}

FIGURE 3.18



When staggered holes are present in shapes other than angles, and the holes are in different elements of the cross section, the shape can still be visualized as a plate, even if it is an I-shape. The AISC Specification furnishes no guidance for gage lines crossing a "fold" when the different elements have different thicknesses. A method for handling this case is illustrated in Figure 3.19. In Example 3.8, all of the holes are in one element of the cross section, so this difficulty does not arise. Example 3.9 illustrates the case of staggered holes in different elements of an S-shape.

3.19



EXAMPLE 3.9

Find the available strength of the S-shape shown in Figure 3.20. The holes are for ¾-inch-diameter bolts. Use A36 steel.

SOLUTION Compute the net area:

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$\text{Effective hole diameter} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

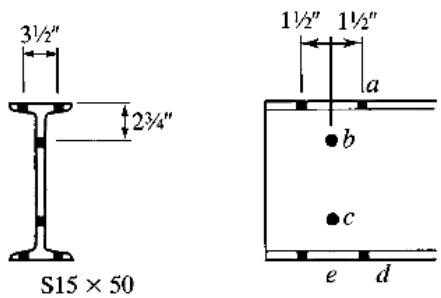
For line *ad*,

$$A_n = 14.7 - 4\left(\frac{7}{8}\right)(0.622) = 12.52 \text{ in.}^2$$

For line *abcd*, the gage distance for use in the $s^2/4g$ term is

$$\frac{g}{2} + g_1 - \frac{t_w}{2} = \frac{3.5}{2} + 2.75 - \frac{0.550}{2} = 4.225 \text{ in.}$$

FIGURE 3.20



Starting at a and treating the holes at b and d as the staggered holes gives

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$= 14.7 - 2(0.622)\left(\frac{7}{8}\right) - (0.550)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right]$$

$$-(0.550)\left(\frac{7}{8}\right) - 2(0.622)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right] = 11.73 \text{ in.}^2$$

Line $abcd$ controls. As all elements of the cross section are connected,

$$A_e = A_n = 11.73 \text{ in.}^2$$

For the net section, the nominal strength is

$$P_n = F_u A_e = 58(11.73) = 680.3 \text{ kips}$$

For the gross section,

$$P_n = F_y A_g = 36(14.7) = 529.2 \text{ kips}$$

LRFD SOLUTION

The design strength based on fracture is

$$\phi_t P_n = 0.75(680.3) = 510 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(529.2) = 476 \text{ kips}$$

Yielding of the gross section controls.

ANSWER

Design strength = 476 kips.

ASD SOLUTION

The allowable stress based on fracture is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_e = 29.0(11.73) = 340$ kips.

The allowable stress based on yielding is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_g = 21.6(14.7) = 318$ kips.

Yielding of the gross section controls.

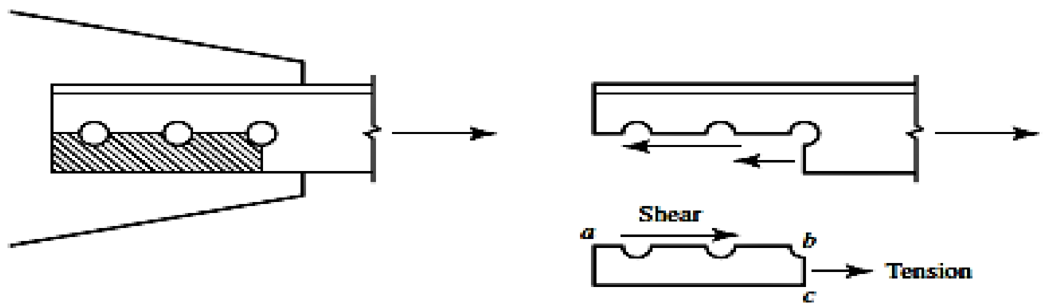
ANSWER

Allowable strength = 318 kips.

3.5 BLOCK SHEAR

For certain connection configurations, a segment or "block" of material at the end of the member can tear out. For example, the connection of the single-angle tension member shown in Figure 3.21 is susceptible to this phenomenon, called block shear.

FIGURE 3.21

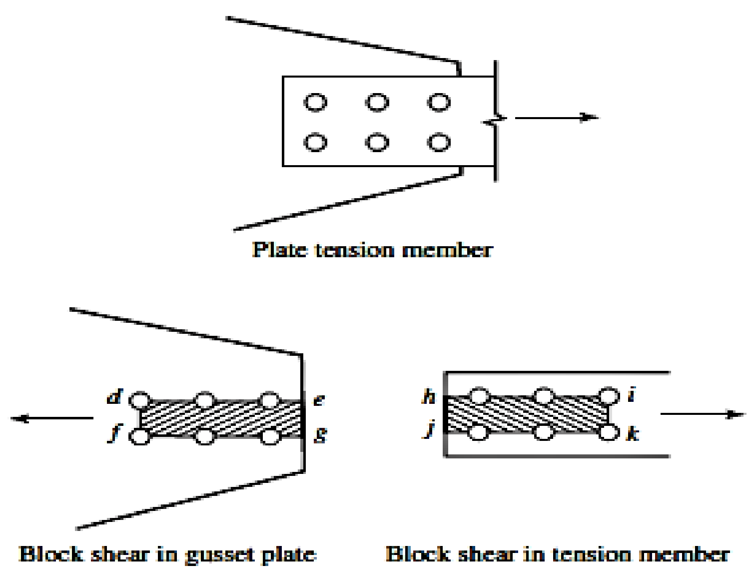


For the case illustrated, the shaded block would tend to fail by shear along the longitudinal section *ab* and by tension on the transverse section *bc*.

For certain arrangements of bolts, block shear can also occur in gusset plates. Figure 3.22 shows a plate tension member connected to a gusset plate. In this connection, block shear could occur in both the gusset plate and the tension member. For the gusset plate, tension failure would be along the transverse section *df*, and shear failure would occur on two longitudinal surfaces, *de* and *fg*. Block shear failure in the plate tension member would be tension on *ik* and shear on both *hi* and *jk*. This topic is not covered explicitly in AISC Chapter D ("Design of Members for Tension"), but the introductory user note directs you to Chapter J ("Design of Connections"), Section J4.3, "Block Shear Strength."

The model used in the AISC Specification assumes that failure occurs by rupture (fracture) on the shear area and rupture on the tension area. Both surfaces contribute to the total strength, and the resistance to block shear will be the sum of the strengths of the two surfaces. The shear rupture stress is taken as 60% of the tensile ultimate

FIGURE 3.22



stress, so the nominal strength in shear is $0.6F_uA_{nv}$ and the nominal strength in tension is F_uA_{nt},

where

A_{nv} = net area along the shear surface or surfaces
A_{nt} = net area along the tension surface

This gives a nominal strength of

$$R_n = 0.6F_uA_{nv} + F_uA_{nt} \qquad (3.3)$$

The AISC Specification uses Equation 3.3 for angles and gusset plates, but for certain types of coped beam connections (to be covered in Chapter 5), the second term is reduced to account for nonuniform tensile stress. The tensile stress is nonuniform when some rotation of the block is required for failure to occur. For these cases,

$$R_n = 0.6F_uA_{nv} + 0.5F_uA_{nt} \qquad (3.4)$$

The AISC Specification limits the $0.6F_uA_{nv}$ term to $0.6F_yA_{gv}$, where

$0.6F_y$ = shear yield stress

A_{gv} = gross area along the shear surface or surfaces

and gives one equation to cover all cases as follows:

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.6F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(AISC Equation J4-5)}$$

where U_{bs} = 1.0 when the tension stress is uniform (angles, gusset plates, and most coped beams) and U_{bs} = 0.5 when the tension stress is nonuniform. A nonuniform case is illustrated in the Commentary to the Specification.

For LRFD, the resistance factor ϕ is 0.75, and for ASD, the safety factor Ω is 2.00. Recall that these are the factors used for the fracture—or rupture—limit state, and block shear is a rupture limit state.

Although AISC Equation J4-5 is expressed in terms of bolted connections, block shear can also occur in welded connections, especially in gusset plates.

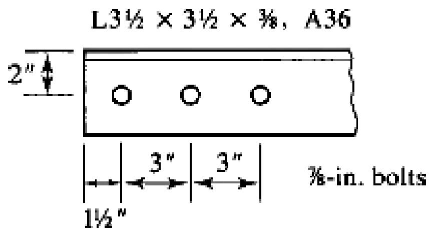
EXAMPLE 3.10

Compute the block shear strength of the tension member shown in Figure 3.23. The holes are for ⅞-inch-diameter bolts, and A36 steel is used.

a. Use LRFD.

b. Use ASD.

FIGURE 3.23



SOLUTION

The shear areas are

$$A_{gv} = \frac{3}{8}(7.5) = 2.813 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = \frac{3}{8}\left[7.5 - 2.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 1.875 \text{ in.}^2$$

The tension area is

$$A_{nt} = \frac{3}{8}\left[1.5 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.3750 \text{ in.}^2$$

(The factor of 0.5 is used because there is one-half of a hole diameter in the tension section.)

Since the block shear will occur in an angle, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$
$$= 0.6(58)(1.875) + 1.0(58)(0.3750) = 87.00 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.813) + 1.0(58)(0.3750) = 82.51 \text{ kips}$$

The nominal block shear strength is therefore 82.51 kips.

ANSWER

a. The design strength for LRFD is $\phi R_n = 0.75(82.51) = 61.9$ kips.

b. The allowable strength for ASD is $\dfrac{R_n}{\Omega} = \dfrac{82.51}{2.00} = 41.3$ kips.

3.6 DESIGN OF TENSION MEMBERS

The design of a tension member involves:

1. finding a member with adequate gross and net areas. If the member has a bolted connection, the selection of a suitable cross section requires an accounting for the area lost because of holes.
2. A secondary consideration in the design of tension members is slenderness. If a structural member has a small cross section in relation to its length, it is said to be slender. For this reason, the user note in AISC D1 suggests a maximum slenderness ratio of 300.

The central problem of all member design, including tension member design, is to find a cross section for which the required strength does not exceed the available strength. For tension members designed by LRFD, the requirement is

$$P_u \leq \phi_t P_n \quad \text{or} \quad \phi_t P_n \geq P_u$$

where P_u is the sum of the factored loads. To prevent yielding,

$$0.90F_y A_g \geq P_u \quad \text{or} \quad A_g \geq \frac{P_u}{0.90F_y}$$

To avoid fracture,

$$0.75F_u A_e \geq P_u \quad \text{or} \quad A_e \geq \frac{P_u}{0.75F_u}$$

For allowable strength design, if we use the allowable *stress* form, the requirement corresponding to yielding is

$$P_a \leq F_t A_g$$

and the required gross area is

$$A_g \geq \frac{P_a}{F_t} \quad \text{or} \quad A_g \geq \frac{P_a}{0.6F_y}$$

For the limit state of fracture, the required effective area is

$$A_e \geq \frac{P_a}{F_t} \quad \text{or} \quad A_e \geq \frac{P_a}{0.5F_u}$$

The slenderness ratio limitation will be satisfied if

$$r \geq \frac{L}{300}$$

where r is the minimum radius of gyration of the cross section and L is the member length.

EXAMPLE 3.11

A tension member with a length of 5 feet 9 inches must resist a service dead load of 18 kips and a service live load of 52 kips. Select a member with a rectangular cross section. Use A36 steel and assume a connection with one line of ⅞-inch-diameter bolts.

LRFD SOLUTION

$$P_u = 1.2D + 1.6L = 1.2(18) + 1.6(52) = 104.8 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{P_u}{0.90F_y} = \frac{104.8}{0.90(36)} = 3.235 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{P_u}{0.75F_u} = \frac{104.8}{0.75(58)} = 2.409 \text{ in.}^2$$

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.235}{1} = 3.235 \text{ in.}$$

Try a 1 × 3½ cross section.

$$A_e = A_n = A_g - A_{hole}$$

$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.409 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$

$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$

$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

ANSWER Use a PL 1 × 3½.

ASD SOLUTION

$$P_a = D + L = 18 + 52 = 70.0 \text{ kips}$$

For yielding, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi, and

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{70}{21.6} = 3.24 \text{ in.}^2$$

For fracture, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi, and

$$\text{Required } A_e = \frac{P_a}{F_t} = \frac{70}{29.0} = 2.414 \text{ in.}^2$$

(The rest of the design *procedure* is the same as for LRFD. The numerical results may be different)

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.241}{1} = 3.241 \text{ in.}$$

Try a 1 × 3 ½ cross section.

$$A_e = A_n = A_g - A_{\text{hole}}$$
$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.414 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{\text{min}} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$
$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

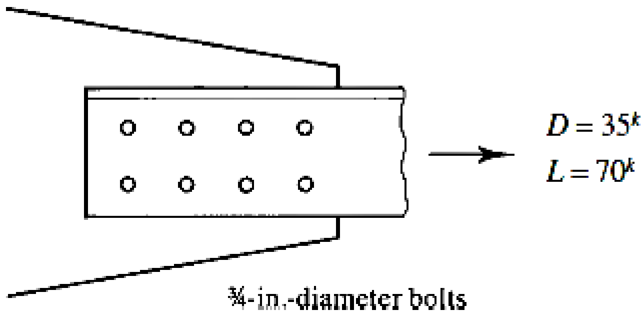
$$r_{\text{min}} = \sqrt{\frac{I_{\text{min}}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$
$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

E R Use a PL 1 × 3½.

EXAMPLE 3.12

Select an unequal-leg angle tension member 15 feet long to resist a service dead load of 35 kips and a service live load of 70 kips. Use A36 steel. The connection is shown in Figure 3.25.

FIGURE 3.25



LRFD SOLUTION

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35) + 1.6(70) = 154 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{154}{0.90(36)} = 4.75 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{154}{0.75(58)} = 3.54 \text{ in.}^2$$

The radius of gyration should be at least

$$\frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

To find the lightest shape that satisfies these criteria, we search the dimensions and properties table for the unequal-leg angle that has the smallest acceptable gross area and then check the effective net area. The radius of gyration can be checked by inspection. There are two lines of bolts, so the connected leg must be at least 5 inches long (see the usual gages for angles in Figure 3.24). Starting at either end of the table, we find that the shape with the smallest area that is at least equal to 4.75 in.^{2} is an L6 × 4 × ½ with an area of 4.75 in.^{2} and a minimum radius of gyration of 0.864 in.

Try L6 × 4 × ½.

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

Because the length of the connection is not known, Equation 3.1 cannot be used to compute the shear lag factor U. Since there are four bolts in the direction of the load, we will use the alternative value of $U = 0.80$.

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

Try the next larger shape from the dimensions and properties tables.

Try L5 × 3½ × ⅝ ($A_g = 4.93$ in.^{2} and $r_{min} = 0.746$ in.)

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$

$$A_e = A_n U = 3.836(0.80) = 3.07 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

(Note that this shape has slightly more gross area than that produced by the previous trial shape, but because of the greater leg thickness, slightly more area is deducted for the holes.) Passing over the next few heavier shapes,

Try L8 × 4 × ½ ($A_g = 5.80$ in.^{2} and $r_{min} = 0.863$ in.)

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.54 \text{ in.}^2 \qquad \text{(OK)}$$

Prob. 3.13 A C8 × 11.5 is connected to a gusset plate with 7/8-inch-diameter bolts as shown in Figure P3.2-7. The steel is A572 Grade 50. If the member is subjected to dead load and live load only, what is the total service load capacity if the live-to-dead load ratio is 3? Assume that *Ae* = 0.85*An*. Use $f_y = 50\ psi$ and $f_u = 65\ psi$ Use LRFD.

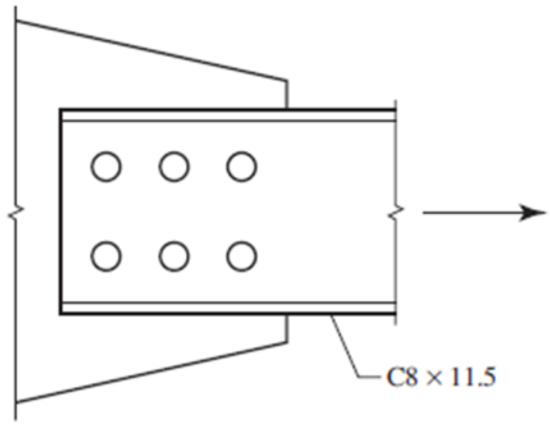


FIGURE P3.2-7

$A_g = 3.37\ in^2$, Effective hole diameter=

$A_n = 3.37 -$ $= 2.93\ in^2$

$A_e =$ $= 2.491\ in^2$

Gross: $P_u =$ $= 151.7\ kips$

Net: $P_u =$ $= 121.4\ kips$

$P_u = 1.2\ DL + 1.6\ LL = 121.4\ kips$

$DL = 20.23\ kips$

$P = DL + LL =$ $= 80.9\ kips$

The tension member shown in Figure P3.3-6 is a C12 × 20.7 of A572 Grade 50 steel. Will it safely support a service dead load of 60 kips and a service live load of 125 kips? Use Equation 3.1 for U.

a. Use LRFD.
b. Use ASD.

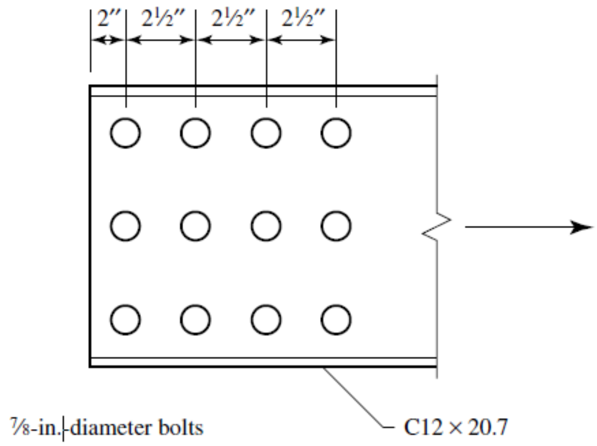


FIGURE P3.3-6

Gross section: $P_n = F_y A_g = 50(6.08) = 304.0$ kips

Net section:

$$A_n = A_g - \Sigma t_w d_h = 6.08 - 3(0.282)\left(\frac{7}{8} + \frac{1}{8}\right) = 5.234 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.698}{3(2.5)} = 0.9069$$

$$A_e = A_n U = 5.234(0.9069) = 4.747 \text{ in.}^2$$

$$P_n = F_u A_e = 65(4.747) = 308.6 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(304.0) = 273.6 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(308.6) = 231.5 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 232$ kips

$$P_u = 1.2D + 1.6L = 1.2(60) + 1.6(125) = 272 \text{ kips} > 232 \text{ kips} \quad \text{(N.G.)}$$

The member is not adequate.

An MC 9 × 23.9 is connected with ¾-inch-diameter bolts as shown in Figure P3.4-3. A572 Grade 50 steel is used.

a. Determine the design strength.

b. Determine the allowable strength.

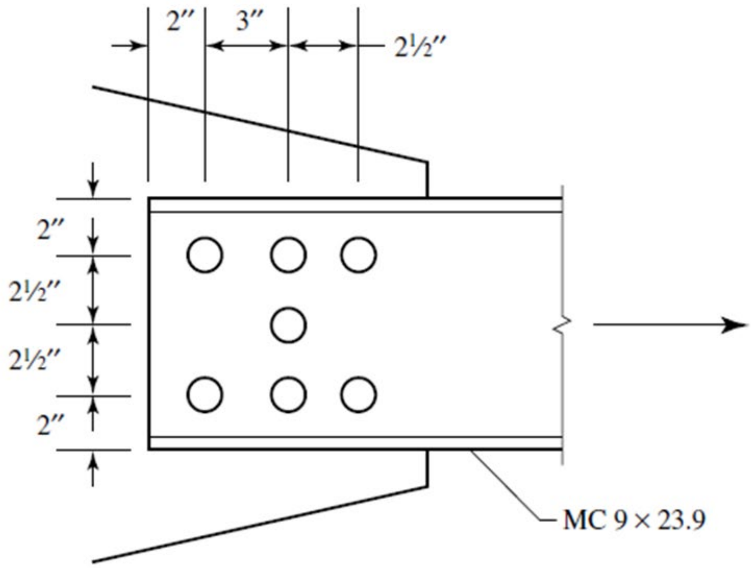


FIGURE P3.4-3

Gross section: $P_n = F_y A_g = 50(7.02) = 351.0$ kips

Net section: Hole diameter $= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_n = 7.02 - 2(7/8)(0.400) = 6.320 \text{ in.}^2$$

or $$[7.02 - 3(7/8)(0.400)] \times \frac{7}{5} = 8.358 \text{ in}^2$$

or $$7.02 - 0.4(7/8) - 0.400\left[\frac{7}{8} - \frac{(2.5)^2}{4(2.5)}\right] \times 2 = 6.47 \text{ in.}^2$$

or $$\left(7.02 - 2(0.4)(7/8) - 0.400\left[\frac{7}{8} - \frac{(2.5)^2}{4(2.5)}\right]\right) \times \frac{7}{6} = 7.256 \text{ in.}^2$$

Use $A_n = 6.320$ in.^{2}

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.981}{5.5} = 0.8216$$

The effective net area is

$$A_e = A_n U = 6.320(0.8216) = 5.193 \text{ in.}^2$$

$$P_n = F_u A_e = 65(5.193) = 337.6 \text{ kips}$$

a. Gross: $\phi_t P_n = 0.90(351.0) = 316$ kips

Net: $\phi_t P_n = 0.75(337.6) = 253$ kips

Net section controls: $\underline{\phi_t P_n = 253 \text{ kips}}$

A C7 × 9.8 tension member is connected to a ⅜-in.-thick gusset plate as shown in Figure P3.5-5. Both the member and the gusset plate are A36 steel.

a. Compute the available block shear strength of the tension member for both LRFD and ASD.
b. Compute the available block shear strength of the gusset plate for both LRFD and ASD.

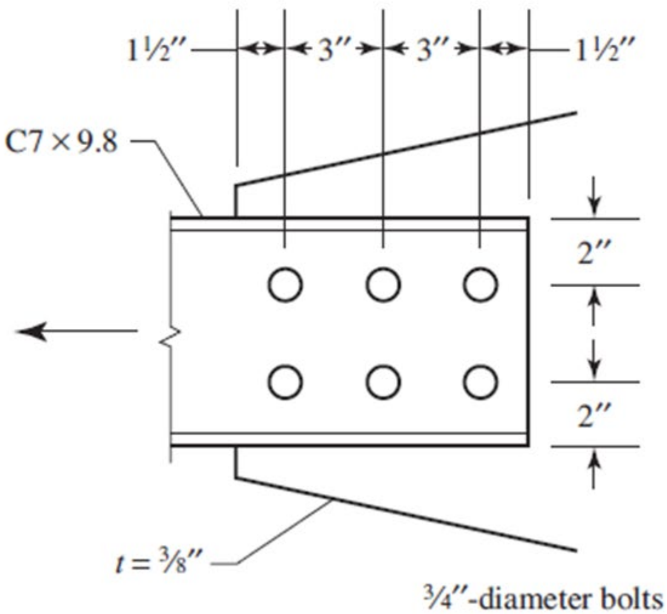
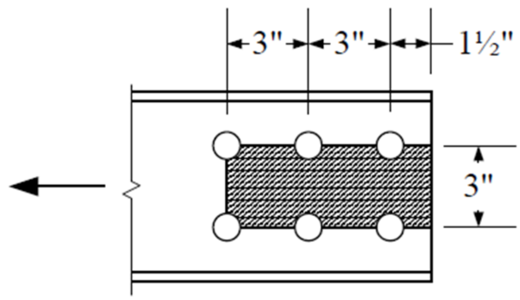


FIGURE P3.5-5



The shear areas are

$$A_{gv} = 0.210(7.5)(2) = 3.15 \text{ in.}^2$$

and since there are 2.5 hole diameters,

$$A_{nv} = 0.210[7.5 - 2.5(7/8)](2) = 2.231 \text{ in.}^2$$

The tension areas are

$$A_{gt} = 0.210(3) = 0.63 \text{ in.}^2, \qquad A_{nt} = 0.210[3 - 1.0(7/8)] = 0.4463 \text{ in.}^2$$

$$F_y = 36 \text{ ksi}, \; F_u = 58 \text{ ksi}$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(2.231) + 1.0(58)(0.4463) = 103.5 \text{ kips}$$

Check upper limit:

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(3.15) + 1.0(58)(0.4463)$$

$$= 93.92 \text{ kips} < 103.5 \text{ kips}$$

LRFD: $\phi R_n = 0.75(93.92) = \underline{70.4 \text{ kips}}$

Use A36 steel and select a double-angle tension member to resist a service dead load of 20 kips and a service live load of 60 kips. Assume that the member will be connected to a ⅜-inch-thick gusset plate with a single line of five ⅞-inch diameter bolts. The member is 15 feet long.

a. Use LRFD.
b. Use ASD.

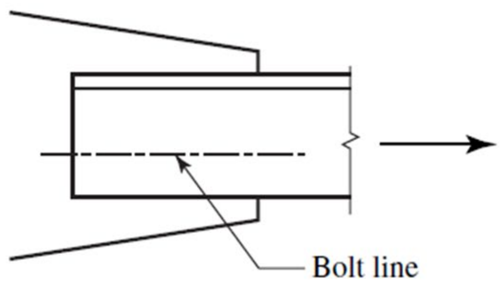


FIGURE P3.6-2

(a) $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(60) = 120.0$ kips

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{120}{0.9(36)} = 3.70 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{120}{0.75(58)} = 2.76 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{15 \times 12}{300} = 0.6 \text{ in.}$$

Try $2L5 \times 3\frac{1}{2} \times \frac{1}{4}$, long legs back-to-back:

$A_g = 2.07 \times 2 = 4.14 \text{ in.}^2 > 3.70 \text{ in.}^2$ (OK)

$r_x = 0.853$ in., $r_y = 1.43$ in., $\therefore r_{\min} = 0.853$ in. > 0.6 in. (OK)

$A_n = 4.14 - 1.0(1/4) = 3.89 \text{ in.}^2$

From Case 8 in AISC Table D3.1, use $U = 0.80$.

$A_e = A_n U = 3.89(0.80) = 3.11 \text{ in.}^2 > 2.76 \text{ in.}^2$ (OK)

$\underline{2L5 \times 3\frac{1}{2} \times \frac{1}{4} \text{ LLBB}}$

Chapter 4: Compression Members

4.1 INTRODUCTION

Compression members are structural elements that are subjected only to axial compressive forces; that is, the loads are applied along a longitudinal axis through the centroid of the member cross section, and the stress can be taken as $f = P/A$, where f is considered to be uniform over the entire cross section. This ideal state is never achieved in reality, however, because some eccentricity of the load is inevitable. Bending will result, but it usually can be regarded as secondary. As we shall see, the AISC Specification equations for compression member strength account for this accidental eccentricity.

The most common type of compression member occurring in buildings and bridges is the *column,* a vertical member whose primary function is to support vertical loads. In many instances, these members are also subjected to bending, and in these cases, the member is a *beam–column.* We cover this topic in Chapter 6. Compression members are also used in trusses and as components of bracing systems. Smaller compression members not classified as columns are sometimes referred to as *struts.*

In many small structures, column axial forces can be easily computed from the reactions of the beams that they support or computed directly from floor or roof loads. This is possible if the member connections do not transfer moment; in other words, if the column is not part of a rigid frame. For columns in rigid frames, there are calculable bending moments as well as axial forces, and a frame analysis is necessary. The AISC Specification provides for three methods of analysis to obtain the axial forces and bending moments in members of a rigid frame:

1. Direct analysis method
2. Effective length method
3. First-order analysis method

4.2 COLUMN THEORY

Consider the long, slender compression member shown in Figure 4.1a. If the axial load P is slowly applied, it will ultimately become large enough to cause the member to become unstable and assume the shape indicated by the dashed line. The member is said to have buckled, and the corresponding load is called the *critical buckling load.* If the member is stockier, as shown in Figure 4.1b, a larger load will be required to bring the member to the point of instability. For extremely stocky members, failure may occur by compressive yielding rather than buckling. Prior to failure, the compressive stress P/A will be uniform over the cross section at any point along the length, whether the failure is by yielding or by buckling. The load at which buckling occurs is a function of slenderness, and for very slender members this load could be quite small.

If the member is so slender (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit—that is, the member is still elastic—the critical buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.1}$$

where E is the modulus of elasticity of the material, I is the moment of inertia of the cross-sectional area with respect to the minor principal axis, and L is the length of the member between points of support. For Equation 4.1 to be valid, the member must be elastic, and its ends must be free to rotate but not translate laterally. This end condition is satisfied by hinges or pins, as shown in Figure 4.2. This remarkable

FIGURE 4.1

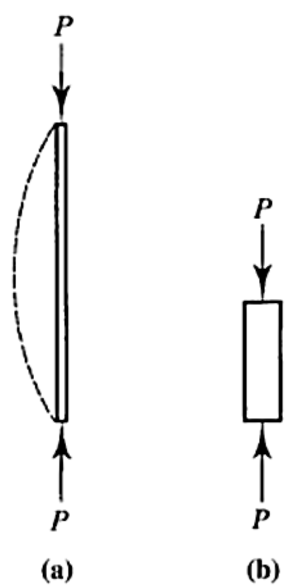
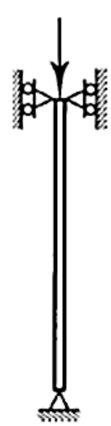


FIGURE 4.2



relationship was first formulated by Swiss mathematician Leonhard Euler and published in 1759. The critical load is sometimes referred to as the *Euler load* or the *Euler buckling load*. The validity of Equation 4.1 has been demonstrated convincingly by numerous tests. Its derivation is given here to illustrate the importance of the end conditions.

For convenience, in the following derivation, the member will be oriented with its longitudinal axis along the x-axis of the coordinate system given in Figure 4.3. The roller support is to be interpreted as restraining the member from translating either up or down. An axial compressive load is applied and gradually increased. If a temporary transverse load is applied so as to deflect the member into the shape indicated by the dashed line, the member will return to its original position when this temporary load is removed if the axial load is less than the critical buckling load. The critical buckling load, P_{cr}, is defined as the load that is just large enough to maintain the deflected shape when the temporary transverse load is removed.

The differential equation giving the deflected shape of an elastic member subjected to bending is

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \tag{4.2}$$

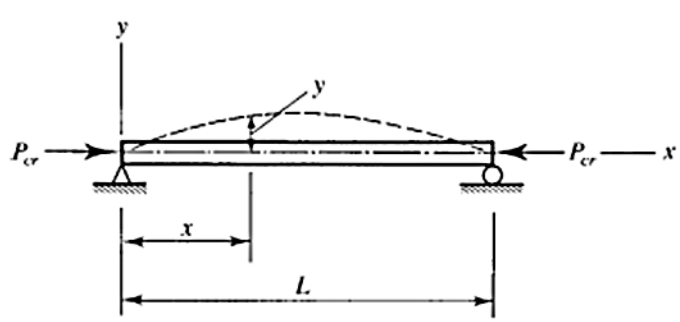
where x locates a point along the longitudinal axis of the member, y is the deflection of the axis at that point, and M is the bending moment at the point. E and I were previously defined, and here the moment of inertia I is with respect to the axis of bending (buckling). This equation was derived by Jacob Bernoulli and independently by Euler, who specialized it for the column buckling problem (Timoshenko, 1953). If we begin at the point of buckling, then from Figure 4.3 the bending moment is $P_{cr}y$. Equation 4.2 can then be written as

$$y'' + \frac{P_{cr}}{EI}y = 0$$

where the prime denotes differentiation with respect to x. This is a second-order, linear, ordinary differential equation with constant coefficients and has the solution

$$y = A\cos(cx) + B\sin(cx)$$

FIGURE 4.3



where

$$c = \sqrt{\frac{P_{cr}}{EI}}$$

and A and B are constants. These constants are evaluated by applying the following boundary conditions:

$$\text{At } x = 0, y = 0: \quad 0 = A\cos(0) + B\sin(0) \qquad A = 0$$
$$\text{At } x = L, y = 0: \quad 0 = B\sin(cL)$$

This last condition requires that $\sin(cL)$ be zero if B is not to be zero (the trivial solution, corresponding to $P = 0$). For $\sin(cL) = 0$,

$$cL = 0, \pi, 2\pi, 3\pi, \ldots = n\pi, \qquad n = 0, 1, 2, 3, \ldots$$

From

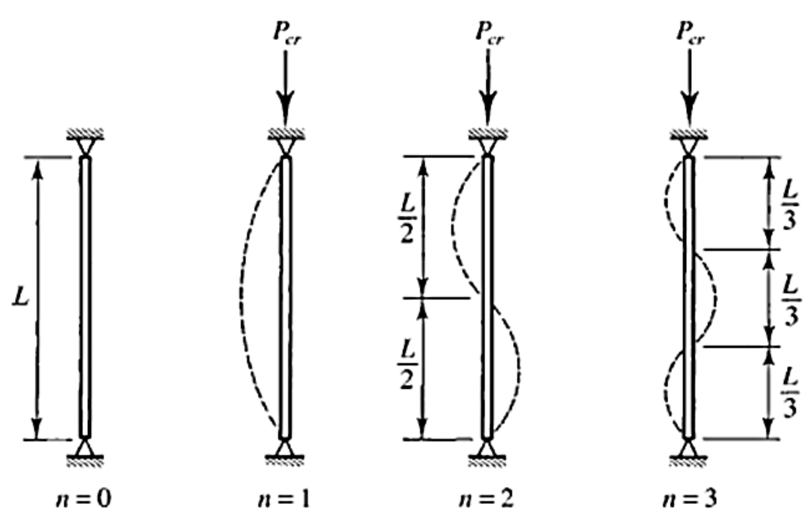
$$c = \sqrt{\frac{P_{cr}}{EI}}$$

we obtain

$$cL = \left(\sqrt{\frac{P_{cr}}{EI}}\right)L = n\pi, \quad \frac{P_{cr}}{EI}L^2 = n^2\pi^2 \quad \text{and} \quad P_{cr} = \frac{n^2\pi^2 EI}{L^2}$$

The various values of n correspond to different buckling modes; $n = 1$ represents the first mode, $n = 2$ the second, and so on. A value of zero gives the trivial case of no load. These buckling modes are illustrated in Figure 4.4. Values of n larger than 1 are not possible unless the compression member is physically restrained from deflecting at the points where the reversal of curvature would occur.

FIGURE 4.4



The solution to the differential equation is therefore

$$y = B \sin\left(\frac{n\pi x}{L}\right)$$

and the coefficient B is indeterminate. This result is a consequence of approximations made in formulating the differential equation; a linear representation of a nonlinear phenomenon was used.

For the usual case of a compression member with no supports between its ends, $n = 1$ and the Euler equation is written as

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (4.3)$$

It is convenient to rewrite Equation 4.3 as

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EAr^2}{L^2} = \frac{\pi^2 EA}{(L/r)^2}$$

where A is the cross-sectional area and r is the radius of gyration with respect to the axis of buckling. The ratio L/r is the slenderness ratio and is the measure of a member's slenderness, with large values corresponding to slender members.

If the critical load is divided by the cross-sectional area, the critical buckling stress is obtained:

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} \quad (4.4)$$

At this compressive stress, buckling will occur about the axis corresponding to r. Buckling will take place as soon as the load reaches the value given by Equation 4.3, and the column will become unstable about the principal axis corresponding to the largest slenderness ratio. This axis usually is the axis with the smaller moment of inertia (we examine exceptions to this condition later). Thus the minimum moment of inertia and radius of gyration of the cross section should ordinarily be used in Equations 4.3 and 4.4.

EXAMPLE 4.1

A W12 × 50 is used as a column to support an axial compressive load of 145 kips. The length is 20 feet, and the ends are pinned. Without regard to load or resistance factors, investigate this member for stability. (The grade of steel need not be known: The critical buckling load is a function of the modulus of elasticity, not the yield stress or ultimate tensile strength.)

SOLUTION For a W12 × 50,

$$\text{Minimum } r = r_y = 1.96 \text{ in.}$$

$$\text{Maximum} \frac{L}{r} = \frac{20(12)}{1.96} = 122.4$$

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2(29{,}000)(14.6)}{(122.4)^2} = 278.9 \text{ kips}$$

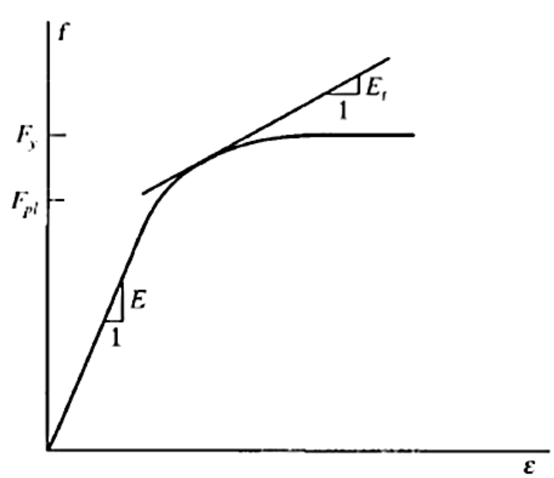
ANSWER Because the applied load of 145 kips is less than P_{cr}, the column remains stable and has an overall factor of safety against buckling of 278.9/145 = 1.92.

Early researchers soon found that Euler's equation did not give reliable results for stocky, or less slender, compression members. The reason is that the small slenderness ratio for members of this type causes a large buckling stress (from Equation 4.4). If the stress at which buckling occurs is greater than the proportional limit of the material, the relation between stress and strain is not linear, and the modulus of elasticity E can no longer be used. (In Example 4.1, the stress at buckling is $P_{cr}/A = 278.9/14.6 = 19.10$ ksi, which is well below the proportional limit for any grade of structural steel.) This difficulty was initially resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, E_t, in Equation 4.3. For a material with a stress–strain curve like the one shown in Figure 4.5, E is not a constant for stresses greater than the proportional limit F_{pl}. The tangent modulus E_t is defined as the slope of the tangent to the stress–strain curve for values of f between F_{pl} and F_y. If the compressive stress at buckling, P_{cr}/A, is in this region, it can be shown that

$$P_{cr} = \frac{\pi^2 E_t I}{L^2} \tag{4.5}$$

Equation 4.5 is identical to the Euler equation, except that E_t is substituted for E.

FIGURE 4.5



Effective Length

Both the Euler and tangent modulus equations are based on the following assumptions:

1. The column is perfectly straight, with no initial crookedness.
2. The load is axial, with no eccentricity.
3. The column is pinned at both ends.

The first two conditions mean that there is no bending moment in the member before buckling. As mentioned previously, some accidental moment will be present, but in most cases it can be ignored.

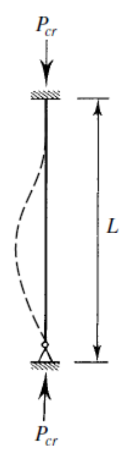
Other end conditions can be accounted for in the derivation of Euler Equation.

The Euler equation for case of column pinned at one end and fixed against rotation and translation at the other, derived in the same manner as Euler Equation, is

$$P_{cr} = \frac{2.05\,\pi^2 EI}{L^2}$$

or

$$P_{cr} = \frac{2.05\,\pi^2 EA}{(L/r)^2} = \frac{\pi^2 EA}{(0.70L/r)^2}$$



Thus this compression member has the same load capacity as a column that is pinned at both ends and is only 70% as long as the given column. Similar expressions can be found for columns with other end conditions.

For convenience, the equations for critical buckling load will be written as:

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 E_t A}{(KL/r)^2}$$

where ***KL*** is the ***effective length,*** and ***K*** is the ***effective length factor***

Values of ***K*** for these and other cases can be determined with the aid of Table C-A-7.1 in the Commentary to AISC Specification Appendix 7.

4.3 AISC REQUIREMENTS

The basic requirements for compression members are covered in Chapter *E* of the AISC Specification. The nominal compressive strength is:

$$P_n = F_{cr} A_g \qquad \text{(AISC Equation E3-1)}$$

For **LRFD**,

$$P_u \leq \phi_c P_n$$

where

P_u = sum of the factored loads

ϕ_c = resistance factor for compression = 0.90

$\phi_c P_n$ = design compressive strength

For **ASD**,

$$P_a \leq P_n / \Omega_c$$

where

P_a = sum of the service loads

Ω_c = safety factor for compression = 1.67

P_n / Ω_c = allowable compressive strength

If an allowable stress formulation is used,

$$f_a \leq F_a$$

where

f_a = computed axial compressive stress = P_a / A_g

F_a = allowable axial compressive stress = $F_{cr} / \Omega_c = F_{cr} / 1.67 = 0.6\, F_{cr}$

In order to present the AISC expressions for the critical stress F_{cr}, we first define the Euler load as:

$$P_e = \frac{\pi^2 EA}{(KL/r)^2}$$

This is the critical buckling load according to the Euler equation. The Euler stress is:

$$F_e = \frac{P_e}{A} = \frac{\pi^2 E}{(KL/r)^2}$$

To obtain the critical stress for elastic columns, the Euler stress is reduced as follows to account for the effects of initial crookedness:

$$F_{cr} = 0.877\, F_e$$

For **inelastic** columns, the tangent modulus equation, Equation 4.6b, is replaced by the exponential equation

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y$$

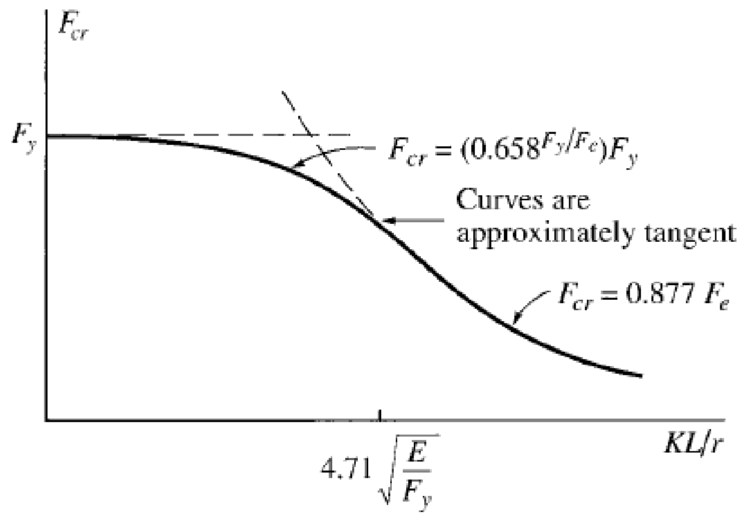
With above Equation, a direct solution for inelastic columns can be obtained, avoiding the trial-and-error approach inherent in the use of the tangent modulus equation. At the boundary between inelastic and elastic columns, the above two Equations give the same value of F_{cr}. This occurs when KL/r is approximately

$$4.71\sqrt{\frac{E}{F_y}}$$

Summary:

When $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$, $F_{cr} = \left(0.658^{F_y/F_e}\right) F_y$

When $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, $F_{cr} = (0.877) F_e$



General Procedure for Analysis of Compression Members

1. Calculate the value of effective length ***KL*** (after specifying effective length factor *K*)
2. Calculate or select the minimum value of radius of gyration *r*
3. Calculate slenderness ratio (or choose larger value of slenderness ratios) ***KL/r.***
4. Calculate the value of

$$\mathbf{4.71}\sqrt{\frac{E}{F_y}}$$

5. Compare the value of ***4.71 E/F_y*** with ***KL/r***

(a) If $\frac{KL}{r} \leq \mathbf{4.71}\sqrt{\frac{E}{F_y}}$, then critical stress will be: $F_{cr} = \left(\mathbf{0.658}^{F_y/F_e}\right)F_y$

(b) If $\frac{KL}{r} > \mathbf{4.71}\sqrt{\frac{E}{F_y}}$, the critical stress will be: $F_{cr} = (\mathbf{0.877})F_e$

Note that: $F_e = \frac{\pi^2 E}{(KL/r)^2}$

6. Calculate the nominal axial force by:

$$P_n = F_{cr} \times A_g$$

7. The design strength according to the **LRFD** will be

$$P_n = \phi_c\ P_n = 0.9\ P_n = 0.9(F_{cr} \times A_g)$$

8. The design strength according to the **ASD** will be

$$P_n = P_n / \Omega_c = P_n / 1.67 = 0.6\ P_n = 0.6\ (F_{cr} \times A_g)$$

EXAMPLE 4.2

A W14 × 74 of A992 steel has a length of 20 feet and pinned ends. Compute the design compressive strength for LRFD and the allowable compressive strength for ASD.

SOLUTION

Slenderness ratio:

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.48} = 96.77 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113$$

Since 96.77 < 113, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(96.77)^2} = 30.56 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/30.56)}(50) = 25.21 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr} A_g = 25.21(21.8) = 549.6 \text{ kips}$$

LRFD SOLUTION

The design strength is

$$\phi_c P_n = 0.90(549.6) = 495 \text{ kips}$$

ASD SOLUTION

From Equation 4.7, the allowable stress is

$$F_a = 0.6 F_{cr} = 0.6(25.21) = 15.13 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 15.13(21.8) = 330 \text{ kips}$$

ANSWER

Design compressive strength = 495 kips. Allowable compressive strength = 330 kips.

4.4 LOCAL STABILITY

The strength corresponding to any *overall* buckling mode, however, such as flexural buckling, cannot be developed if the elements of the cross section are so thin that ***local* buckling** occurs. The measure of this susceptibility is the width-to-thickness ratio of each cross-sectional element.

Limiting values of width-to-thickness ratios are given in AISC B4.1, “Classification of Sections for Local Buckling.” For compression members, shapes are classified as *slender* or *non-slender.*

If a shape is slender, its strength limit state is local buckling, and the corresponding reduced strength must be computed. The width-to-thickness ratio is given the generic symbol λ.

AISC Table B4.1a shows the upper limit, λr, for nonslender members of various cross-sectional shapes.

1. If $\lambda \leq \lambda r$, the shape is nonslender.
2. Otherwise, the shape is slender.

Using AISC notation gives:

$$= \frac{b}{t} = \frac{b_f/2}{t_f} = \frac{b_f}{2t_f}$$

where b_f and t_f are the width and thickness of the flange. The upper limit is:

$$= 0.56\sqrt{\frac{E}{F_y}}$$

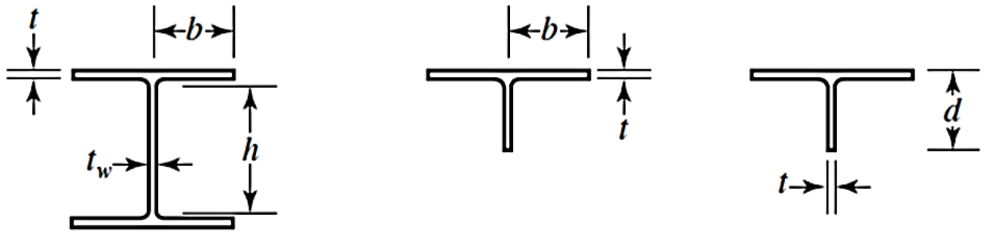
The webs of I shapes are stiffened elements, and the stiffened width is the distance between the roots of the flanges. The width-to-thickness parameter is:

$$= h/t_w$$

where h is the distance between the roots of the flanges, and t_w is the web thickness. The upper limit is

$$= 1.49\sqrt{\frac{E}{F_y}}$$

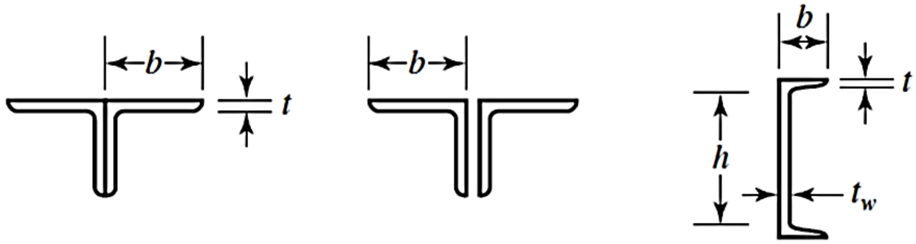
Stiffened and unstiffened elements of various cross-sectional shapes are illustrated in the Figure below. The appropriate compression member limit, λr, from AISC B4.1 is given for each case.



$b/t \le 0.56\sqrt{E/F_y}$ $\quad h/t_w \le 1.49\sqrt{E/F_y}$

$b/t \le 0.56\sqrt{E/F_y}$

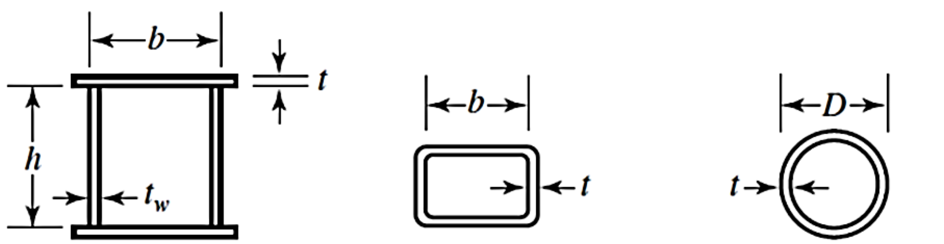
$d/t \le 0.75\sqrt{E/F_y}$



$b/t \le 0.56\sqrt{E/F_y}$

$b/t \le 0.45\sqrt{E/F_y}$

$b/t \le 0.56\sqrt{E/F_y}$ $\quad h/t_w \le 1.49\sqrt{E/F_y}$



$b/t \le 1.40\sqrt{E/F_y}$ $\quad h/t_w \le 1.40\sqrt{E/F_y}$

$b/t \le 1.40\sqrt{E/F_y}$

$D/t \le 0.11\, E/F_y$

EXAMPLE 4.3

Investigate the column of Example 4.2 for local stability.

SOLUTION For a W14 × 74, $b_f = 10.1$ in., $t_f = 0.785$ in., and

$$\frac{b_f}{2t_f} = \frac{10.1}{2(0.785)} = 6.43$$

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.5 > 6.43 \qquad \text{(OK)}$$

$$\frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{14.2 - 2(1.38)}{0.450} = 25.4$$

where k_{des} is the *design* value of *k*. (Different manufacturers will produce this shape with different values of *k*. The *design* value is the smallest of these values. The *detailing* value is the largest.)

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.9 > 25.4 \qquad \text{(OK)}$$

ANSWER Local instability is not a problem.

4.5 TABLES FOR COMPRESSION MEMBERS

The *Manual* contains many useful tables for analysis and design. For compression members whose strength is governed by flexural buckling (that is, not local bucking), Table 4-22 in Part 4 of the *Manual,* "Design of Compression Members," can be used. This table gives values of $\phi_c F_{cr}$ (for LRFD) and F_{cr}/Ω_c (for ASD) as a function of KL/r for various values of F_y. This table stops at the recommended upper limit of $KL/r = 200$.

Procedure for using Table 4-22

1. Calculate KL/r for the column
2. Enter table by KL/r and F_y values

3. Select Critical Stress $\boldsymbol{F_{cr}}$ from intersecting ***KL/r*** with $\boldsymbol{F_y}$ under the method required (ASD or LRFD)
4. The design strength will be: $\boldsymbol{P_n = F_{cr} \times A_g}$

The available strength tables, however, are the most useful. These tables, which we will refer to as the "**column load tables**," give the available strengths of selected shapes, both $\boldsymbol{\phi_c P_n}$ for LRFD and $\boldsymbol{P_n/\Omega_c}$ for ASD, as a function of the effective length KL. These tables include values of KL up to those corresponding to $KL/r = 200$.

Procedure for using column load tables

1. Calculate **the effective length** value (**KL**)
2. Enter tables by: KL, Fy and Section type then select the available axial load under the method required (ASD or LRFD).

EXAMPLE 4.5

Compute the available strength of the compression member of Example 4.2 with the aid of (a) Table 4-22 from Part 4 of the *Manual* and (b) the column load tables.

LRFD SOLUTION

a. From Example 4.2, $KL/r = 96.77$ and $F_y = 50$ ksi. Values of $\phi_c F_{cr}$ in Table 4-22 are given only for integer values of KL/r; for decimal values, KL/r may be rounded *up* or linear interpolation may be used. For uniformity, we use interpolation in this book for all tables unless otherwise indicated. For $KL/r = 96.77$ and $F_y = 50$ ksi,

$$\phi_c F_{cr} = 22.67 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 22.67(21.8) = 494 \text{ kips}$$

b. The column load tables in Part 4 of the *Manual* give the available strength for selected W-, HP-, single-angle, WT-, HSS, pipe, double-angle, and composite shapes. (We cover composite construction in Chapter 9.) The tabular values for the symmetrical shapes (W, HP, HSS and pipe) were calculated by using the minimum radius of gyration for each shape. From Example 4.2, $K = 1.0$, so

$$KL = 1.0(20) = 20 \text{ ft}$$

For a W14 × 74, $F_y = 50$ ksi and $KL = 20$ ft,

$$\phi_c P_n = 495 \text{ kips}$$

ASD SOLUTION

a. From Example 4.2, $KL/r = 96.77$ and $F_y = 50$ ksi. By interpolation, for $KL/r = 96.77$ and $F_y = 50$ ksi,

$$F_{cr}/\Omega_c = 15.07 \text{ ksi}$$

Note that this is the allowable stress, $F_a = 0.6F_{cr}$. Therefore, the allowable strength is

$$\frac{P_n}{\Omega_c} = F_a A_g = 15.07(21.8) = 329 \text{ kips}$$

b. From Example 4.2, $K = 1.0$, so

$$KL = 1.0(20) = 20 \text{ ft}$$

From the column load tables, for a W14 × 74 with $F_y = 50$ ksi and $KL = 20$ ft,

$$\frac{P_n}{\Omega_c} = 329 \text{ kips}$$

4.6 DESIGN OF COMPRESSION MEMBERS

The selection of an economical rolled shape to resist a given compressive load is simple with the aid of the column load tables. Enter the table with the effective length and move horizontally until you find the desired available strength (or something slightly larger). In some cases, you must continue the search to be certain that you have found the lightest shape. Usually the category of shape (W, WT, etc.) will have been decided upon in advance. Often the overall nominal dimensions will also be known because of architectural or other requirements. As pointed out earlier, all tabulated values correspond to a slenderness ratio of 200 or less.

Design by Column Load Table

1. Calculate the compression service load or compression factored load.
2. Calculate the effective length (KL) value
3. Enter "column table" by compression load and KL and select the section

Notes:

1. If the section was specified (such as W10, W12… etc) then you can choose the section directly.
2. If section not specified (such as W) the select many section provides the applied load capacity and then choose the lighter one.

EXAMPLE 4.6

A compression member is subjected to service loads of 165 kips dead load and 535 kips live load. The member is 26 feet long and pinned at each end. Use A992 steel and select a W14 shape.

LRFD SOLUTION

Calculate the factored load:

$$P_u = 1.2D + 1.6L = 1.2(165) + 1.6(535) = 1054 \text{ kips}$$

$\therefore$ Required design strength $\phi_c P_n = 1054$ kips.

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 145 has a design strength of 1230 kips.

ANSWER Use a W14 × 145.

ASD SOLUTION

Calculate the total applied load:

$$P_a = D + L = 165 + 535 = 700 \text{ kips}$$

$\therefore$ Required allowable strength $\dfrac{P_n}{\Omega_c} = 700$ kips

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 132 has an allowable strength of 702 kips.

ANSWER Use a W14 × 132.

XAMPLE 4.7

Select the lightest W-shape that can resist a service dead load of 62.5 kips and a service live load of 125 kips. The effective length is 24 feet. Use ASTM A992 steel.

SOLUTION The appropriate strategy here is to find the lightest shape for each nominal depth in the column load tables and then choose the lightest overall.

LRFD SOLUTION The factored load is

$$P_u = 1.2D + 1.6L = 1.2(62.5) + 1.6(125) = 275 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $\phi_c P_n \geq 275$ kips.
W10: W10 × 54, $\phi_c P_n = 282$ kips
W12: W12 × 58, $\phi_c P_n = 292$ kips
W14: W14 × 61, $\phi_c P_n = 293$ kips

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

ANSWER Use a W10 × 54.

ASD SOLUTION The total applied load is

$$P_a = D + L = 62.5 + 125 = 188 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $P_n/\Omega_c \geq 188$ kips.
W10: W10 × 54, $\dfrac{P_n}{\Omega_c} = 188$ kips

For shapes not in the column load tables, a trial-and-error approach must be used. The general procedure is to assume a shape and then compute its strength. If the strength is too small (unsafe) or too large (uneconomical), another trial must be made. A systematic approach to making the trial selection is as follows:

1. Assume a value for the critical buckling stress F_{cr}. Examination of AISC Equations E3-2 and E3-3 shows that the theoretically maximum value of F_{cr} is the yield stress F_y.
2. Determine the required area. For LRFD,

$$\phi_c F_{cr} A_g \geq P_u$$

$$A_g \geq \frac{P_u}{\phi_c F_{cr}}$$

For ASD,

$$0.6F_{cr} \geq \frac{P_a}{A_g}$$

$$A_g \geq \frac{P_a}{0.6F_{cr}}$$

3. Select a shape that satisfies the area requirement.
4. Compute F_{cr} and the strength for the trial shape.
5. Revise if necessary. If the available strength is very close to the required value, the next tabulated size can be tried. Otherwise, repeat the entire procedure, using the value of F_{cr} found for the current trial shape as a value for Step 1.
6. Check local stability (check the width-to-thickness ratios). Revise if necessary.

EXAMPLE 4.8

Select a W18 shape of A992 steel that can resist a service dead load of 100 kips and a service live load of 300 kips. The effective length KL is 26 feet.

LRFD SOLUTION

$P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600$ kips

Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.9 \text{ in.}^2 > 20.2 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{KL}{r_{min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(7.455)(20.9) = 140 \text{ kips} < 600 \text{ kips} \qquad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \quad \text{(OK)}$$

$$\frac{KL}{r_{min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.65)(35.1) = 589 \text{ kips} < 600 \text{ kips} \qquad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.3 \text{ in.}^2$$
$$\frac{KL}{r_{min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \qquad \text{(OK)}$$
$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(115.6)^2} = 21.42 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.79)(38.3) = 648 \text{ kips} > 600 \text{ kips} \qquad \text{(OK.)}$$

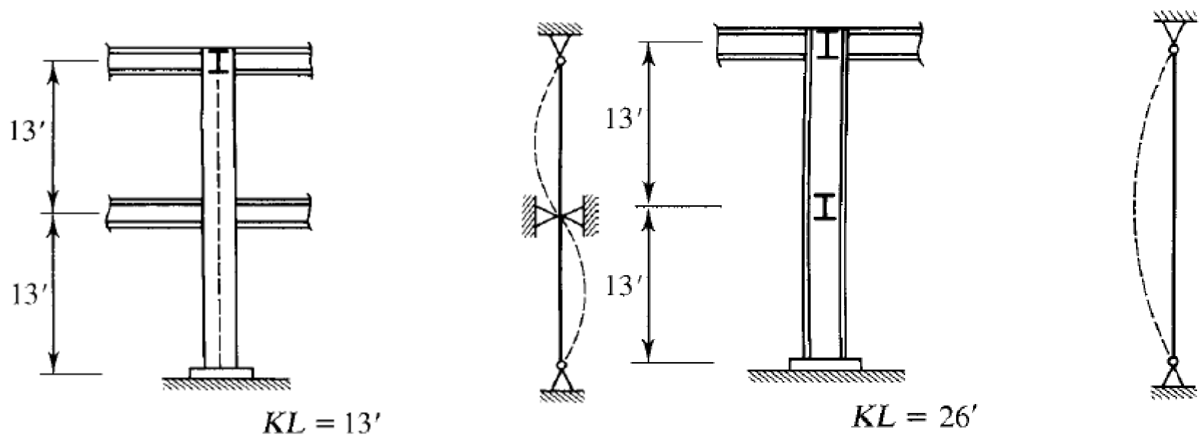
This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

ANSWER Use a W18 × 130.

4.7 MORE ON EFFECTIVE LENGTH

We introduced the concept of effective length in Section 4.2, "Column Theory." All compression members are treated as pin-ended regardless of the actual end conditions but with an effective length KL that may differ from the actual length. With this modification, the load capacity of compression members is a function of only the slenderness ratio and modulus of elasticity. For a given material, the load capacity is a function of the slenderness ratio only.

If a compression member is supported differently with respect to each of its principal axes, the effective length will be different for the two directions. In Figure below, a W-shape is used as a column and is braced by horizontal members in two perpendicular directions at the top. These members prevent translation of the column in all directions, but the connections, the details of which are not shown, permit small rotations to take place. Under these conditions, the member can be treated as pin-connected at the top.



(a) Minor Axis Buckling **(b) Major Axis Buckling**

Again, the connection prevents translation, but no restraint against rotation is furnished. This brace prevents translation perpendicular to the weak axis of the cross section but provides no restraint perpendicular to the strong axis. As shown schematically in Figure above, if the member were to buckle about the major axis, the effective length would be 26 feet, whereas buckling about the minor axis would have to be in the second buckling mode, corresponding to an effective length of 13 feet.
Because its strength decreases with increasing *KL*/*r,* a column will buckle in the direction corresponding to the largest slenderness ratio, so K_xL/r_x must be compared with K_yL/r_y. In Figure above, the ratio

$26(12)/r_x$ must be compared with $13(12)/r_y$ (where r_x and r_y are in inches), and the larger ratio would be used for the determination of the axial compressive strength.

EXAMPLE 4.9

A W12 × 58, 24 feet long, is pinned at both ends and braced in the weak direction at the third points, as shown in Figure 4.11. A992 steel is used. Determine the available compressive strength.

SOLUTION

$$\frac{K_x L}{r_x} = \frac{24(12)}{5.28} = 54.55$$

$$\frac{K_y L}{r_y} = \frac{8(12)}{2.51} = 38.25$$

$K_x L/r_x$, the larger value, controls.

LRFD SOLUTION From Table 4-22 from Part 4 of the *Manual* and with $KL/r = 54.55$,

$$\phi_c F_{cr} = 36.24 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 36.24(17.0) = 616 \text{ kips}$$

ANSWER Design strength = 616 kips.

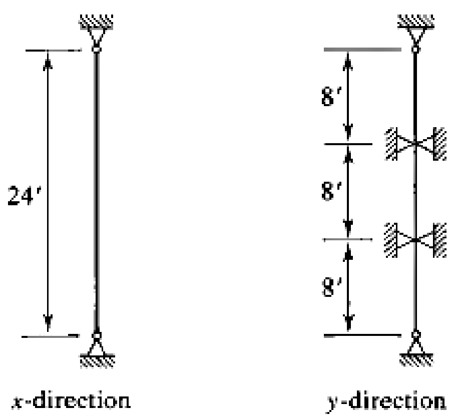
ASD SOLUTION From Table 4-22 with $KL/r = 54.55$,

$$\frac{F_{cr}}{\Omega_c} = 24.09 \text{ ksi}$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr}}{\Omega_c} A_g = 24.09(17.0) = 410 \text{ kips}$$

ANSWER Allowable strength = 410 kips.

FIGURE 4.11



The available strengths given in the column load tables are based on the effective length with respect to the *y*-axis. A procedure for using the tables with $K_x L$, however, can be developed by examining how the tabular values were obtained.
Starting with a value of *KL,* the strength was obtained by a procedure similar to the following:

- *KL* was divided by r_y to obtain KL/r_y.
- F_{cr} was computed.
- The available strengths, $\phi_c P_n$ for LRFD and P_n/Ω_c for ASD, were computed.

Thus the tabulated strengths are based on the values of *KL* being equal to $K_y L$. If the capacity with respect to *x*-axis buckling is desired, the table can be entered with

$$KL = \frac{K_x L}{r_x/r_y}$$

and the tabulated load will be based on

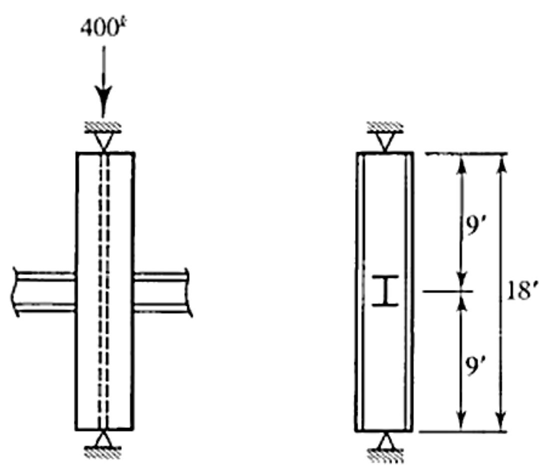
$$\frac{KL}{r_y} = \frac{K_x L/(\frac{r_x}{r_y})}{r_y} = \frac{K_x L}{r_x}$$

The ratio r_x/r_y is given in the column load tables for each shape listed.

EXAMPLE 4.10

The compression member shown in Figure 4.12 is pinned at both ends and supported in the weak direction at midheight. A service load of 400 kips, with equal parts of dead and live load, must be supported. Use F_y = 50 ksi and select the lightest W-shape.

FIGURE 4.12



LRFD SOLUTION

Factored load = $P_u = 1.2(200) + 1.6(200) = 560$ kips

Assume that the weak direction controls and enter the column load tables with $KL = 9$ feet. Beginning with the smallest shapes, the first one found that will work is a W8 × 58 with a design strength of 634 kips.

Check the strong axis:

$$\frac{K_xL}{r_x/r_y} = \frac{18}{1.74} = 10.34 \text{ ft} > 9 \text{ ft}$$

$\therefore K_xL$ controls for this shape.

Enter the tables with $KL = 10.34$ feet. A W8 × 58 has an interpolated strength of

$$\phi_c P_n = 596 \text{ kips} > 560 \text{ kips} \quad \text{(OK)}$$

Next, investigate the W10 shapes. Try a W10 × 49 with a design strength of 568 kips.

Check the strong axis:

$$\frac{K_xL}{r_x/r_y} = \frac{18}{1.71} = 10.53 \text{ ft} > 9 \text{ ft}$$

$\therefore K_xL$ controls for this shape.

Enter the tables with $KL = 10.53$ feet. A W10 × 54 is the lightest W10, with an interpolated design strength of 594 kips.

Continue the search and investigate a W12 × 53 ($\phi_c P_n = 611$ kips for $KL = 9$ ft):

$$\frac{K_xL}{r_x/r_y} = \frac{18}{2.11} = 8.53 \text{ ft} < 9 \text{ ft}$$

$\therefore K_yL$ controls for this shape, and $\phi_c P_n = 611$ kips.

Determine the lightest W14. The lightest one with a possibility of working is a W14 × 61. It is heavier than the lightest one found so far, so it will not be considered.

ANSWER Use a W12 × 53.

Whenever possible, the designer should provide extra support for the weak direction of a column. Otherwise, the member is inefficient: It has an excess of strength in one direction. When K_xL and K_yL are different, K_yL will control unless r_x/r_y is smaller than K_xL/K_yL. When the two ratios are equal, the column has equal strength in both directions. For most of the W-shapes in the column load tables, r_x/r_y ranges between 1.6 and 1.8, but it is as high as 3.1 for some shapes.

EXAMPLE 4.11

The column shown in Figure 4.13 is subjected to a service dead load of 140 kips and a service live load of 420 kips. Use A992 steel and select a W-shape.

SOLUTION $K_xL = 20$ ft and maximum $K_yL = 8$ ft. The effective length K_xL will control whenever

$$\frac{K_xL}{r_x/r_y} > K_yL$$

or

$$r_x/r_y < \frac{K_xL}{K_yL}$$

In this example,

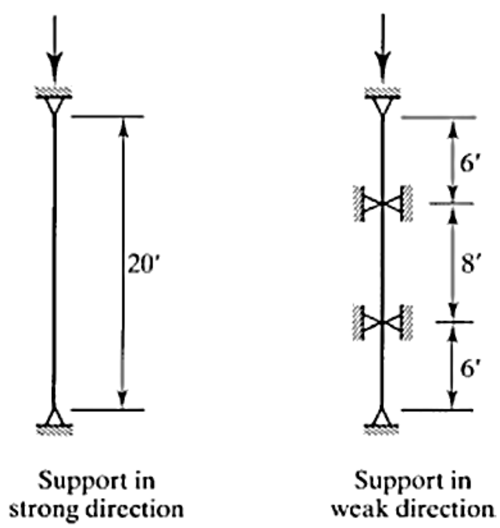
$$\frac{K_xL}{K_yL} = \frac{20}{8} = 2.5$$

so K_xL will control if $r_x/r_y < 2.5$. Since this is true for almost every shape in the column load tables, K_xL probably controls in this example.

Assume $r_x/r_y = 1.7$:

$$\frac{K_xL}{r_x/r_y} = \frac{20}{1.7} = 11.76 > K_yL$$

FIGURE 4.13



LRFD SOLUTION

$P_u = 1.2D + 1.6L = 1.2(140) + 1.6(420) = 840$ kips

Enter the column load tables with KL = 12 feet. There are no W8 shapes with enough load capacity.

Try a W10 × 88 ($\phi_c P_n = 940$ kips):

$$\text{Actual}\ \frac{K_x L}{r_x/r_y} = \frac{20}{1.73} = 11.56\ \text{ft} < 12\ \text{ft}$$

$$\therefore \phi_c P_n > \text{required 840 kips}$$

(By interpolation, $\phi_c P_n = 955$ kips.)

Check a W12 × 79:

$$\frac{K_x L}{r_x/r_y} = \frac{20}{1.75} = 11.43\ \text{ft.}$$

$$\phi_c P_n = 900\ \text{kips} > 840\ \text{kips} \qquad \text{(OK)}$$

Investigate W14 shapes. For $r_x/r_y = 2.44$ (the approximate ratio for all likely possibilities),

$$\frac{K_x L}{r_x/r_y} = \frac{20}{2.44} = 8.197\ \text{ft} > K_y L = 8\ \text{ft}$$

For $KL = 9$ ft, a W14 × 74, with a capacity of 854 kips, is the lightest W14-shape. Since 9 feet is a conservative approximation of the actual effective length, this shape is satisfactory.

ANSWER Use a W14 × 74 (lightest of the three possibilities).

4.9 BUILT-UP MEMBERS

If the cross-sectional properties of a built-up compression member are known, its analysis is the same as for any other compression member, provided the component parts of the cross section are properly connected. AISC E6 contains many details related to this connection, with separate requirements for members composed of two or more rolled shapes and for members composed of plates or a combination of plates and shapes. Before considering the connection problem, we will review the computation of cross-sectional properties of built-up shapes.

The design strength of a built-up compression member is a function of the slenderness ratio *KL*/*r*. Hence the principal axes and the corresponding radii of gyration about these axes must be determined. For homogeneous cross sections, the principal axes coincide with the centroidal axes. The procedure is illustrated in Example 4.17. The components of the cross section are assumed to be properly connected.

EXAMPLE 4.17

The column shown in Figure 4.19 is fabricated by welding a ⅜-inch by 4-inch cover plate to the flange of a W18 × 65. Steel with $F_y = 50$ ksi is used for both components. The effective length is 15 feet with respect to both axes. Assume that the components are connected in such a way that the member is fully effective and compute the strength based on flexural buckling.

FIGURE 4.19

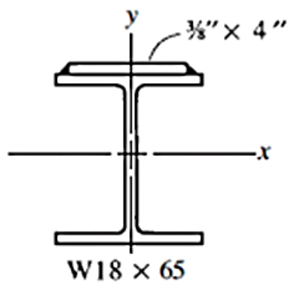


TABLE 4.2

Component	A	y	Ay
Plate	1.500	0.1875	0.2813
W	19.10	9.575	182.9
	20.60		183.2

SOLUTION With the addition of the cover plate, the shape is slightly unsymmetrical, but the flexural-torsional effects will be negligible.

The vertical axis of symmetry is one of the principal axes, and its location need not be computed. The horizontal principal axis will be found by application of the *principle of moments:* The sum of moments of component areas about any axis (in this example, a horizontal axis along the top of the plate will be used) must equal the moment of the total area. We use Table 4.2 to keep track of the computations.

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{183.2}{20.60} = 8.893 \text{ in.}$$

With the location of the horizontal centroidal axis known, the moment of inertia with respect to this axis can be found by using the *parallel-axis theorem:*

$$I = \bar{I} + Ad^2$$

where

$\bar{I}$ = moment of inertia about the centroidal axis of a component area
A = area of the component
I = moment of inertia about an axis parallel to the centroidal axis of the component area
d = perpendicular distance between the two axes

The contributions from each component area are computed and summed to obtain the moment of inertia of the composite area. These computations are shown in Table 4.3, which is an expanded version of Table 4.2. The moment of inertia about the x-axis is

$$I_x = 1193 \text{ in.}^4$$

TABLE 4.3

Component	A	y	Ay	$\bar{I}$	d	$\bar{I}+Ad^2$
Plate	1.500	0.1875	0.2813	0.01758	8.706	113.7
W	19.10	9.575	182.9	1070	0.6820	1079
	20.60		183.2			1193

For the vertical axis,

$$I_y = \frac{1}{12}\left(\frac{3}{8}\right)(4)^3 + 54.8 = 56.80 \text{ in.}^4$$

Since $I_y < I_x$, the y-axis controls.

$$r_{min} = r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{56.80}{20.60}} = 1.661 \text{ in.}$$

$$\frac{KL}{r_{min}} = \frac{15 \times 12}{1.661} = 108.4$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(108.4)^2} = 24.36 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/24.36)}(50) = 21.18 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 21.18(20.60) = 436.3 \text{ kips}$$

LRFD SOLUTION

The design strength is

$$\phi_c P_n = 0.90(436.3) = 393 \text{ kips}$$

ASD SOLUTION

From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(21.18) = 12.71 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 12.71(20.60) = 262 \text{ kips}$$

ANSWER

Design compressive strength = 393 kips. Allowable compressive strength = 262 kips.