# Chapter 1: Introduction to Two-Way Slab Systems

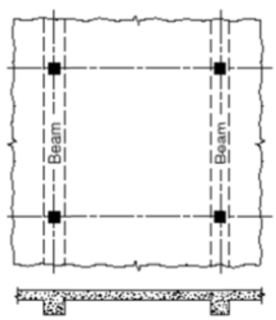
## Types of Slabs

A reinforced concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel. It may be supported by RC beams, by masonry or RC walls, by structural steel members, directly by columns, or continuously by the ground.

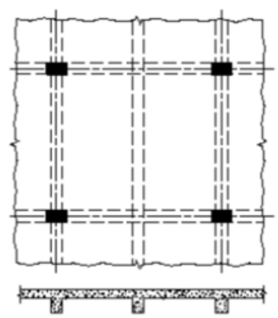
General slab systems that may be designed according to this course include flat slabs, flat plates, two-way slabs, and waffle slabs. Slabs-on-ground are excluded.

### 1. One-way slab:

A slab supported on two sides only (fig. a), or if the ratio of length to width of one slab panel is > 2 (fig. c)




(*a*) One-way slab



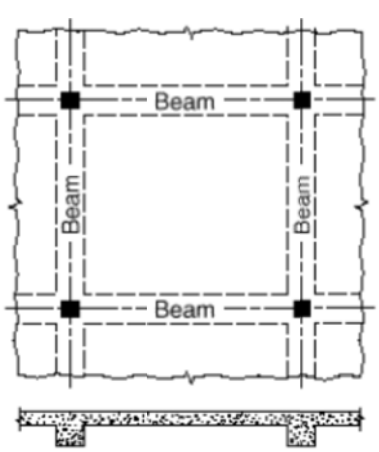
(*c*) One-way slab

In this type, loads are carried by the slab in *one* direction perpendicular to the supporting beam (in the short direction).

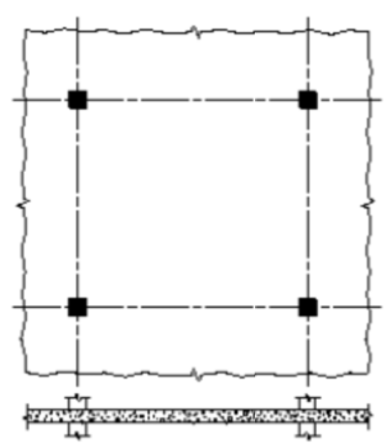
### 2. Two way slab:

The ratio of length to width of one slab panel is < 2

- It may be with beams on all four sides (fig. b),

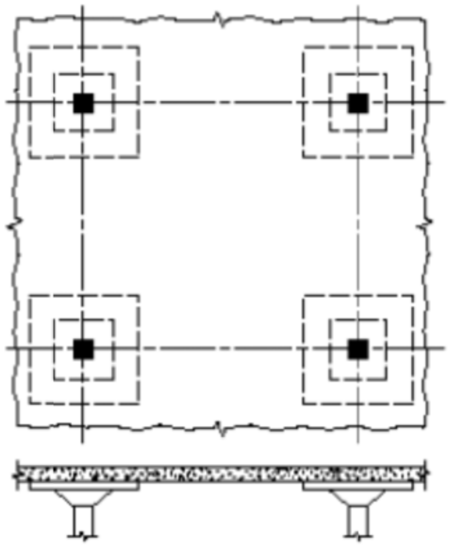



(*b*) Two-way slab

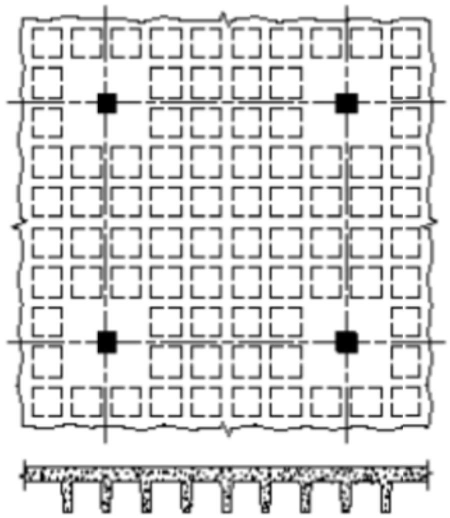


(*d*) Flat plate

- Or beamless:
- Flat plate, (fig. d), spans are not large and loads not heavy.
- Flat slab, (fig. e), incorporates drop panels and column capitals to reduce stresses due to shear and negative bending around the columns, and
- Waffle (grid) slab, (fig. f), a two way joist (ribbed) slab in which voids are formed in a rectilinear pattern to reduce the dead load of a solid-slab construction.

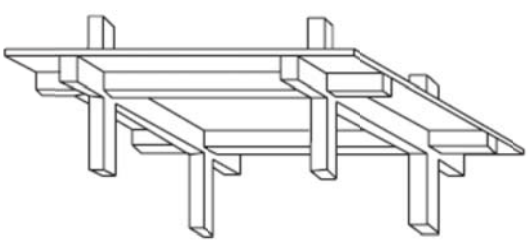


(*e*) Flat slab

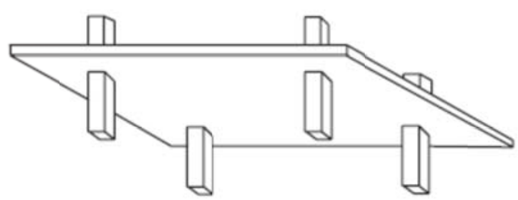


(*f*) Grid or waffle slab

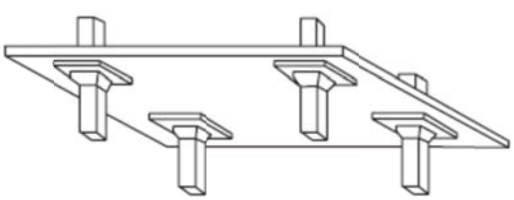
## 3D figures of two-way slabs:



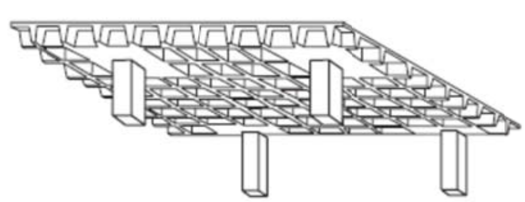
(a) Two-Way Beam Supported Slab



(b) Flat Plate

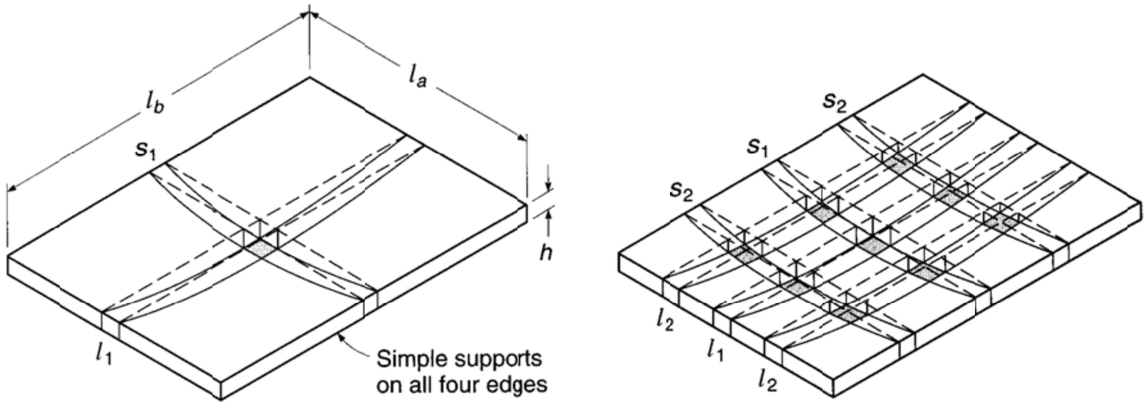


(c) Flat Slab



(d) Waffle Slab (Two-Way Joist Slab)

## **Behavior of TW slabs**




(*a*) (*b*)

**Fig.(a);** two center strips of a rectangular slab with short span $\boldsymbol{l_a}$ and long span $\boldsymbol{l_b}$, loaded with uniform load $\boldsymbol{q}$ (kN/m$^2$). Their deflections at the intersection point are the same:

$$\frac{5q_a l_a^4}{384EI} = \frac{5q_b l_b^4}{384EI} \qquad\qquad \frac{q_a}{q_b} = \frac{l_b^4}{l_a^4}$$

It is seen that the larger share of the load is carried in the short direction.

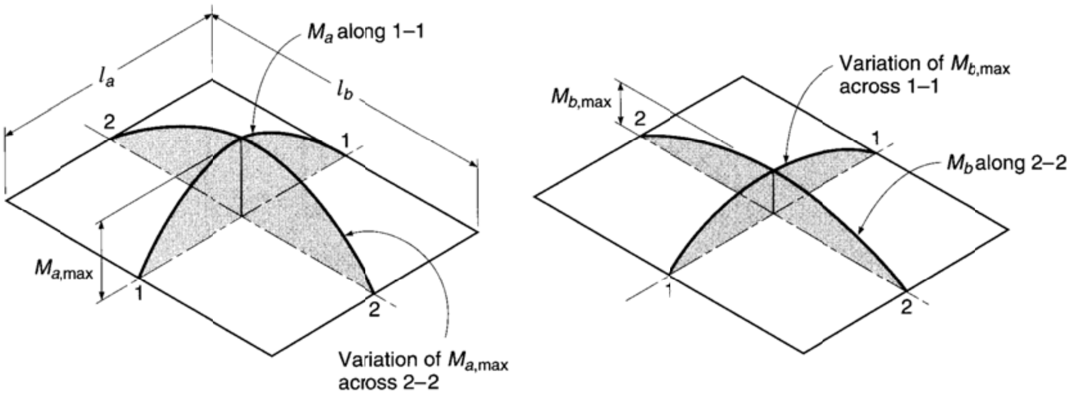
The actual behavior is more complex;

**Fig.(b);** grid model of the slab.

The outer strips $s_2$ and $l_2$ are not only bent but also twisted. It means that bending moments are less due to the share of torsional moments.

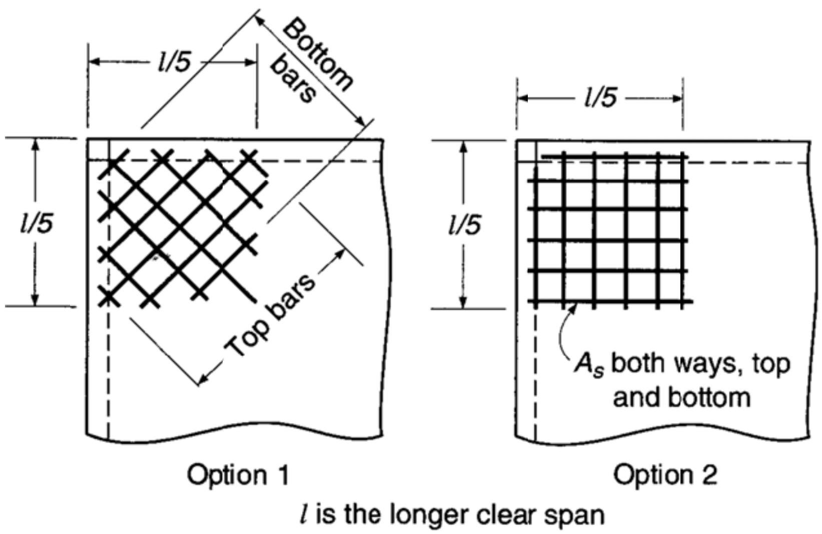
The max moments occur at the sharpest curvature. $\boldsymbol{M}$ along $s_1 >$ $\boldsymbol{M}$ along $s_2$

The moments vary across short and long directions, as shown below for a SS slab;

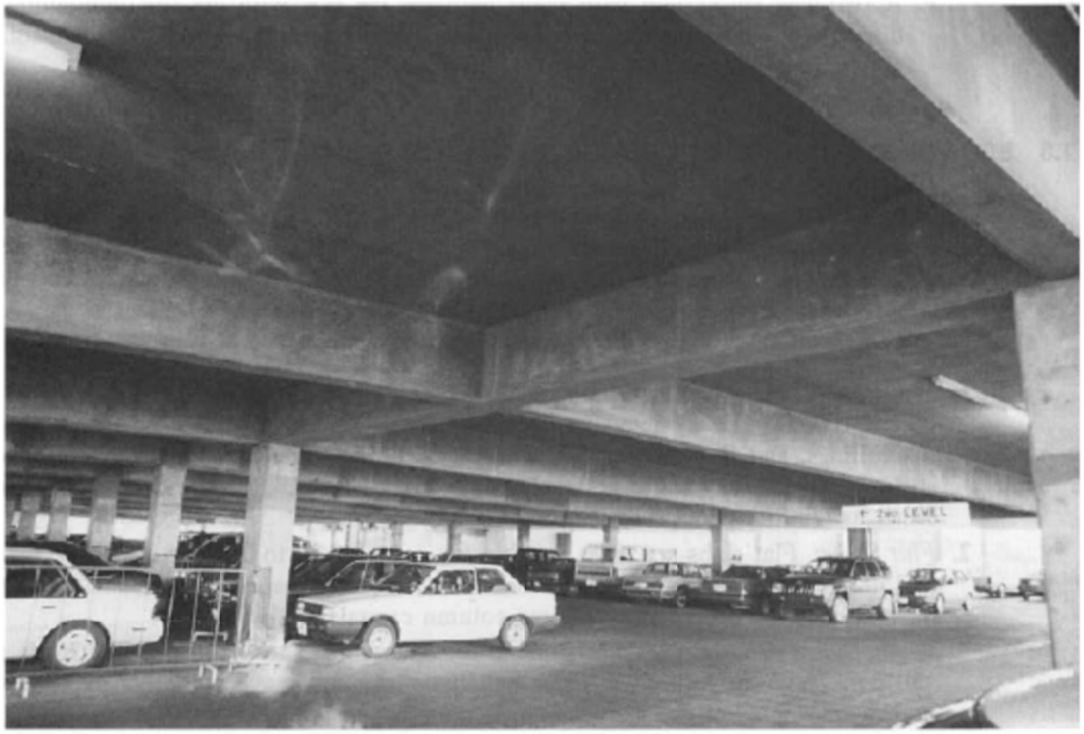



(*a*) (*b*)

**Special reinforcement against twisting moments** is required at exterior corners of a beam-supported two-way slab system; where they tend to crack the slab at the bottom // diagonal, and at the top ⊥ diagonal, as follows;



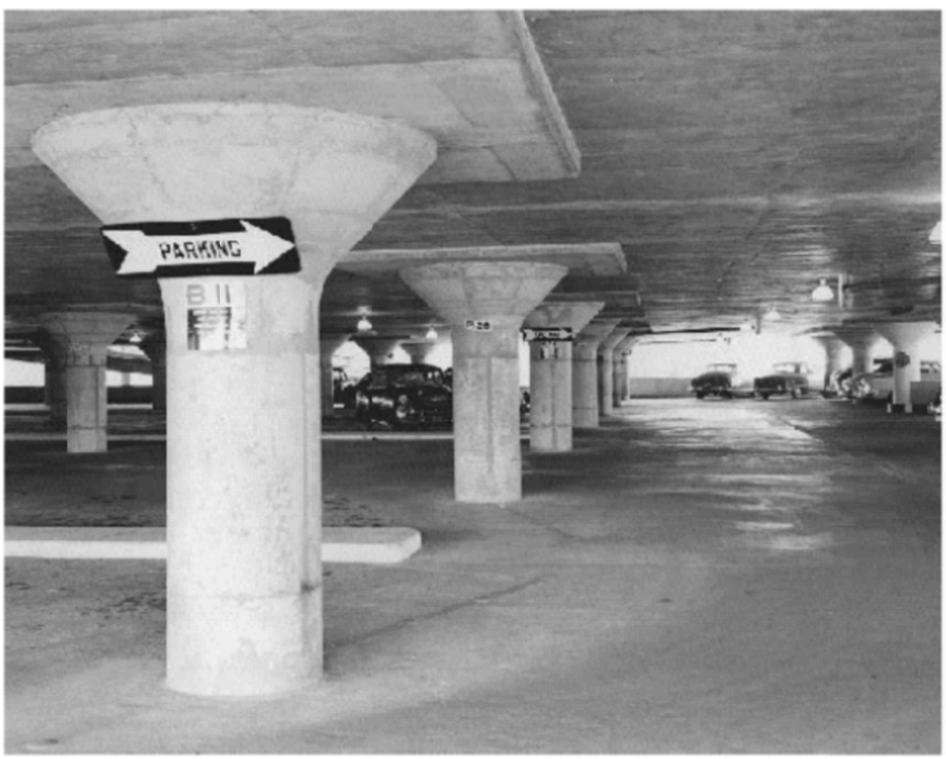

***Special A***$_s$ = max positive $\boldsymbol{A_s}$ in the panel.



Slab on beams.

Flat-plate floor system.

**Flat Slab floor system**




**Flat Slab floor system (with column capitals)**

Waffle slab with light fixtures at the centers of the squares.

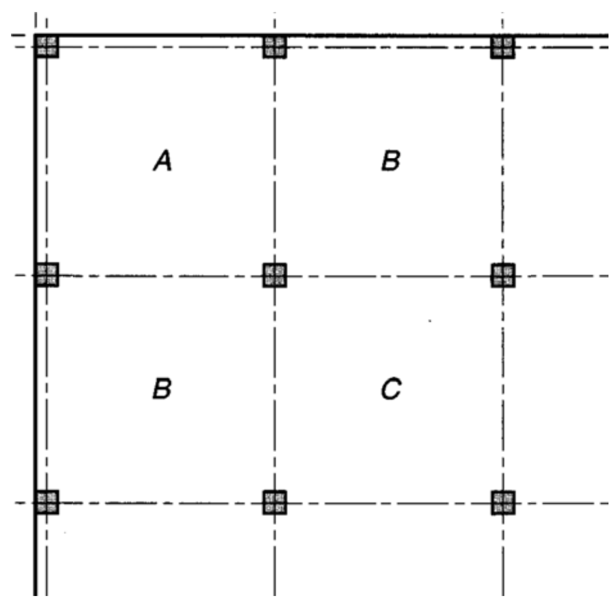
# Chapter 2: Deflection Control of T-W Slab Systems

## <u>ACI Slab Thickness Limitations (*ACI Code*)</u>

ACI Code (318-19) 8.3.1.1 establishes minimum thicknesses for TW slabs designed according to ACI chapter 8 (Two-Way Slabs).

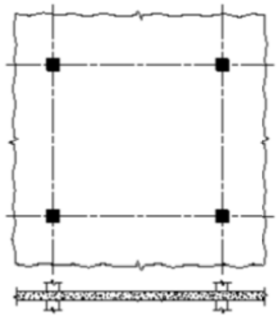
These min thicknesses were found practically satisfactory.

### <u>Exterior / Interior Panels:</u>

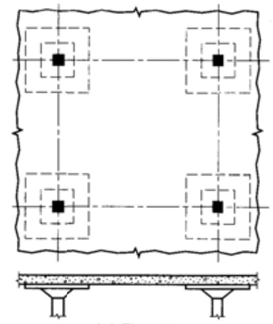



Panel A = Exterior, corner panel
Panel B = Exterior, edge panel
Panel C = Interior panel

# 1. Slabs without Interior Beams (Flat Plates / Flat Slabs):



(*d*) Flat plate



(*e*) Flat slab

The min thickness (***h***) must not be less than the values given in ACI Table 8.3.1.1:

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (in.)[1]**

| $f_y$, psi[2] | Without drop panels[3] | | | With drop panels[3] | | |
|---|---|---|---|---|---|---|
| | Exterior panels | | Interior panels | Exterior panels | | Interior panels |
| | Without edge beams | With edge beams[4] | | Without edge beams | With edge beams[4] | |
| 40,000 | $\ell_n/33$ | $\ell_n/36$ | $\ell_n/36$ | $\ell_n/36$ | $\ell_n/40$ | $\ell_n/40$ |
| 60,000 | $\ell_n/30$ | $\ell_n/33$ | $\ell_n/33$ | $\ell_n/33$ | $\ell_n/36$ | $\ell_n/36$ |
| 80,000 | $\ell_n/27$ | $\ell_n/30$ | $\ell_n/30$ | $\ell_n/30$ | $\ell_n/33$ | $\ell_n/33$ |

[1]$\ell_n$ is the clear span in the long direction, measured face-to-face of supports (in.).

[2]For $f_y$ between the values given in the table, minimum thickness shall be calculated by linear interpolation.

[3]Drop panels as given in 8.2.4.

[4]Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if $\alpha_f$ is less than 0.8.

40,000 psi = 280 MPa, 60,000 psi = 420 MPa, 80,000 psi = 560 MPa.

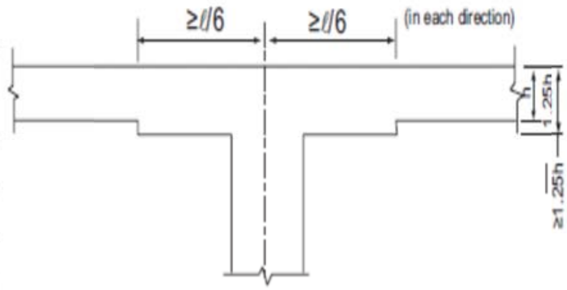
$l_n$ = length of clear span in the long direction, face to face of columns, or beams in slabs with interior beams.

In all cases, the min thickness (***h***) must not be less than the following:

**125 mm** for slabs without drop panels.
**100 mm** for slabs with drop panels.

## ACI 8.2.4. Drop Panel min Dimensions:

(a) The drop panel shall project below the slab at least one-fourth of the adjacent slab thickness.
(b) The drop panel shall extend in each direction from the centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

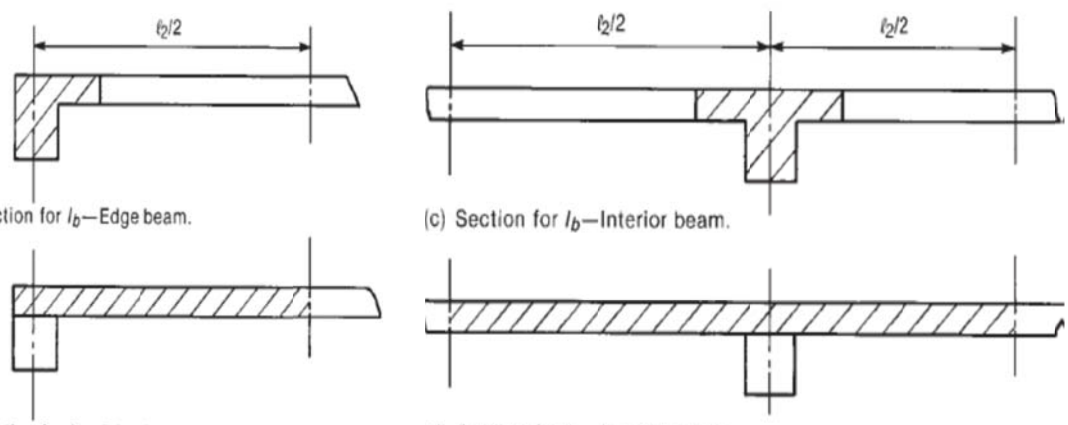



## Relative Beam / Slab Stiffness Ratio; $\alpha_f$

This important parameter is defined as

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

$\boldsymbol{\alpha_{f1}}$ : computed $\boldsymbol{\alpha_f}$ in the direction of $\boldsymbol{l_1}$
$\boldsymbol{\alpha_{f2}}$ **:** computed $\boldsymbol{\alpha_f}$ in the direction of $\boldsymbol{l_2}$

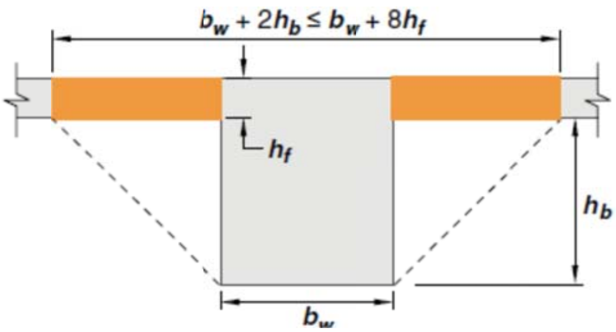


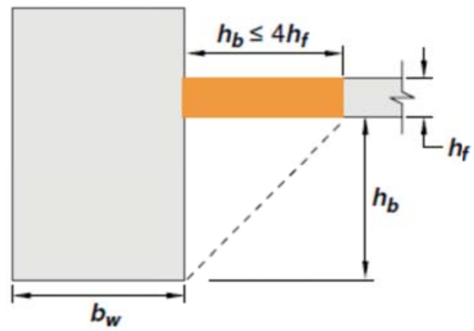

(a) Section for $I_b$—Edge beam.
(b) Section for $I_s$—Edge beam.
(c) Section for $I_b$—Interior beam.
(d) Section for $I_s$—Interior beam.

## ACI Effective Beam Section:

ACI Code 8.4.1.8 defines the effective beam section as follows:

**For Interior Beams:** **For Edge Beams:**




$$I_{beam} = K\,(b_w H^3/12) \qquad I_{slab} = l\,h^3/12$$

***H*** = Total depth of beam: ($h_b$ + $h_f$),

***l*** = width of slab between centerlines of panels on each side of beam.

$K = 1 + (x - 1)y^3 + 3\,[(1 - y)^2 y(x - 1) / \{1 + y(x - 1)\}]$

Where

$x = B / b_w$ ***B*** = width of flange

$y = h_f / H$

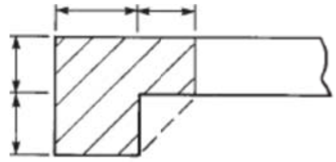
For simplicity take $K = 2$ for interior beams, and $K = 1.5$ for edge beams.

### Example1 : Calculation of $\alpha_f$

An **200 mm** thick slab of length **6 m** (c.c.) is provided with edge beam of total depth of **400 mm** and width of **300 mm**. The slab and beam were cast monolithically, have the same concrete strength (*and thus the same $E_c$*). Compute $\alpha_f$.

Solution:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$




*Beam section*

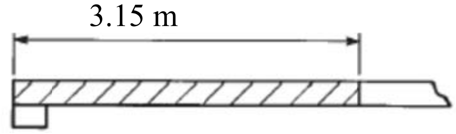
$\alpha_f = I_b / I_s$

*Centroid is located 175 mm from top of slab (Check!)*

$I_b = (300\times400^3/12) + (300\times400)\times25^2 + (200\times200^3/12) + (200\times200)\times75^2 = 2.033\times10^9 mm^4$

[*Or* $I_{beam} = K\,(b_w H^3/12) = 1.5\,(300\times400^3/12) = 2.4\times10^9 mm^4$]

slab section:




$I_s = 3150\times200^3/12 = 2.1\times10^9$ mm$^4$

$\alpha_f = I_b / I_s = 2.033 / 2.1 = 0.968$ *Ans.*

<u>**Example 2**</u>: A flat plate of square panels and clear spans = **6 m**. If $\boldsymbol{f_y}$ **= 560 MPa**
1. Estimate the min thickness for interior and exterior panels.
2. If drop panels according to ACI 8.4.2 are used, what will be the thicknesses?
3. If edge beams with $\alpha_f$ = 0.8 are used, what will be the thicknesses?

Solution: Table 8.3.1.1
1. int. $h = l_n / 30$ = 6000/30 = <u>200 mm</u> > 125 mm
Ext. $h = l_n / 27$ = 6000/27 = 223 mm use <u>225 mm</u> > 125 mm
2. int. $h = l_n / 33$ = 182 mm use <u>185 mm</u> > 100 mm
Ext $h = l_n / 30$ = <u>200 mm</u> > 100 mm
3. int. $h$ = ext. $h = l_n / 30$ = <u>200 mm</u> > 125 mm

## <u>2. Two Way Sabs with Interior Beams</u>:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

$\boldsymbol{\alpha_{fm}}$ **=** average value of $\boldsymbol{\alpha_f}$ for all beams on the edges of a given panel.

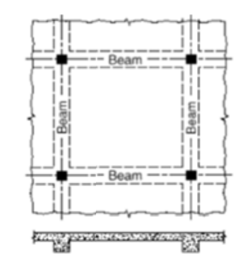
**Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides**

| $\alpha_{fm}$[1] | Minimum $h$, in. | | |
|---|---|---|---|
| $\alpha_{fm} \le 0.2$ | 8.3.1.1 applies | | (a) |
| $0.2 < \alpha_{fm} \le 2.0$ | Greater of: | $\dfrac{\ell_n\left(0.8+\dfrac{f_y}{200,000}\right)}{36+5\beta(\alpha_{fm}-0.2)}$ | (b)[2],[3] |
| | | 5.0 | (c) |
| $\alpha_{fm} > 2.0$ | Greater of: | $\dfrac{\ell_n\left(0.8+\dfrac{f_y}{200,000}\right)}{36+9\beta}$ | (d)[2],[3] |
| | | 3.5 | (e) |

[1]$\alpha_{fm}$ is the average value of $\alpha_f$ for all beams on edges of a panel and $\alpha_f$ shall be calculated in accordance with 8.10.2.7.

[2]$\ell_n$ is the clear span in the long direction, measured face-to-face of beams (in.).

[3]$\beta$ is the ratio of clear spans in long to short directions of slab.



(*b*) Two-way slab

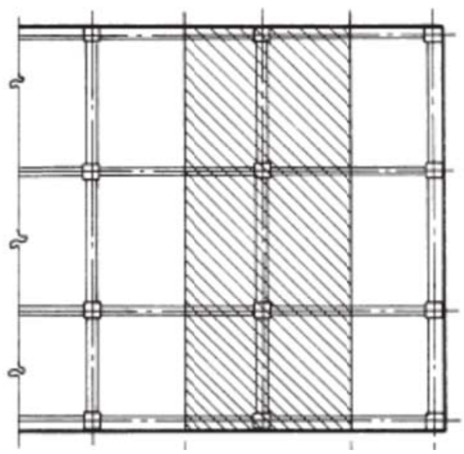
**Notes:**
1. Put **(1400)** instead of **(200,000)** and **($f_y$)** in **MPa** to get (**$h$)** in SI units (mm)
2. **5 in. = 125 mm,** and **3.5 in. = 90 mm**

<u>At discontinuous edge</u>, an edge beam must have $\boldsymbol{\alpha_f \ge 0.8}$, **otherwise increase min *h* by at least 10% in the panel with the discontinuous edge.**

**Example 3:** Thickness of TW slab with interior beams

Determine slab thickness for the following floor satisfactory to control deflections.
Given;
Edge beam dims: 350 × 650 mm
Interior beam dims: 350 × 500 mm
Column dims: 450 × 450 mm
Use preliminary slab thickness $h$ = 150 mm
c.c. spans in N-S direction are 5.50 m
c.c. spans in E-W direction are 6.50 m
$f'_c$ = 28 MPa, $f_y$ = 420 MPa

| 6.5m | 6.5m |

Solution:

1. Find $\alpha_f$ for beams

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s}$$

N-S edge beams:

$I_b = K\,(b_w H^3/12) = 1.5(350 \times 650^3)/12 = 1.2 \times 10^{10}\ \text{mm}^4$

$I_s = l\,h^3/12$ , $l = 6.5/2 + 0.35/2 = 3.425$ m

$= 3425\,(150)^3 / 12 = 9.63 \times 10^8\ \text{mm}^4$

$\alpha_f = I_b / I_s = 1.2 / 0.0963 =$ **12.46**

N-S interior beams:

$I_b = K\,(b_w H^3/12) = 2.0(350 \times 500^3)/12 = 7.29 \times 10^9\ \text{mm}^4$

$I_s = l\,h^3/12$ , $l = 6.5$m

$= 6500\,(150)^3 / 12 = 1.83 \times 10^9\ \text{mm}^4$

$\alpha_f = I_b / I_s = 7.29 / 1.83 =$ **3.98**

E-W edge beams:

$I_b = K\,(b_w H^3/12) = 1.5(350 \times 650^3)/12 = 1.2 \times 10^{10}\ \text{mm}^4$

$I_s = l\,h^3/12$ , $l = 5.5/2 + 0.35/2 = 2.925$ m

$= 2925\,(150)^3 / 12 = 8.23 \times 10^8\ \text{mm}^4$

$\alpha_f = I_b / I_s = 1.2 / 0.0823 =$ **14.48**

E-W interior beams:

$I_b = K\,(b_w H^3/12) = 2(350 \times 500^3)/12 = 7.29 \times 10^9\ \text{mm}^4$

$I_s = l\,h^3/12$ , $l = 5.5$m

$= 5500\,(150)^3 / 12 = 1.55 \times 10^9\ \text{mm}^4$

$\alpha_f = I_b / I_s$ = 7.29 / 1.55 = **4.70**

2. Find $\alpha_{fm}$ for panels

Corner Panel: $\alpha_{fm}$ = (12.46 + 3.98 + 14.48 + 4.7) / 4 = 8.905
Exterior Panel(N-S) : $\alpha_{fm}$ = (14.48 + 4.70 + 3.98 + 3.98) / 4 = 6.785
Exterior Panel(E-W) : $\alpha_{fm}$ = (12.46 + 3.98 + 4.7 + 4.7) / 4 = 6.46
Interior Panel: $\alpha_{fm}$ = (3.98 + 3.98 + 4.7 + 4.7) / 4 = 4.34

3. Find **β** for panels *(note: clear spans face to face of columns)*

Corner Panel: β = (6.5 – 0.45) / (5.5 – 0.45) = 6.05 / 5.05 = 1.198
Exterior Panel(N-S) : β = 1.198
Exterior Panel(E-W) : β = 1.198
Interior Panel: β = 1.198

4. Find min $h$ for panels

Since $\alpha_{fm}$ for all panels are > 2.0, ACI equation (d) controls:

$$h = l_n (0.8 + f_y / 1400) / (36 + 9\beta) \qquad \geq 90 \text{ mm}$$

$= l_n$ (0.8 + 420/1400) / (36 + 9×1.198)

$= l_n$ / 42.53 = 6150 / 42.53 = 145 mm < 150 mm used, OK

Use 150 mm slab thickness

*CW 1:*
*Remove the interior beams only and estimate the slab thickness. (Flat plate with edge beams)*
*CW 2:*
*Remove all the beams and estimate the slab thickness. (Flat Plate w/o edge beams)*
*HW:*
*For Ex.2, reduce dimensions of all beams to 250×400 mm, and estimate the minimum slab thickness. Take prelim h = 150 mm*

# Chapter 3: Direct Design Method of T-W Slab Systems

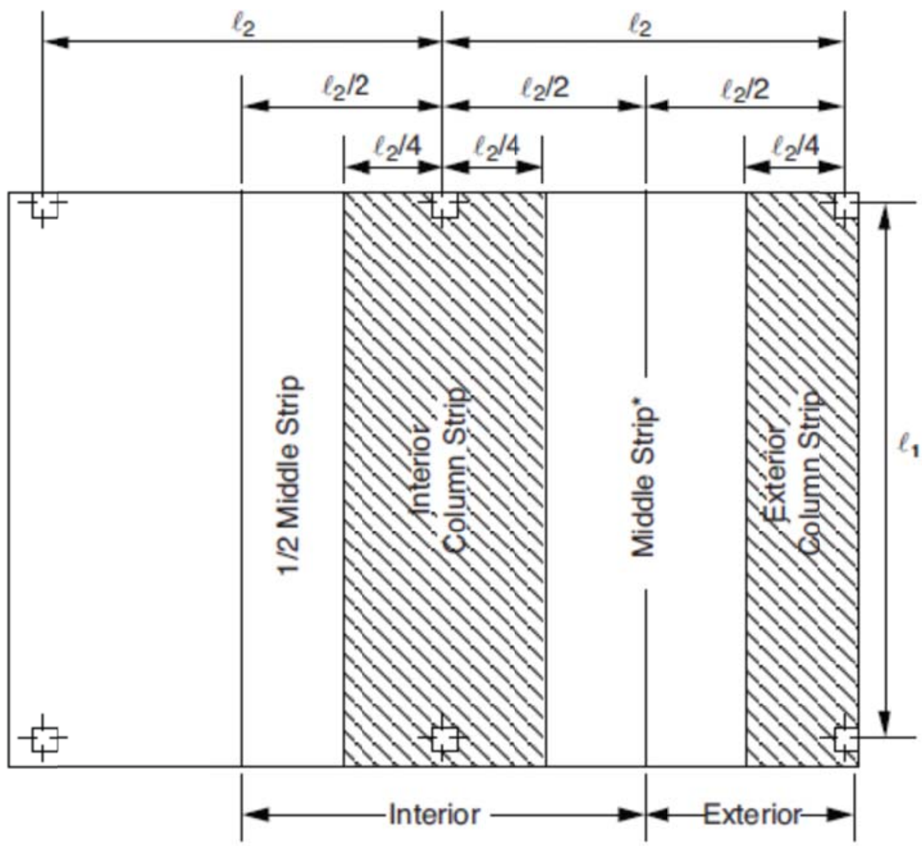
## Introduction

For purposes of analysis and design, ACI Code (318-19) chapter 8 deals in a unified way all the two-way slab systems: flat plates, flat slabs, slabs supported by beams, and two-way joist (ribbed) slabs.
Specific reference is made in textbooks to two alternative approaches:

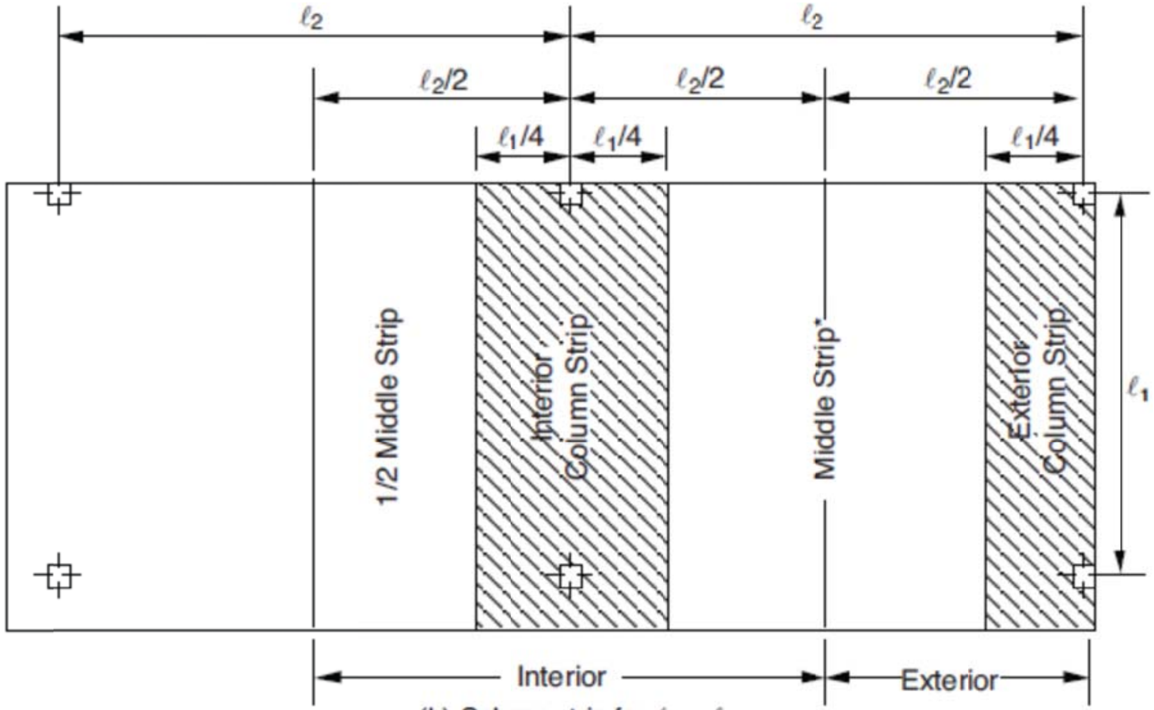
- *Direct Design Method* (DDM) ; a semi empirical method, and
- *Equivalent Frame Method* (EFM) based on approximate elastic analysis.

In either case, the slab is treated as follows:

A typical panel is divided into ***column strips*** and ***middle strips.***
**Column strip:** a strip of slab having width on each side of the column centerline equal to (**1/4**) the smallest of the panel dimensions $l_1$ and $l_2$.
**Middle strip:** a strip bounded by two column strips.
$l_1$ = span in the direction of moment analysis. (c.c. of supports)
$l_2$ = span in the lateral direction of $l_1$. (c.c. of supports)

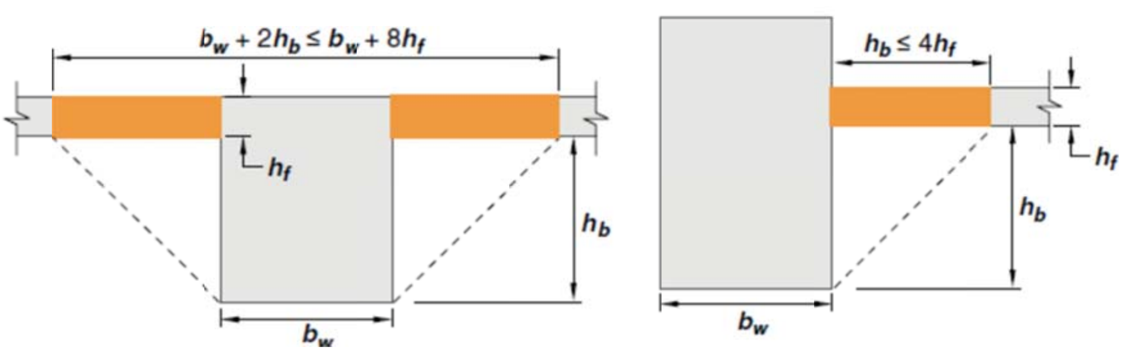


(a) Column strip for $\ell_2 \leq \ell_1$




(b) Column strip for $\ell_2 > \ell_1$

**Beams** are defined (as before) to include that part of the slab on each side of beam extending a distance equal to the projection of the beam above and below the slab $h_b$ (whichever is greater) but not greater than 4 times the slab thickness.




Symmetric beam(Interior beam) One-sided beam (Edge beam)

# Direct Design Method

## Limitations of the DDM:

1. There must be a minimum of three continuous spans in each direction. Thus, a nine panel structure (3 by 3) is the smallest that can be considered.
2. Rectangular panels must have a long-span/short-span ratio that is not greater than 2.
3. Successive span lengths in each direction shall not differ by more than one-third of the longer span.
4. Columns may be offset from the basic rectangular grid of the building by up to 10% of the span parallel to the offset.
5. All loads must be due to gravity only and uniformly distributed over an entire panel. The service live load shall not exceed two times the service dead load.
6. For a panel with beams between supports on all sides, the relative stiffness of the beams in the two perpendicular directions shall not be less than 0.2 or greater than 5:

$$\mathbf{0.2 \leq \{\alpha_{f1}\, l_2^2 \,/\, \alpha_{f2}\, l_1^2\} \leq 5}$$

Subscription 1 refers to direction 1

Subscription 2 refers to direction 2 (⊥ dir 1)

## DDM:

1. **Determine $M_o$, the total static bending moment in a panel**
2. **Moments assigned to critical sections**
3. **Lateral distribution of moments (*to cs, ms, and beams if any*)**
4. **Moments assigned to columns**
5. **Shear in slab systems with beams**

## 1. Determine $M_o$ in panels

The total static BM in a span, for a strip bounded laterally by the centerline of the panel on each side of the centerline of supports, is

$$M_o = q_u \, l_2 \, l_n^2 \, / \, 8$$

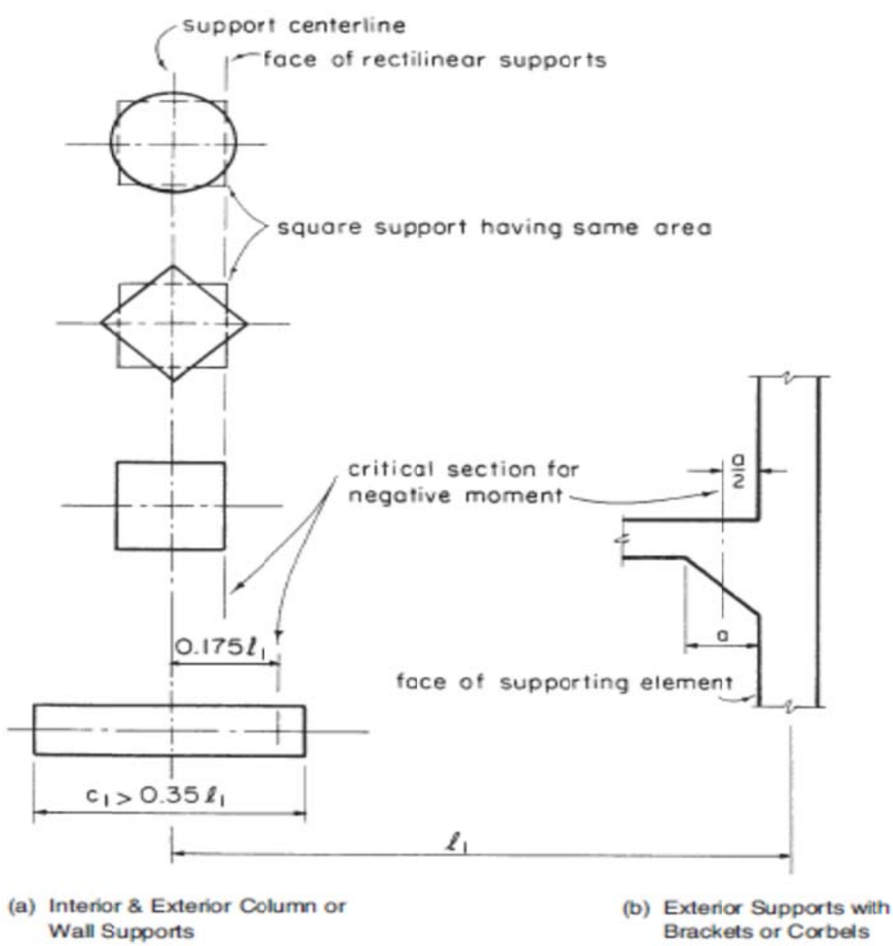
Where;

$q_u = 1.2\,D + 1.6\,L$ (kN/m$^2$)

$l_2$ = the span (centerline-to-centerline) transverse to $l_n$; however, when the span adjacent and parallel to an edge is being considered, the distance from edge of slab to panel centerline is used for $l_2$ in calculation of $M_o$

$l_n$ = clear span in the direction of analysis (dir. 1) face to face of supports.

$\geq 0.65\,l_1$ , *see fig. below:*

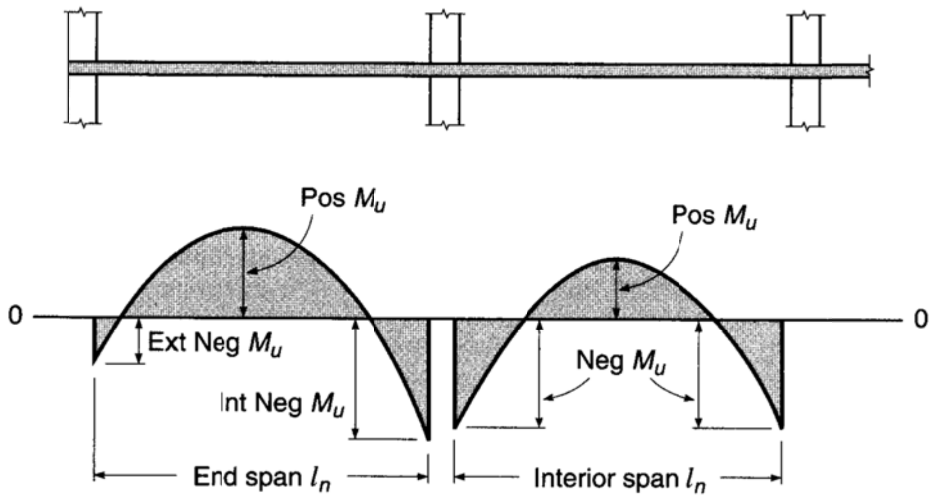



*This fig. shows critical sections for design negative moments, and defines the face of support.*

## 2. Moments assigned to critical sections

### a. For interior spans (ACI 8.10.4.1)

$$M^{-} = 0.65\, M_o$$
$$M^{+} = 0.35\, M_o$$



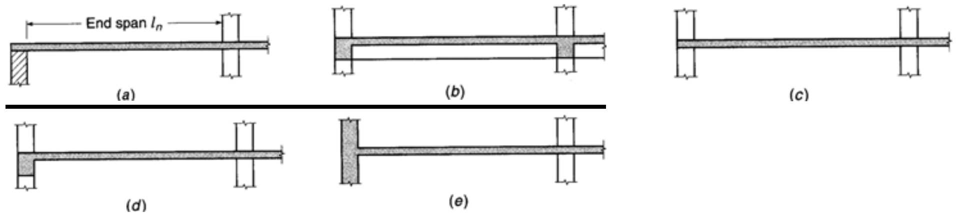

### b. For end spans

Moments are assigned according to **Table 8.10.4.2:**

$$M = (\textit{factor from Table}) \times M_o$$

**Table 8.10.4.2—Distribution coefficients for end spans**

| | Exterior edge unrestrained | Slab with beams between all supports | Slab without beams between interior supports: Without edge beam | Slab without beams between interior supports: With edge beam | Exterior edge fully restrained |
|---|---|---|---|---|---|
| Interior negative | 0.75 | 0.70 | 0.70 | 0.70 | 0.65 |
| Positive | 0.63 | 0.57 | 0.52 | 0.50 | 0.35 |
| Exterior negative | 0 | 0.16 | 0.26 | 0.30 | 0.65 |




a) Ext edge unrestrained; e.g., a masonry wall
b) Slab with beams between all supports
c) Slab w/o beams, i.e., flat plate
d) Flat plate with edge beams
**e)** Exterior edge fully restrained; e.g., a monolithic RC wall.

## 3. <u>Lateral Distribution of Moments</u> (*to cs, ms, and beams*)

### a. Interior M− :

**Table 8.10.5.1—Portion of interior negative $M_u$ in column strip**

| $\alpha_{f1}\ell_2/\ell_1$ | $\ell_2/\ell_1$ | | |
|---|---|---|---|
| | **0.5** | **1.0** | **2.0** |
| 0 | 0.75 | 0.75 | 0.75 |
| ≥1.0 | 0.90 | 0.75 | 0.45 |

Note: Linear interpolations shall be made between values shown.

### b. Exterior M− :

**Table 8.10.5.2—Portion of exterior negative $M_u$ in column strip**

| $\alpha_{f1}\ell_2/\ell_1$ | $\beta_t$ | $\ell_2/\ell_1$ | | |
|---|---|---|---|---|
| | | **0.5** | **1.0** | **2.0** |
| 0 | 0 | 1.0 | 1.0 | 1.0 |
| | ≥2.5 | 0.75 | 0.75 | 0.75 |
| ≥1.0 | 0 | 1.0 | 1.0 | 1.0 |
| | ≥2.5 | 0.90 | 0.75 | 0.45 |

Note: Linear interpolations shall be made between values shown. $\beta_t$ is calculated using Eq. (8.10.5.2a), where $C$ is calculated using Eq. (8.10.5.2b).

### c. Positive M+:

**Table 8.10.5.5—Portion of positive $M_u$ in column strip**

| $\alpha_{f1}\ell_2/\ell_1$ | $\ell_2/\ell_1$ | | |
|---|---|---|---|
| | **0.5** | **1.0** | **2.0** |
| 0 | 0.60 | 0.60 | 0.60 |
| ≥1.0 | 0.90 | 0.75 | 0.45 |

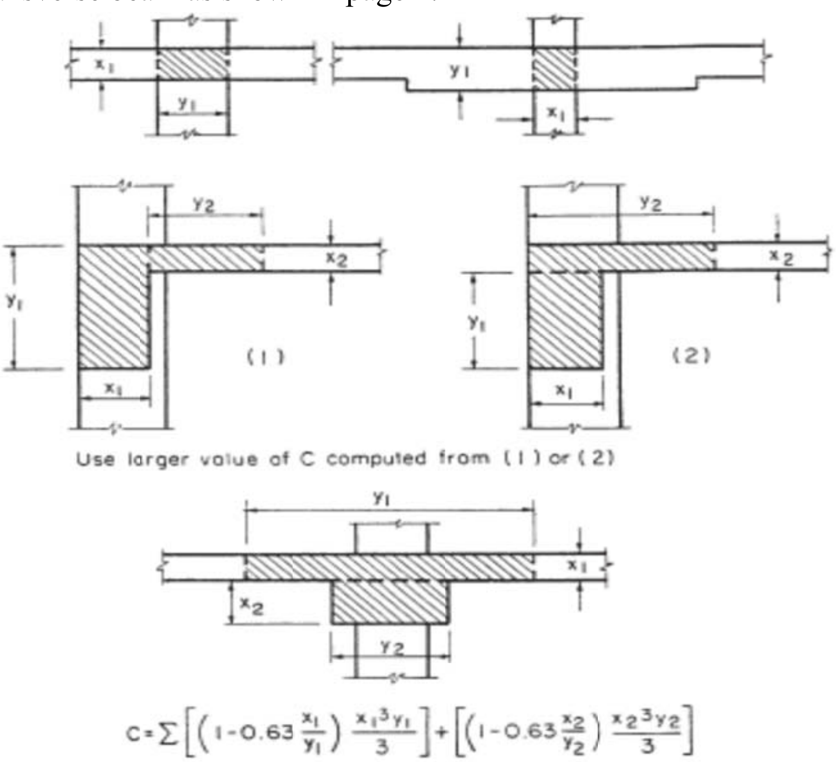
Note: Linear interpolations shall be made between values shown.

.**<u>Parameter $\beta_t$:</u>** The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter $\beta_t$**,** defined as

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad C = \sum\left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3y}{3}$$

$x$ = smaller dimension, $y$ = larger dimension.
Slab moment of inertia**,** $I_s = l_2\,h^3/12$
Constant **C** = torsional rigidity of beam effective cross section, taken as the largest of the following:

1. A portion of the slab having a width equal to that of the column, bracket, or capital in the direction in which moments are taken
2. The portion in 1 plus that part of any transverse beam above and below the slab
3. The transverse beam as shown in page 2.




*Cross-Sectional Constant C, Defining Torsional Properties of a Torsional Member*

$\beta_t = 0$ for flat plate w/o edge beams , and if support is a masonry wall.
$\beta_t = 2.5$ for RC wall.

***Two-half Middle strip*** shall take the moments not assigned to column strip. *An exception to this is a middle strip adjacent to and parallel with an edge supported by a wall, where the moment to be resisted is **twice** the factored moment assigned to the half middle strip corresponding to the first row of interior supports:*

$M_{ms\,(ext\ strip\ supp\ by\ wall)} = 2\,M_{ms\,(neighbor\ int\ strip)}$

**Moments to beams**:

When $(\alpha_{f1} l_2 / l_1) \geq 1.0$, $M_{beam} = 85\ \%\ M_{cs}$

When $0 \leq (\alpha_{f1} l_2 / l_1) \leq 1.0$, $0 \leq M_{beam} \leq 85\ \%\ M_{cs}$ ;
$M_{beam}$ is found by linear interpolation

In addition, the beam section must resist the effects of loads applied directly to the beam, including weight of beam stem projecting above or below the slab.

**Moments to columns and walls:**
Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.
*At interior supports*,

$$M_{sc} = 0.07[(q_{Du} + 0.5q_{Lu})\ell_2\ell_n^2 - q_{Du}'\ell_2'(\ell_n')^2] \qquad (8.10.7.2)$$

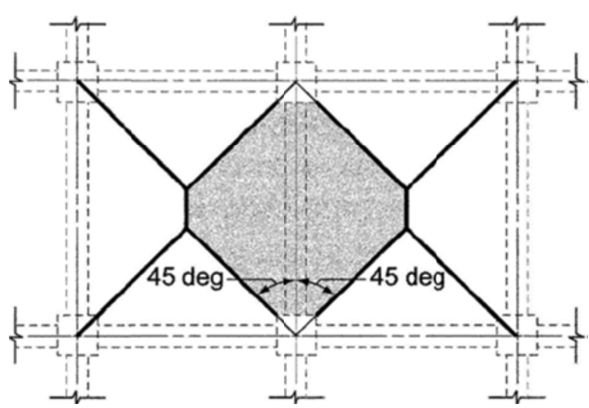
where $q_{Du}'$, $\ell_2'$, and $\ell_n'$ refer to the shorter span.

*At exterior column or wall supports*, the total exterior negative factored moment from the slab system (Table **8.10.4.2**) is transferred directly to the supporting members.

Columns above and below slab shall resist moments in direct proportion of their stiffness;

$M_{col\ above} = [K_{col\ above} / (K_{col\ above} + K_{col\ below})]\ M_{col}$ , where $K_{col} = 4\ EI_{col} / l_{col}$

**Shear in Slab systems with Beams**
Beams with $(\alpha_{f1} l_2 / l_1) \geq 1.0$, shall resist shear caused by factored loads on tributary areas, (*shown shaded in fig*.), which are bounded by 45$^o$ lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.

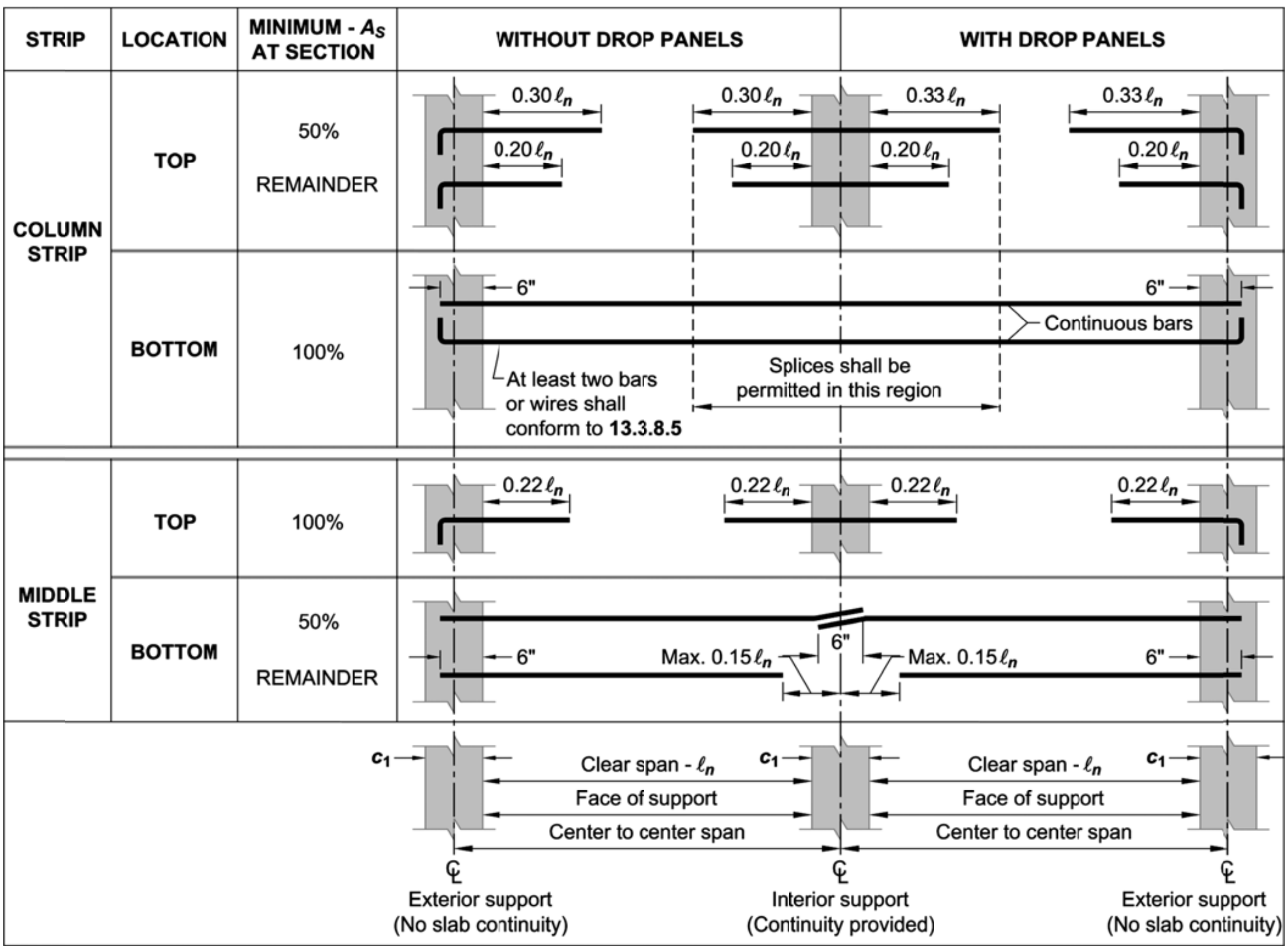



In addition, beams shall resist shears caused by factored loads applied directly on beams.

**Reinforcement for TW Slabs:** (ACI 8.7)

- Reinforcement in TW slabs is placed in an orthogonal grid, with bars parallel to the sides of the panels. This will cause the inner steel will have an effective depth 1 $d_b$ less than the outer steel.
  $d_{\text{inner steel}} = d_{\text{outer steel}} - d_b$
  For TW slabs with beams, short dir bars have the larger ***d***.
  For beamless TW slabs, long dir bars have the larger ***d***.
- Straight bars are generally used, although in some cases *+veM* steel is bent up where no longer needed, to provide for part or all the *–veM* steel.
- Max bar spacing = **2 *h*** (*to ensure crack control*).
- Min cover = **20 mm**
- $A_{s,min}$ (*for shrinkage and temperature crack control*) = **0.0018 *bh***

- Bars cutoff, see figure below for beamless slabs:



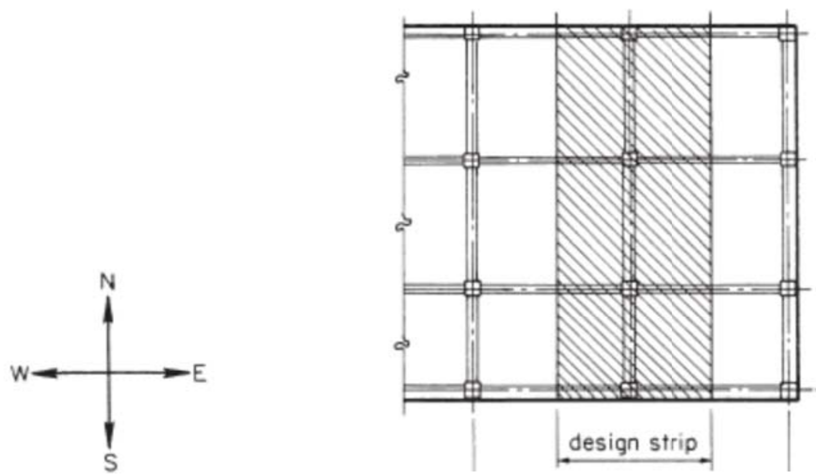
## Example: Direct Design Method; Slab with beams

Use the DDM to determine the design moments for the slab system in the NS direction (*shaded design strip*) in an itermediate floor.
Given;
Edge beam dims: 350 × 650 mm
Interior beam dims: 350 × 500 mm
Column dims: 450 × 450 mm
Slab thickness $h$ = 150 mm
c.c. spans in N-S direction are 5.50 m
c.c. spans in E-W direction are 6.50 m
$f'_c$ = 28 MPa, $f_y$ = 420 MPa
Service live load $L$ = 5 kN/m$^2$

## Solution:

***Check slab thickness*** $h$ = 150 mm is satisfactory for deflection control, see example in chapter 2.

| 6.5m | 6.5m |




***Check the limitations of the DDM***: (*see page 3*)
Limitations 1, 2,3,4,5 are readily satisfactory
Check the ratio $\mathbf{0.2 \le \{\alpha_{f1} l_2^2 / \alpha_{f2} l_1^2\} \le 5}$ for panels
*Calculate $\alpha_f$ for beams, see example in chapter 2,*
Corner panel: $(12.46+3.98)6.5^2 / (14.48+4.7)5.5^2 = 1.2$, $\mathbf{0.2} < 1.2 < \mathbf{5}$ OK
Exterior panel NS; $(3.98+3.98)6.5^2 / (14.48+4.7)5.5^2 = 0.58$ OK
Exterior panel EW; $(12.46+3.98)6.5^2 / (4.7+4.7)5.5^2 = 2.44$ OK
Interior panel; $(3.98+3.98)6.5^2 / (4.7+4.7)5.5^2 = 1.18$ OK
Therefore, DDM is applicable.

***Factored loads:***
Average wt. of beam stem = (0.35×0.35/6.5)(24 kN/m$^3$) = 0.45 kN/m$^2$
wt. of slab = (0.15)(24 kN/m$^3$) = 3.6 kN/m$^2$
Total $D$ = 0.45 + 3.6 = 4.05 kN/m$^2$

Factored load, $w_u$ = 1.2(4.05) + 1.6(5) = 12.86 kN/m$^2$

***Total static BM, $M_o$:***

$l_n$ = 5.5 – 0.45 = 5.05 m *(face to face of columns)*

$M_o = w_u l_2 l_n^2 / 8$ = 12.86×6.5×5.05$^2$ / 8 = **266.47 kNm**

***Moments at critical sections:***

**Interior spans**

Interior $M^- = 0.65 M_o$ = 0.65×266.47 = 173.21 kNm

$M^+ = 0.35 M_o$ = 0.35×266.47 = 93.26 kNm

**End span**

Exterior $M^- = 0.16 M_o$ = 0.16×266.47 = 42.64 kNm

$M^+ = 0.57 M_o$ = 0.57×266.47 = 151.89 kNm

Interior $M^- = 0.70 M_o$ = 0.70×266.47 = 186.53 kNm

***Disribution to col strip, mid strip and beams***

$l_2 / l_1$ = 6.5 / 5.5 = 1.18, $(\alpha_{f1} l_2 / l_1)$ = 3.98×6.5/5.5 = 4.7 ≥ 1.0

***Interior negative moments***

% to col strip = 75 – [(1.18 – 1)/(2 – 1)] (75 – 45) = 69.6%

Interior spans $M_{cs}^-$ = 69.6% (173.21) = 120.55 kNm

End span, Interior $M_{cs}^-$ = 69.6% (186.53) = 129.82 kNm

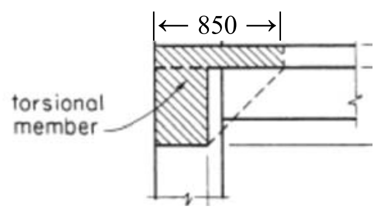
***Positive moments***

% to col strip = 75 – [(1.18 – 1)/(2 – 1)] (75 – 45) = 69.6%

Interior spans $M_{cs}^+$ = 69.6% (93.26) = 64.91 kNm

End span $M_{cs}^+$ = 69.6% (151.89) = 105.72 kNm

***Exterior negative moments***




Edge Beam dims: $b_w$ = 350, $H$ = 650, $h_f$ =150, then flange $b = b_w + (H - h_f)$ = 350 + (650 – 150) = 850 mm

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3}$$

$C_1$ = (1 – 0.63×350/500)350$^3$×500/3 + (1 – 0.63×150/850)150$^3$×850/3
= 3.995×10$^9$ + 8.50×10$^8$ = 4.845×10$^9$ mm$^3$

$C_2$ = (1 – 0.63×350/650)350$^3$×650/3 + (1 – 0.63×150/500)150$^3$×500/3
= 6.138×10$^9$ + 4.56×10$^8$ = <u>6.594×10$^9$ mm$^3$</u> Use (larger C)

$I_s = l_2 h^3 /12$ = 6500(150)$^3$/ 12 = 1.828×10$^9$ mm$^3$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$$

$= C / 2I_s$ = 6.594 / (2×1.828) = <u>1.80</u>

1st interpolation @βt =0 100%
3rd interpolation @βt =1.8 **78%** ← 69.6 + (100 – 69.6)(2.5 – 1.8)/(2.5 – 0) =78%
2nd interpolation @βt =2.5 69.6%
End span, exterior $M_{cs}^{-}$ = 78% (42.64) = 33.26 kNm

## ***Summary, and moments to mid strip and beams***

| | $M_u$ (kNm) | % to cs | $M_{cs}$ (kNm) * | 2-half ms $M_{ms}$ (kNm) |
|---|---|---|---|---|
| *End Span* | | | | |
| Ext. negative | 42.64 | 78 | 33.26 | 9.38 |
| Positive | 151.89 | 69.6 | 105.72 | 46.17 |
| Int. negative | 186.53 | 69.6 | 129.82 | 56.71 |
| *Interior Span* | | | | |
| Negative | 173.21 | 69.6 | 120.55 | 52.66 |
| Positive | 93.26 | 69.6 | 64.91 | 28.35 |

* Beams shall take 85% of $M_{cs}$ [ $(\alpha_{f1} l_2 / l_1)$ > 1.0], and slab $M_{cs}$ shall take 15% of it.

## ***Slab Reinforcement***

### ***Column strip:***

| | Slab$M_{cs}$ (kNm) 15% of $M_{cs}$ * | $R = M_u / \varphi b d^2$ ** | ρ Table A.5 | $A_s$ mm²/m*** |
|---|---|---|---|---|
| *End Span* | | | | |
| Ext. negative | 15%(33.26)= 4.99 | 0.129 | 0.0005 | ~~63~~ /use 270 |
| Positive | 15.86 | 0.410 | 0.0010 | ~~125~~ /use 270 |
| Int. negative | 19.47 | 0.503 | 0.0012 | ~~150~~ /use 270 |
| *Interior Span* | | | | |
| Negative | 18.08 | 0.468 | 0.0011 | ~~138~~ /use 270 |
| Positive | 9.74 | 0.252 | 0.0006 | ~~75~~ /use 270 |

* Beams shall take 85% of $M_{cs}$ [ $(\alpha_{f1} l_2 / l_1)$ > 1.0], and slab $M_{cs}$ shall take 15% of it.
** $b_{cs} = l_1/2 = 2750$ mm, $d = 125$ mm
*** $A_s = \rho b d$ , $A_{s\,min} = 0.0018bh$ =270 mm²/m, use No.13@300 (430 mm²/m)everywhere
max spacing = $2h$ = 2(150) = 300 mm OK

### ***Middle strip:***

| | 2-half ms $M_{ms}$ (kNm) | $R = M_u / \varphi b d^2$ * | ρ Table A.5 | $A_s$ mm²/m** |
|---|---|---|---|---|
| *End Span* | | | | |
| Ext. negative | 9.38 | 0.178 | 0.0005 | ~~63~~ /use 270 |
| Positive | 46.17 | 0.875 | 0.0021 | ~~263~~/use 270 |
| Int. negative | 56.71 | 1.075 | 0.0026 | 325 |
| *Interior Span* | | | | |
| Negative | 52.66 | 0.999 | 0.0024 | 300 |
| Positive | 28.35 | 0.538 | 0.0013 | ~~163~~ /use 270 |

* $b_{ms} = l_2 - b_{cs} = 6500 - 2750 = 3750$ mm, $d = 125$ mm
** $A_s = \rho b d$ , $A_{s\,min} = 0.0018\,bh$ =270 mm²/m, use No.13@300 (430 mm²/m)everywhere
max spacing = $2h$ = 2(150) = 300 mm

## ***Moments to columns***

***Interior columns***

***Int.*** $M_{col} = 0.07[(q_{Du} + 0.5\times q_{Lu})l_2 l_n^2 - 1.2\, q_{Du}{}'l'_2(l'_n)^2]$

$= 0.07(0.5\times q_{Lu})l_2 l_n^2$ *(adjacent spans are equal)*

$= 0.07(0.5\times 1.6\times 5)6.5(5.05)^2 = 46.41$ kNm

***At exterior column***, *the total exterior negative factored moment from the slab system (Table **8.10.4.2**) is transferred directly to the column:*

***Ext.*** $M_{col} = 0.16\, M_o = 0.16\times 266.47 = 42.64$ kNm

Columns above and below slab shall resist moments in proportion of their stiffnesses;

$M_{\text{col above}} = [K_{\text{col above}} / (K_{\text{col above}} + K_{\text{col below}})]\, M_{col}$

$M_{\text{col above}} = M_{\text{col below}} = (1/2)\, M_{col}$ (*columns above and below are identical*)

$= (1/2)(46.41) = 23.21$ kNm for interior columns

$= (1/2)(42.64) = 21.32$ kNm for exterior columns

## ***Shear in Beams*** (*only interior beams are checked because the carry higher shear than edge beams*)

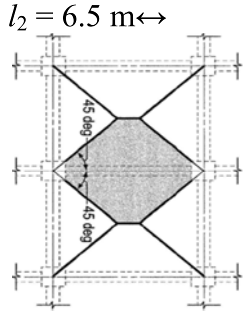
NS beams (short dir.): $V_u = w_u\,[(1/2)l_1 l_1/2] = w_u\,[(1/4)l_1^2]$

$= 12.86[(1/4)5.5^2] = 97.3$ kN

EW beams (long dir.): $V_u = w_u\,[(1/4)l_1^2 + (1/2)l_1(l_2 - l_1)]$

$= 12.86[(1/4)5.5^2 + (1/2)5.5(6.5 - 5.5)] = 132.6$ kN

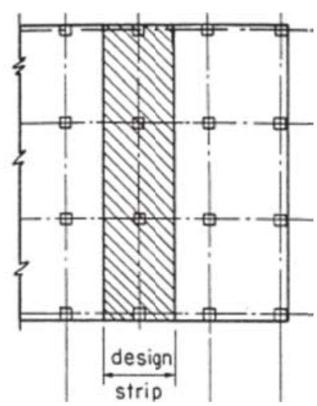
$l_1 = 5.5$ m ↕ $l_2 = 6.5$ m↔




## *HW: DDM of Flat Plate Floor*

Use the DDM to determine the design moments for the flat plate slab system in the NS direction (*shaded design strip*) in an itermediate floor. Given;

No Edge beams, column dims: 400 × 400 mm, c.c. spans in N-S direction are 5.50 m, c.c. spans in E-W direction are 4.25 m, $f'_c = 28$ MPa, $f_y = 420$ MPa, Service live load $L = 2$ kN/m$^2$, Partition weight = 1 kN/m$^2$ ,Story height = 3.5 m

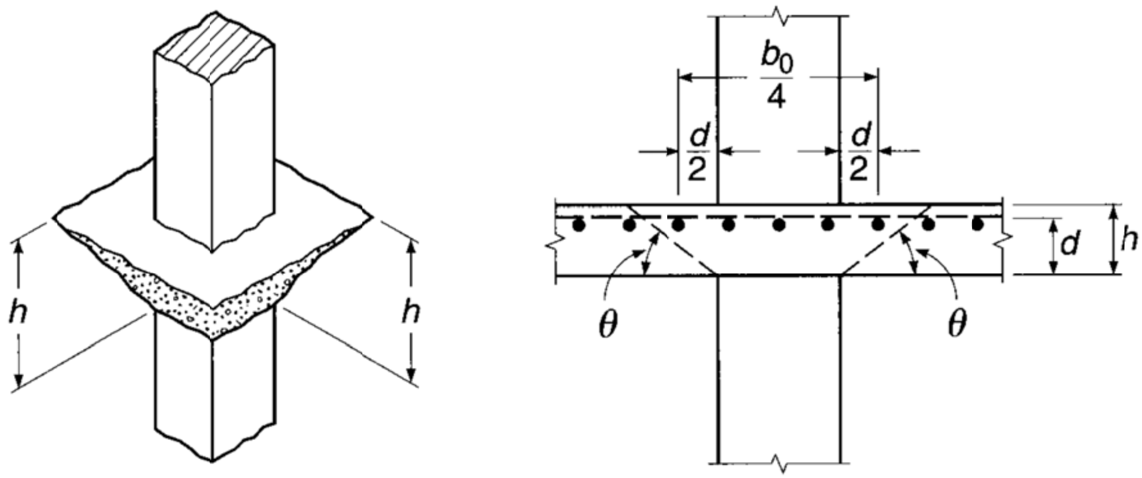


# Chapter 4: Shear Strength of TW Slabs w/o Beams

## TW Shear of Slabs

When TW slabs are supported directly by columns, as in *flat slabs* and *flat plates*, or when slabs carry concentrated loads, as in *footings*, shear near columns is of critical importance.

Failure may occur by ***punching shear***, with the diagonal crack following the surface of a truncated cone or pyramid around the column, capital, or drop panel as shown below (fig. a).




(a) (b)

The failure surface extends from the bottom of slab, at the support, diagonally upward to the top surface. The angle of inclination with the horizontal *θ, fig. b,* depends upon the nature and amount of reinforcement. It may range between 20 and 45°.

## Critical section for TW shear:

Critical section for shear is taken perpendicular to the plane of the slab and a distance ***d*/2** from the periphery of the support, as shown in *fig. b.* The perimeter of the critical section is $b_o$ . The perimeter $b_o$ is located at distance ***d*/2** from edges or corners of columns, concentrated loads, or reactions, or from changes in slab thickness such as edges of capitals, drop panels, or shear caps (*ACI 22.6.4*); see figure;

## TW Shear Force $V_u$:

TW shear force $V_u$ can be calculated as the total factored load on the area bounded by panel centerlines around the column less the load applied within the area defined by the critical shear perimeter, *unless significant moments must be transferred from the slab to the column.*

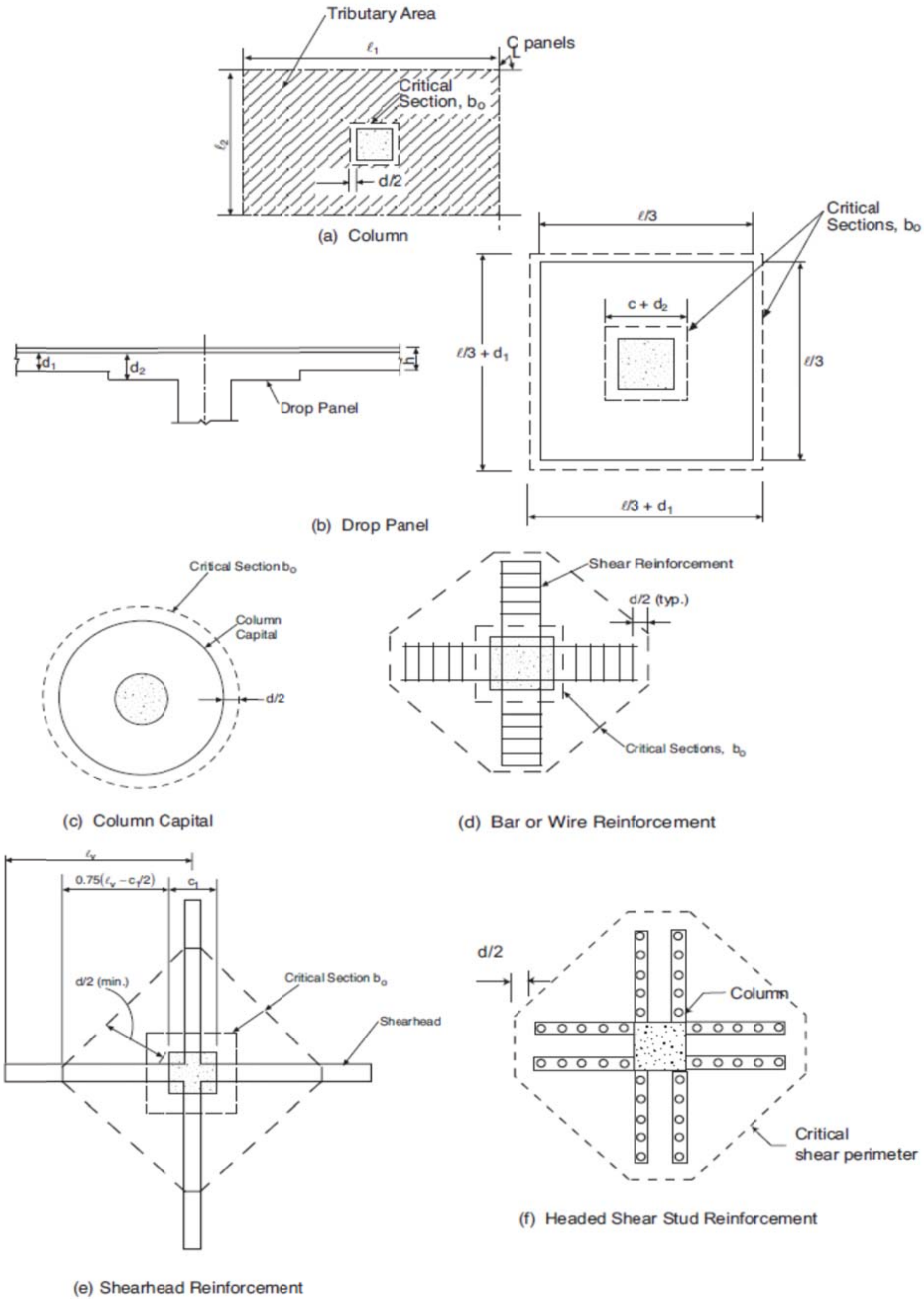



***Fig. : Critical section for TW shear***

## TW Shear strength, $V_c$

The TW shear strength (*stress in psi, $f_c$' in psi*) shall be the smallest of the following three equations:

**Table 22.6.5.2—$v_c$ for two-way members without shear reinforcement**

| $v_c$ | | |
|---|---|---|
| Least of (a), (b), and (c): | $4\lambda_s\lambda\sqrt{f'_c}$ | (a) |
| | $\left(2+\frac{4}{\beta}\right)\lambda_s\lambda\sqrt{f'_c}$ | (b) |
| | $\left(2+\frac{\alpha_s d}{b_o}\right)\lambda_s\lambda\sqrt{f'_c}$ | (c) |

Notes:
(i) $\lambda_s$ is the size effect factor given in 22.5.5.1.3.
(ii) $\beta$ is the ratio of long to short sides of the column, concentrated load, or reaction area.
(iii) $\alpha_s$ is given in 22.6.5.3.

The TW shear strength (*force in N, $f_c$' in MPa*) shall be the smallest of the following three equations:

$$V_c = (1/3)\ [\lambda_s \lambda \sqrt{f'_c}\ b_o d\ ] \qquad \textbf{(ACI 22.6.5.2a)}$$

$$V_c = (1/12)\ [\ (2 + 4/\beta)\ \lambda_s \lambda \sqrt{f'_c}\ b_o d\ ] \qquad \textbf{(ACI 22.6.5.2b)}$$

Where $\beta$ = long side / short side of the column
$\lambda$ = lightweight concrete factor = **1.0** for normal weight concrete
= **0.75** for lightweight concrete.

$$V_c = (1/12)\ [(2 + \alpha_s d / b_o)\ \lambda_s \lambda \sqrt{f'_c}\ b_o d\ ] \qquad \textbf{(ACI 22.6.5.2c)}$$

**Where,** $\alpha_s$ = **40** for interior column, **(ACI 22.6.5.3)**
= **30** for edge column, and
= **20** for corner column.

$\lambda_s$ = size effect factor

$$\lambda_s = \sqrt{\frac{2}{1+0.004 \cdot d}} \le 1.0$$

***Notes:***

For $\beta \le 2.0$, eq. (**a**) governs. For an interior column with $b_o/d \le 20$, eq. (**a**) governs

## Shear Strength Requirements

$$\varphi V_n = V_u = \varphi\,(V_c + V_s) \quad , \quad \varphi = 0.75 \qquad \textbf{(ACI 22.6)}$$

- **When $V_u \le \varphi\, V_c$ OK, no shear reinforcement is required.**
- **When $V_u > \varphi\, V_c$ N.G.**

Shear strength may be increased by:
i. increasing concrete strength $f_c'$
ii. increasing slab thickness at column support, i.e., using a drop panel
iii. providing shear reinforcement

**If shear reinforcement is required:**

**In this case, reduce $V_c$ to:**

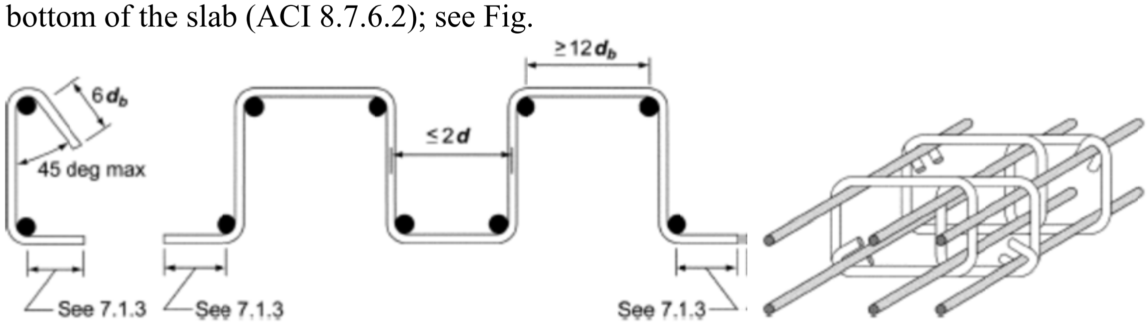
$$V_c = (1/6)\,[\,\lambda \sqrt{f'_c}\, b_o\, d\,]$$

**and max nominal strength:**

$$\varphi\,(V_c + V_s) \le (1/2)\,[\,\lambda \sqrt{f'_c}\, b_o\, d\,]$$

## TW Shear reinforcement in slabs

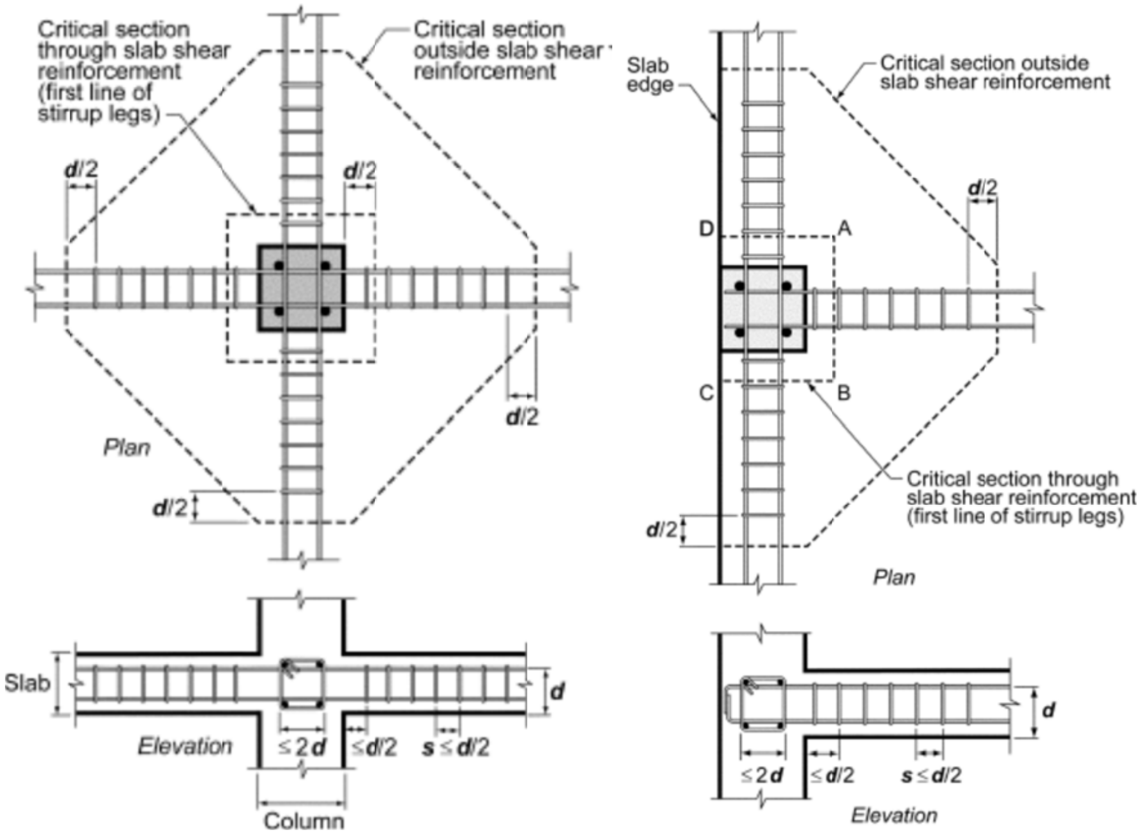
The use of bars, wires, or single or multiple-leg stirrups as shear reinforcement in slabs is permitted provided that $d \ge$ **150 mm**, and **16 $d_b$.**
Suggested rebar shear reinforcement consist of properly anchored single-leg, multiple-leg, or closed stirrups that are engaging longitudinal reinforcement at both the top and bottom of the slab (ACI 8.7.6.2); see Fig.




a)Single-leg b) multiple-leg stirrup or bar c) closed stirrups

**Arrangement of stirrup shear reinforcement:**

a) **Interior column** b) **edge column**




**Calculation of the area of shear reinforcement:**

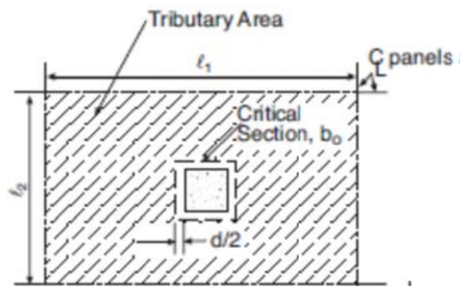
$$A_v = V_s\, s / f_y\, d$$

$A_v$ = cross sec area of all the legs of shear reinforcement
Spacing limits are shown in figs above.
The shear reinforcement can be terminated when $V_u \le \varphi\,(1/6)\,\lambda\,\sqrt{f_c'}\,b_o\,d$ (ACI 22.6.6-7).

## Example 4-1:

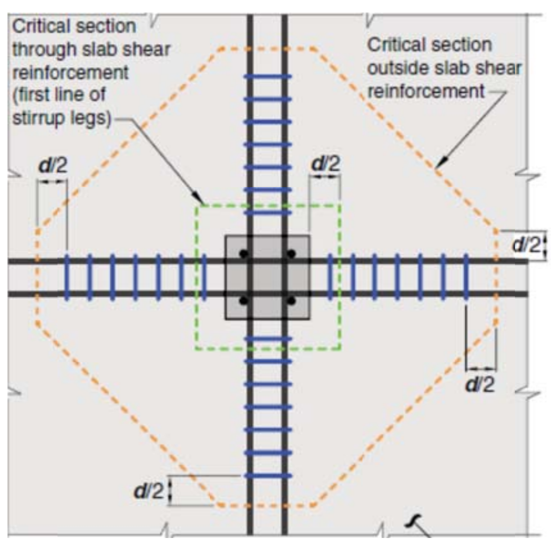
Consider an interior panel of a flat plate slab system supported by a **300 mm** square column. Panel size $l_1 = l_2 = 6.5$ **m**. Determine shear strength of slab at column support, and if not adequate, increase the shear strength by considering closed stirrups option. Overall slab thickness $h$ = 190 mm ($d$ = **150 mm**). $f_c'$ = **28 MPa**, normal weight concrete, $f_y$ = **420 MPa**.
Live load = **5 kN/m**$^2$

**Solution:**




$q_u$ = 1.2×(0.19×24) + 1.6×5 = 5.5 + 8 = 13.5 kN/m$^2$
$c + d$ = 300 + 150 = 450 mm
$V_u = q_u$ **[tributary area]** = 13.5[6.5×6.5 – 0.45×0.45] = 567.64 kN
$b_o$ = 4(450) = 1800 mm
$\beta$ = 300 / 300 = 1 < 2.0
$b_o / d$ = 1800 / 150 = 12 < 20, equation *c* governs.
$\varphi V_c = \varphi(1/3)[\lambda\sqrt{f'_c} b_o d]$ = 0.75(1/3)[1√28×1800×150]= 357.18 kN< $V_u$ =567.64 kN N.G.

*Determine distance from sides of column where stirrups may be terminated (distance a)*
$\varphi V_c = \varphi(1/6)[\lambda\sqrt{f'_c} b_o d]$ = 178.6 kN

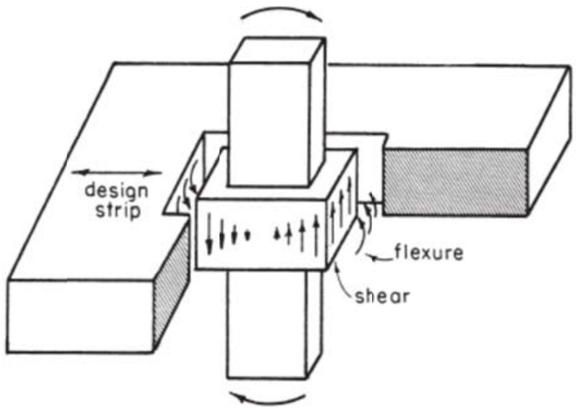



|← *a* →|*300mm*|
↔ ***Col side***

$V_u \le \varphi V_c = \varphi(1/6)[\lambda\sqrt{f'_c} b_o d]$
For square column: $b_o$ **= 4 (col side +** $a\sqrt{2}$ **)**
567.64 ×10$^3$ = 0.75 (1/6)×√28× 4 (300 + $a$ √2 ) × 150
Solving, $a$ = 799.3 mm, say $a$ = 800 mm
Stirrups are terminated at $d/2$ = 75 mm inside the critical perimeter $b_o$.
Trying $s$ = 75 mm;
***One-leg*** $A_v = (1/8) V_s\, s / f_y\, d = (V_u - \varphi V_c)\; s / 8 f_y\, d$
= (567.64 – 178.6)10$^3$(75) / [8×420×150] = 58 mm$^2$; Use No.10 ($A_v$ = 71 mm$^2$)
(Use 10 No.10 stirrups @ 75mm along each arm/column line, with 4 No. 10 longitudinal bars at corners).

## Effects of Moment Transfer

Unbalanced moment $M_u$ can occur at the slab-column connections. For slabs without beams between supports, shear strength at an exterior slab-column connection (without spandrel beam) is especially critical, because the total exterior negative moment must be transferred to the column, which is in addition to the direct shear due to gravity loads; see Fig.




$M_u$ at connection is transferred to column by:

1. Flexure; $\gamma_f M_u$
2. Shear; $\gamma_v M_u$

i,e,

$M_u = \gamma_f M_u + \gamma_v M_u$

- The fraction of unbalanced moment transferred by flexure $\gamma_f$ is:

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

*ACI Eq(8.4.2.2.2)*

$\gamma_f M_u$ is transferred within an effective width = **1.5 *h*** on each side of column, outside opposite faces of the column or capital. (***h*** = slab thickness including drop panel if any):

**Table 8.4.2.2.3—Dimensional limits for effective slab width**

| | Distance on each side of column or capital | |
|---|---|---|
| Without drop panel or shear cap | Lesser | 1.5*h* of slab |
| | | Distance to edge of slab |
| With drop panel or shear cap | Lesser | 1.5*h* of drop or cap |
| | | Distance to edge of the drop or cap plus 1.5*h* of slab |

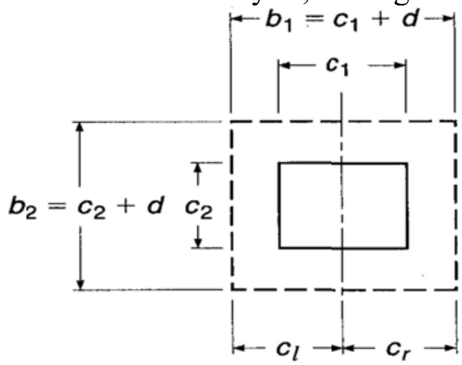
- The fraction of unbalanced moment transferred by eccentricity of shear is:

$$\gamma_v = 1 - \gamma_f$$

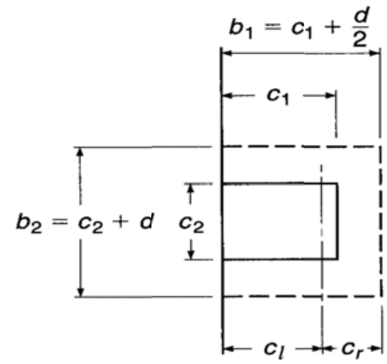
*ACI Eq. (8.4.4.2.2)*

$M_u$ **= 0.30** $M_o$ ***(if DDM is used)***
$M_u$ ***= computed frame moment (if EFM is used)***

where $b_1$ and $b_2$ are the dimensions of the perimeter of the critical section, with $b_1$ parallel to the direction of analysis; see Fig.

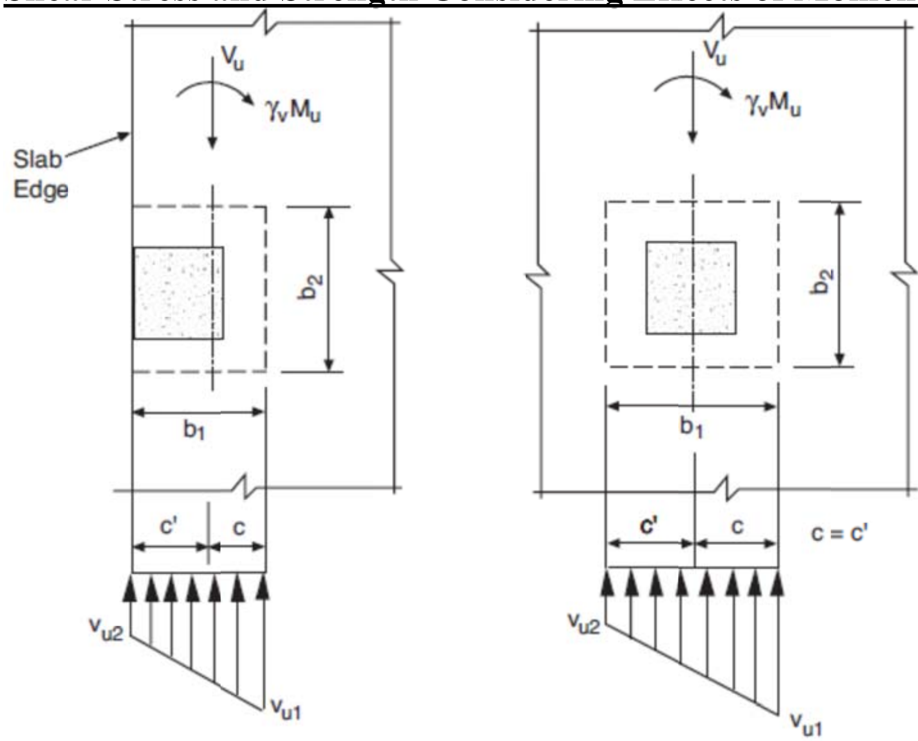


Critial section for interior column



Critial section for edge column

## Shear Stress and Strength Considering Effects of Moment Transfer



(a) Edge Column (b) Interior Column

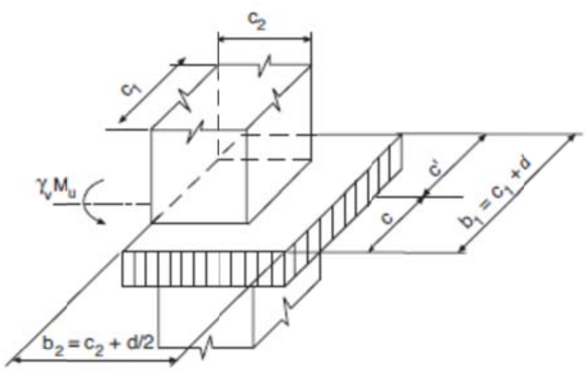
$$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$

$$v_{u2} = \frac{V_u}{A_c} - \frac{\gamma_v M_u c'}{J}$$

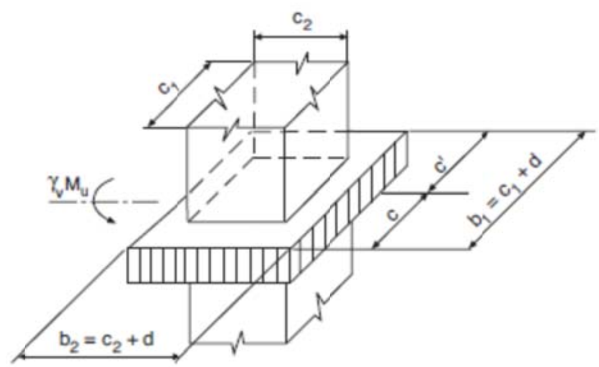
where: $A_c$ = area of concrete section resisting shear transfer, $A_c = b_o d$
$J$ = property of critical section analogous to polar moment of inertia of segments forming area $A_c$.
$c$ and $c'$ = distances from centroidal axis of critical section to the perimeter of the critical section in the direction of analysis.

## Expressions for $A_c$, $c$, $c'$, $J/c$, and $J/c'$ ,
## *For Rectangular columns;*

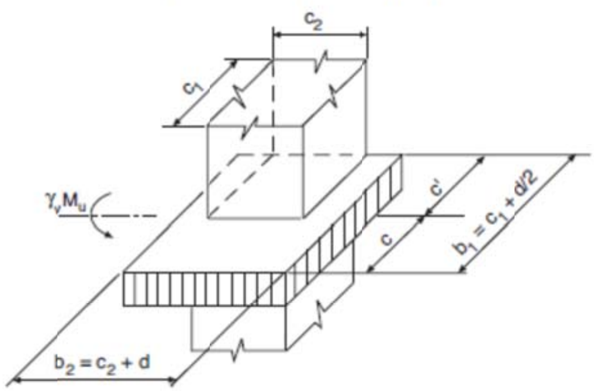
**Case A:** Edge Column (Bending parallel to edge)



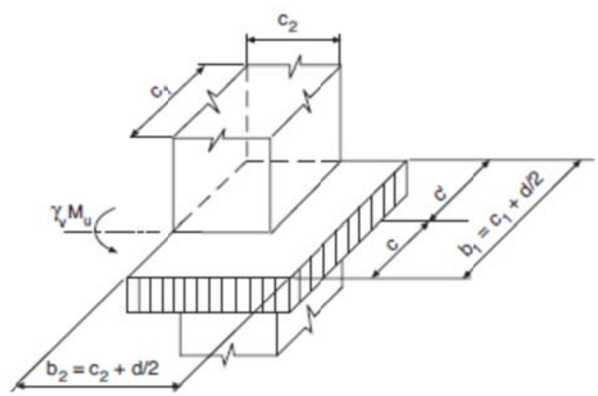

**Case B:** Interior Column




**Case C:** Edge Column (Bending perpendicular to edge)




**Case D:** Corner Column




| Case | Area of critical section, $A_c$ | Modulus of critical section | | c | c' |
|---|---|---|---|---|---|
| | | J/c | J/c' | | |
| A | $(b_1+2b_2)d$ | $\frac{b_1d(b_1+6b_2)+d^3}{6}$ | $\frac{b_1d(b_1+6b_2)+d^3}{6}$ | $\frac{b_1}{2}$ | $\frac{b_1}{2}$ |
| B | $2(b_1+b_2)d$ | $\frac{b_1d(b_1+3b_2)+d^3}{3}$ | $\frac{b_1d(b_1+3b_2)+d^3}{3}$ | $\frac{b_1}{2}$ | $\frac{b_1}{2}$ |
| C | $(2b_1+b_2)d$ | $\frac{2b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6b_1}$ | $\frac{2b_1^2d(b_1+2b_2)+d^3(2b_1+b_2)}{6(b_1+b_2)}$ | $\frac{b_1^2}{2b_1+b_2}$ | $\frac{b_1(b_1+b_2)}{2b_1+b_2}$ |
| D | $(b_1+b_2)d$ | $\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6b_1}$ | $\frac{b_1^2d(b_1+4b_2)+d^3(b_1+b_2)}{6(b_1+2b_2)}$ | $\frac{b_1^2}{2(b_1+b_2)}$ | $\frac{b_1(b_1+2b_2)}{2(b_1+b_2)}$ |

## Example 4-2: Shear Strength of Slab with Transfer of Moment

Consider an exterior (edge) panel of a flat plate slab system supported by a **400-mm** square column. Determine shear strength for transfer of direct shear and moment between slab and column support. Overall slab thickness
**$h$ = 180 mm**. (**$d \approx 150$ mm**). Assume that the Direct Design Method is used for analysis of the slab. Consider
**$V_u$ = 260 kN, $M_o$ = 230 kNm, $f_c'$ = 28 MPa, $f_y$ = 420 MPa.**

**Solution:**

Edge column bending perpendicular to edge (Case C),
$b_1 = c_1 + d/2 = 400 + 75 = 475$ mm
$b_2 = c_2 + d = 400 + 150 = 550$ mm
$b_o = 2\,(475) + 550 = 1500$ mm
$c = b_1^2 / (2b_1 + b_2) = 150.4$ mm
$A_c = (2b_1 + b_2)\, d = 225{,}000$ mm2
$J/c = [2b_1^2 d(b_1 + 2b_2) + d^3(2b_1 + b_2)] / 6b_1 = 3.918\times10^7$ mm$^3$
$\gamma_f = 1/[1 + (2/3)\sqrt{(b_1/b_2)}] = 0.635$
$\gamma_v = 1 - \gamma_f = 0.365$
Unbalanced $M_u = 0.3\, M_o = 69$ kNm

$$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$

$= 260\times10^3/225000 + 0.365(69\times10^6)/3.918\times10^7$
$= 1.156 + 0.643 = 1.799$ MPa

**$\varphi v_c = \varphi(1/3)\sqrt{f'_c}$** $= 0.75(1/3)\sqrt{28} = 1.323$ MPa < 1.799 MPa N.G.
Shear reinforcement must be provided

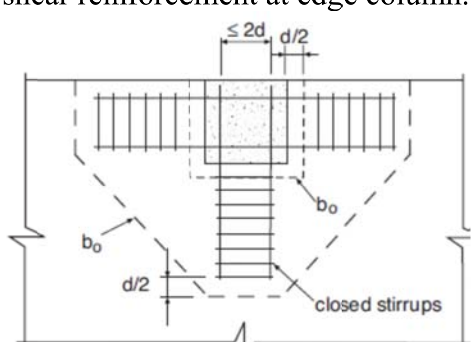
- Check effective depth, ***d***

Assuming No. 10 stirrups ($d_b$ = 9.5 mm),
$d$ = 150 mm OK (min 150 mm and 16 $d_b$ =150 mm) OK

- Check maximum shear stress permitted with bar reinforcement.

**$v_{u1} \le \varphi(1/2)\sqrt{f'_c}$** $= 0.75(1/2)\sqrt{28} = 1.984$ MPa >1.799 MPa OK Shear reinforcement can be provided. (*Otherwise, increase slab thickness*)

**HW1:** Provide closed stirrups as slab shear reinforcement at edge column.



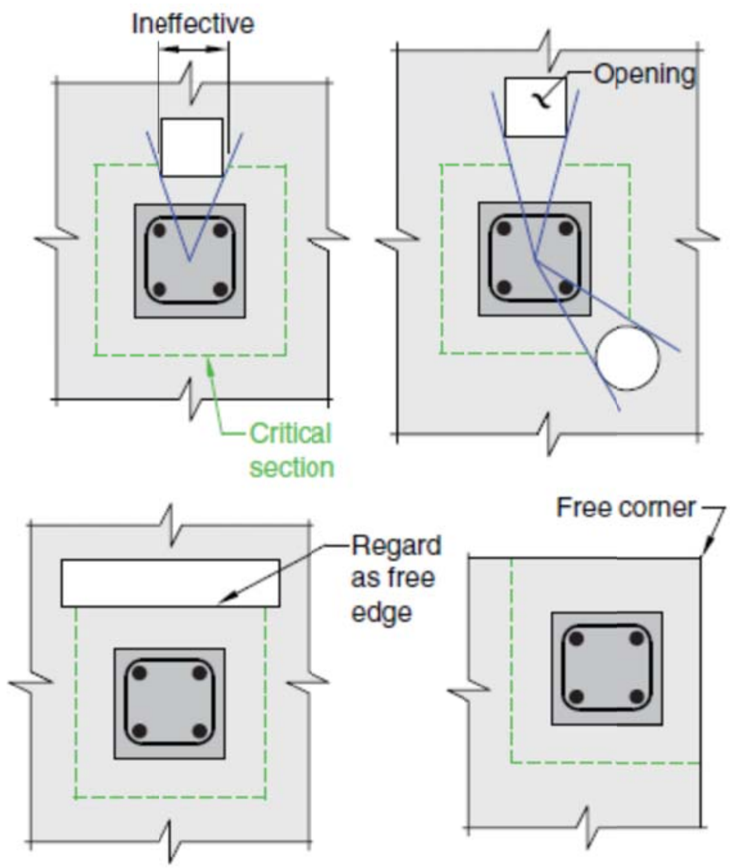

**HW2:** Rework Ex.1 considering the effects of moment transfer. Take unbalanced **$M_u$ = 40 kNm**

## Effect of opening on shear strength

The effect of openings in slabs on concrete shear strength shall be considered when the opening is located within **4 *h*** from a column periphery, a concentrated load or reaction area.

Slab opening effect is evaluated by *reducing the perimeter of the critical section* $\boldsymbol{b_o}$ *by a length* equal to the projection of the opening enclosed by two-lines extending from the centroid of the column and tangent to the opening; see Fig (a).

For slabs with shear reinforcement, the ineffective portion of the perimeter $\boldsymbol{b_o}$ is one-half of that without shear reinforcement; see Fig. (b).




**Note:** Openings shown are located within 4***h*** of the column periphery.

*Fig. R22.6.4.3—Effect of openings and free edges (effective perimeter shown with dashed lines).*

# Chapter 5: Equivalent Frame Method of TW Slab Systems

## Introduction

The DDM for TW slabs is useful if each of the *six limitations* is satisfied, otherwise, a more general method is needed, namely the ***Equivalent Frame Method*** (EFM).
The EFM is based on the analysis by the ***moment distribution method***.

By the EFM, the structure is divided into continuous frames centered on column lines and extending both longitudinally and transversely, as shown:

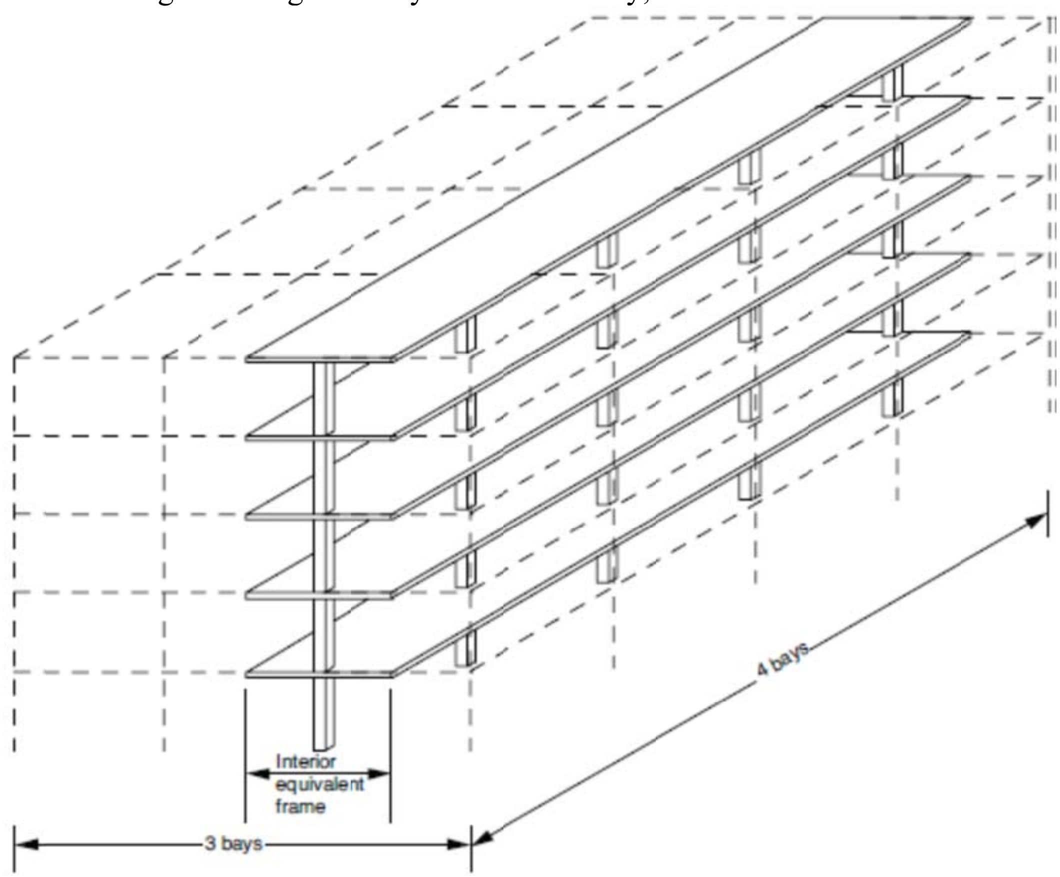
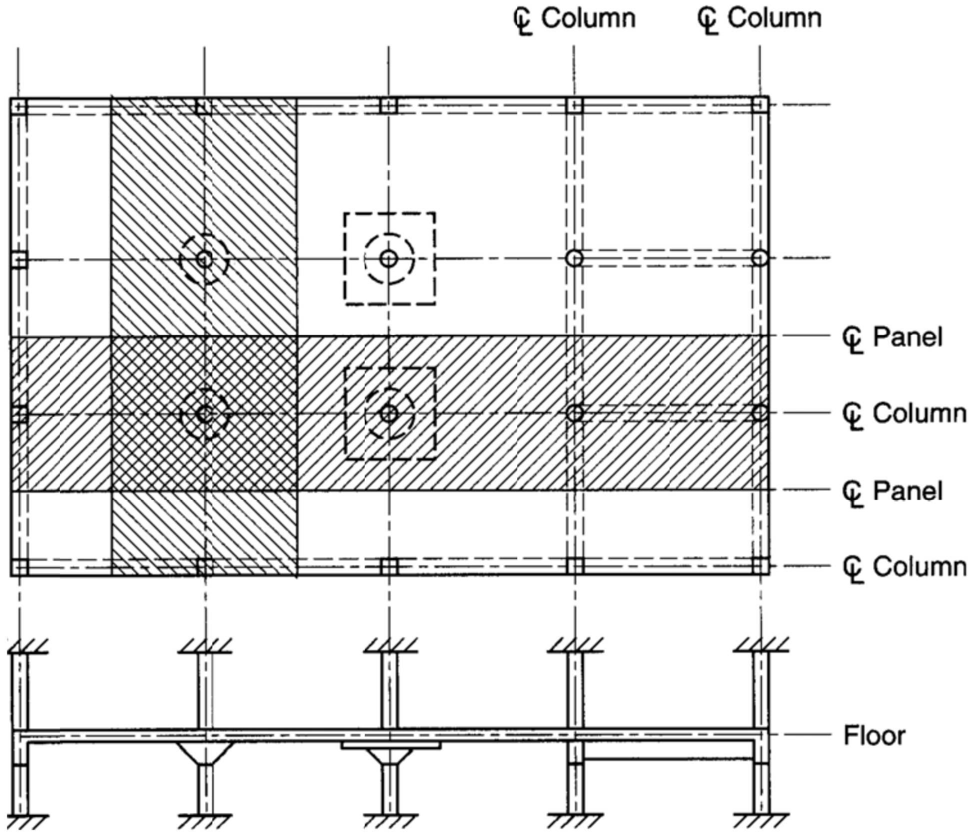


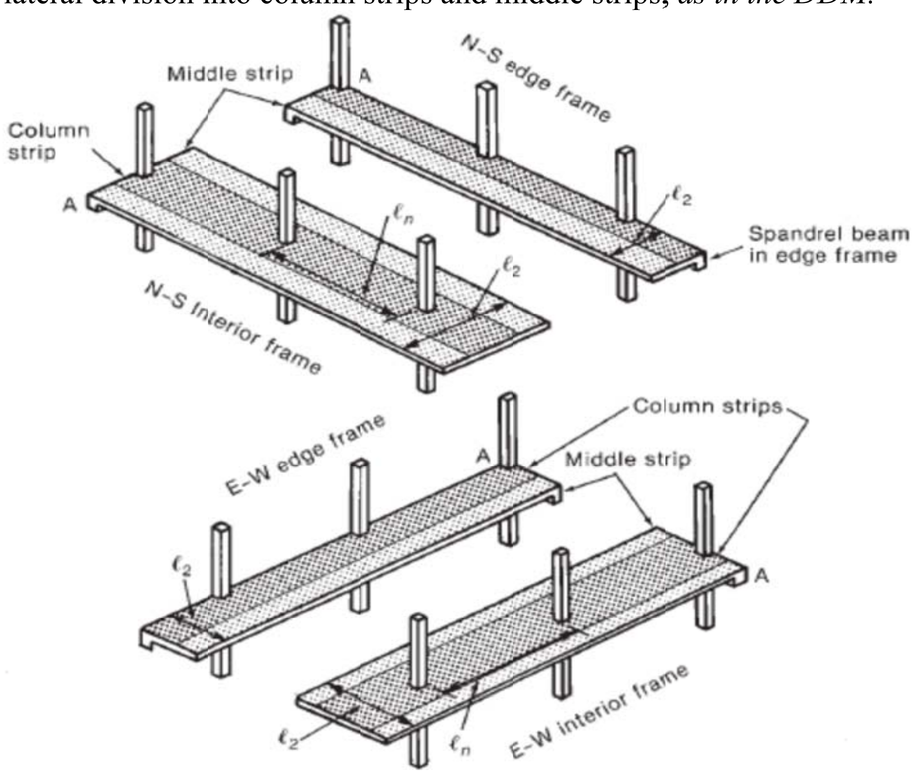

*Figure shows equivalent frame in a 5-story building.*

Each frame is composed of columns and a broad continuous beam, named ***slab-beam***. This slab-beam includes the portion of the slab bounded by panel centerlines on either side of the columns.

For vertical loading, each floor with its columns may be analyzed separately, with the columns assumed fixed at the floors above and below:




The fig below shows the division of building into 'equivalent frames'. Note the same lateral division into column strips and middle strips, *as in the DDM.*



## The Slab-beam Stiffnes: $K_{sb}$

The slab beam moment of inertia $I_{sb}$ is uniform thru the span but it changes at the support and drop panels if any.

According to ACI 8.11.3; $I_{sb,}$**(inside support)** $= I_{sb} / (1 - c_2/l_2)^2$
$c_2$ = size of column or capital in direction of $l_2$
$l_2$ = panel width

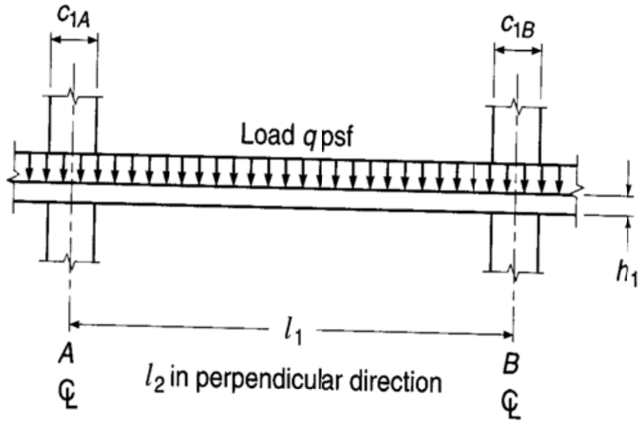
Analysis by the moment distribution method needs stiffness factors $k$, carry-over factors ***COF***, and fixed-end moment factors $M$.
These are given in Table **A13a** for slabs w/o drop panels and Table **A13b** for slabs with drop panels:

$$K_{sb} = k\,(EI_{sb} / l_1)$$

$$\text{Fixed-end Moment} = (factorM)(q_u\, l_2\, l_1^2)$$

**TABLE A.13A**

**Coefficients for slabs with variable moment of inertia**[a]

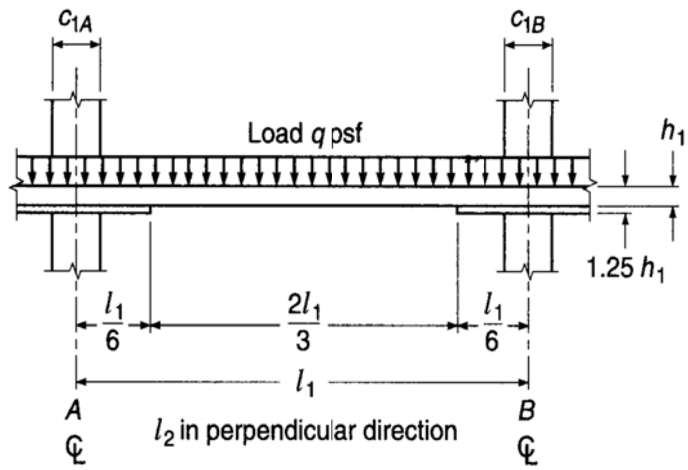



| Column Dimension | | Uniform Load FEM = Coeff.$(ql_2l_1^2)$ | | Stiffness Factor[b] | | Carryover Factor | |
|---|---|---|---|---|---|---|---|
| $c_{1A}/l_1$ | $c_{1B}/l_1$ | $M_{AB}$ | $M_{BA}$ | $k_{AB}$ | $k_{BA}$ | $COF_{AB}$ | $COF_{BA}$ |
| 0.00 | 0.00 | 0.083 | 0.083 | 4.00 | 4.00 | 0.500 | 0.500 |
| | 0.05 | 0.083 | 0.084 | 4.01 | 4.04 | 0.504 | 0.500 |
| | 0.10 | 0.082 | 0.086 | 4.03 | 4.15 | 0.513 | 0.499 |
| | 0.15 | 0.081 | 0.089 | 4.07 | 4.32 | 0.528 | 0.498 |
| | 0.20 | 0.079 | 0.093 | 4.12 | 4.56 | 0.548 | 0.495 |
| | 0.25 | 0.077 | 0.097 | 4.18 | 4.88 | 0.573 | 0.491 |
| 0.05 | 0.05 | 0.084 | 0.084 | 4.05 | 4.05 | 0.503 | 0.503 |
| | 0.10 | 0.083 | 0.086 | 4.07 | 4.15 | 0.513 | 0.503 |
| | 0.15 | 0.081 | 0.089 | 4.11 | 4.33 | 0.528 | 0.501 |
| | 0.20 | 0.080 | 0.092 | 4.16 | 4.58 | 0.548 | 0.499 |
| | 0.25 | 0.078 | 0.096 | 4.22 | 4.89 | 0.573 | 0.494 |
| 0.10 | 0.10 | 0.085 | 0.085 | 4.18 | 4.18 | 0.513 | 0.513 |
| | 0.15 | 0.083 | 0.088 | 4.22 | 4.36 | 0.528 | 0.511 |
| | 0.20 | 0.082 | 0.091 | 4.27 | 4.61 | 0.548 | 0.508 |
| | 0.25 | 0.080 | 0.095 | 4.34 | 4.93 | 0.573 | 0.504 |
| 0.15 | 0.15 | 0.086 | 0.086 | 4.40 | 4.40 | 0.526 | 0.526 |
| | 0.20 | 0.084 | 0.090 | 4.46 | 4.65 | 0.546 | 0.523 |
| | 0.25 | 0.083 | 0.094 | 4.53 | 4.98 | 0.571 | 0.519 |
| 0.20 | 0.20 | 0.088 | 0.088 | 4.72 | 4.72 | 0.543 | 0.543 |
| | 0.25 | 0.086 | 0.092 | 4.79 | 5.05 | 0.568 | 0.539 |
| 0.25 | 0.25 | 0.090 | 0.090 | 5.14 | 5.14 | 0.563 | 0.563 |

[a]Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.
[b]Stiffness is $K_{AB} = k_{AB}E(l_2h_1^3/12l_1)$ and $K_{BA} = k_{BA}E(l_2h_1^3/12l_1)$.

**TABLE A.13B**

## Coefficients for slabs with variable moment of inertia[a]



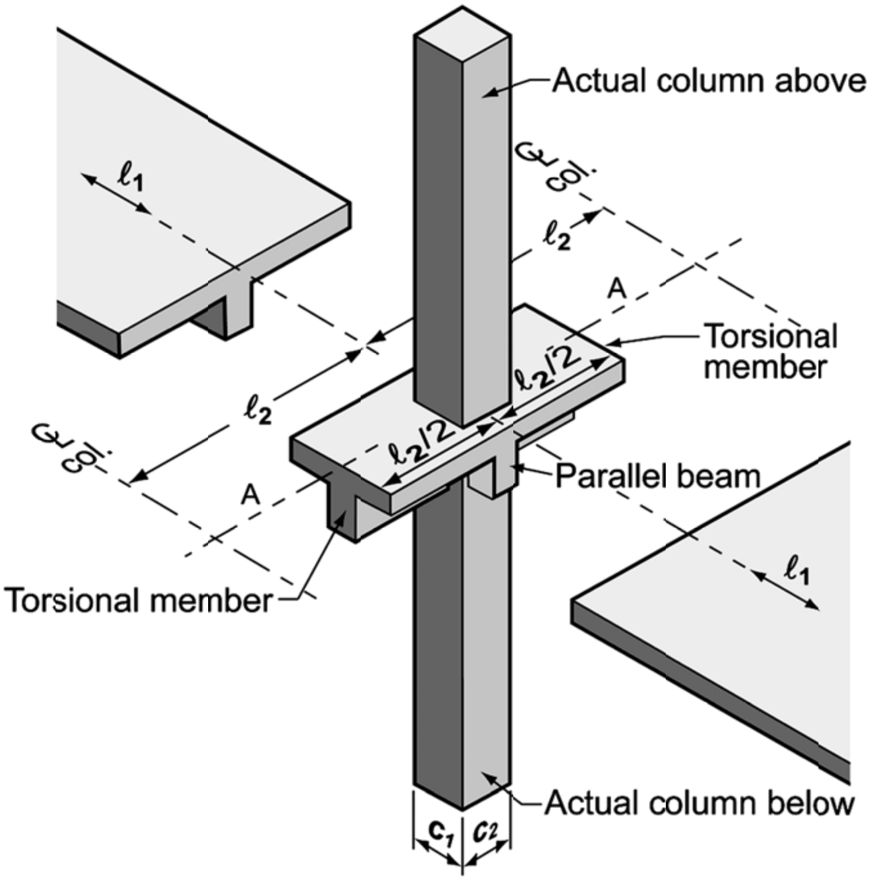

| Column Dimension | | Uniform Load FEM = Coeff.$(ql_2l_1^2)$ | | Stiffness Factor[b] | | Carryover Factor | |
|---|---|---|---|---|---|---|---|
| $c_{1A}/l_1$ | $c_{1B}/l_1$ | $M_{AB}$ | $M_{BA}$ | $k_{AB}$ | $k_{BA}$ | $COF_{AB}$ | $COF_{BA}$ |
| 0.00 | 0.00 | 0.088 | 0.088 | 4.78 | 4.78 | 0.541 | 0.541 |
| | 0.05 | 0.087 | 0.089 | 4.80 | 4.82 | 0.545 | 0.541 |
| | 0.10 | 0.087 | 0.090 | 4.83 | 4.94 | 0.553 | 0.541 |
| | 0.15 | 0.085 | 0.093 | 4.87 | 5.12 | 0.567 | 0.540 |
| | 0.20 | 0.084 | 0.096 | 4.93 | 5.36 | 0.585 | 0.537 |
| | 0.25 | 0.082 | 0.100 | 5.00 | 5.68 | 0.606 | 0.534 |
| 0.05 | 0.05 | 0.088 | 0.088 | 4.84 | 4.84 | 0.545 | 0.545 |
| | 0.10 | 0.087 | 0.090 | 4.87 | 4.95 | 0.553 | 0.544 |
| | 0.15 | 0.085 | 0.093 | 4.91 | 5.13 | 0.567 | 0.543 |
| | 0.20 | 0.084 | 0.096 | 4.97 | 5.38 | 0.584 | 0.541 |
| | 0.25 | 0.082 | 0.100 | 5.05 | 5.70 | 0.606 | 0.537 |
| 0.10 | 0.10 | 0.089 | 0.089 | 4.98 | 4.98 | 0.553 | 0.553 |
| | 0.15 | 0.088 | 0.092 | 5.03 | 5.16 | 0.566 | 0.551 |
| | 0.20 | 0.086 | 0.094 | 5.09 | 5.42 | 0.584 | 0.549 |
| | 0.25 | 0.084 | 0.099 | 5.17 | 5.74 | 0.606 | 0.546 |
| 0.15 | 0.15 | 0.090 | 0.090 | 5.22 | 5.22 | 0.565 | 0.565 |
| | 0.20 | 0.089 | 0.094 | 5.28 | 5.47 | 0.583 | 0.563 |
| | 0.25 | 0.087 | 0.097 | 5.37 | 5.80 | 0.604 | 0.559 |
| 0.20 | 0.20 | 0.092 | 0.092 | 5.55 | 5.55 | 0.580 | 0.580 |
| | 0.25 | 0.090 | 0.096 | 5.64 | 5.88 | 0.602 | 0.577 |
| 0.25 | 0.25 | 0.094 | 0.094 | 5.98 | 5.98 | 0.598 | 0.598 |

[a] Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

[b] Stiffness is $K_{AB} = k_{AB}E(l_2h_1^3/12l_1)$ and $K_{BA} = k_{BA}E(l_2h_1^3/12l_1)$.

## **The Equivalent Column Stiffness: $K_{ec}$**

The columns are considered to be attached to the slab-beam by torsional members that are transverse to the direction of the span. The torsional member extends to the panel centerlines.




The principle is that the total flexibility (inverse of stiffness) of the ***equivalent column*** is the sum of the flexibilities of the ***actual column*** and the ***torsional member***;

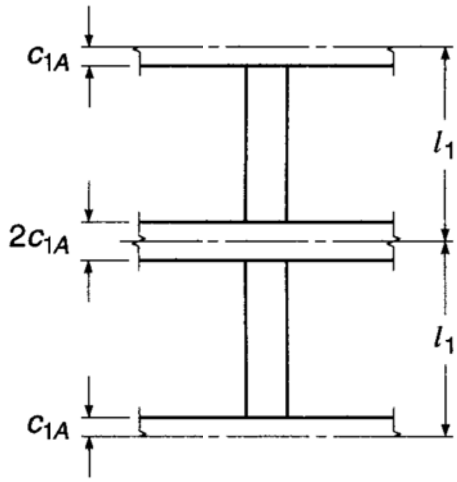
$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t}$$

where $K_{ec}$ = flexural stiffness of equivalent column
$K_c$ = flexural stiffness of actual column
$K_t$ = torsional stiffness of edge beam

$\boldsymbol{K_c = k_c\ (EI_c\ /\ l_c)}$

$\boldsymbol{k_c}$ *is given in* ***<u>Table A13c :</u>***

**TABLE A.13C**
**Stiffness factors for columns with variable moment of inertia**[a]




| Slab Half-depth $c_{1A}/l_1$ | Stiffness Factor $k_{AB}$ | Slab Half-depth $c_{1A}/l_1$ | Stiffness Factor $k_{AB}$ |
|---|---|---|---|
| 0.00 | 4.00 | 0.14 | 9.43 |
| 0.02 | 4.43 | 0.16 | 11.01 |
| 0.04 | 4.94 | 0.18 | 13.01 |
| 0.06 | 5.54 | 0.20 | 15.56 |
| 0.08 | 6.25 | 0.22 | 18.87 |
| 0.10 | 7.11 | 0.24 | 23.26 |
| 0.12 | 8.15 | | |

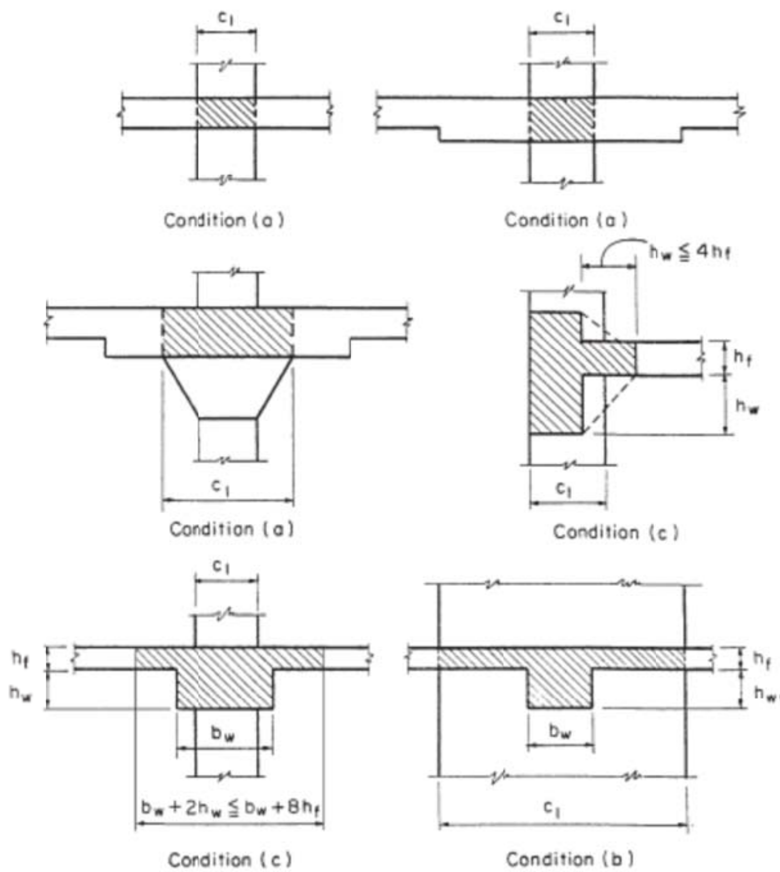
**Torsinal Member Stiffness: $K_t$**

$$K_t = \sum \frac{9E_{cs}C}{l_2(1 - c_2/l_2)^3}$$

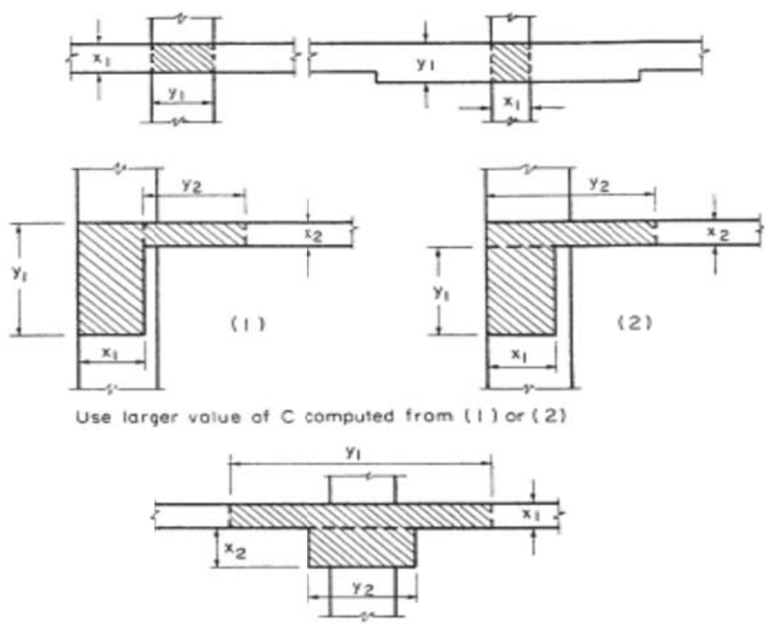
If a beam parallel to direction $l_1$ is contained in the panel, $K_t$ is increased:

$K_t$ (with parallel beam) = $K_t$ ($I_s$, with beam / $I_s$, w/o beam)

C = cross-sec torsional constant, as discussed in DDM:




<u>Fig above shows the torsional members (attaching transversely to slab-beam joint)</u>



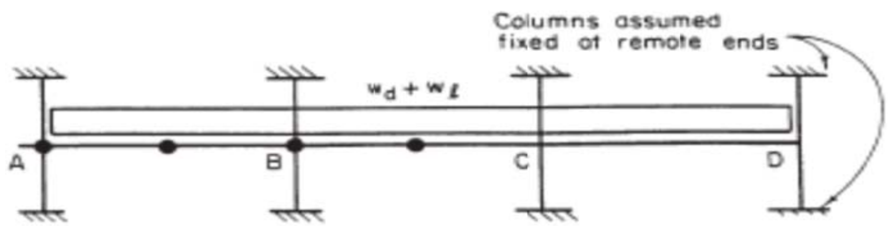

$$C = \Sigma\left[\left(1-0.63\frac{x_1}{y_1}\right)\frac{x_1^3 y_1}{3}\right]+\left[\left(1-0.63\frac{x_2}{y_2}\right)\frac{x_2^3 y_2}{3}\right]$$

*Cross-Sectional Constant C, Defining Torsional Properties of a Torsional Member*

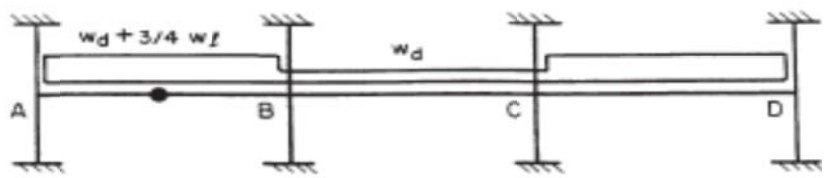
## Moments Analysis

With the slab-beam and equivalent column stiffnesses ($K_{sb}$ and $K_{ec}$) found, the analysis of the equivalent frame can proceed by the ***moment distribution method***.

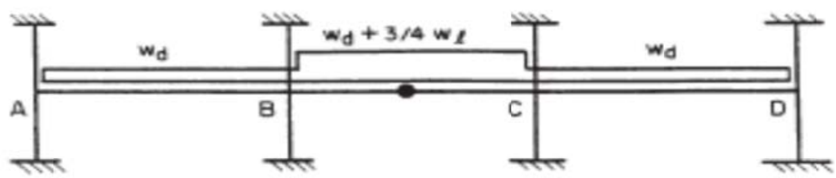
1. When **service** $L \le (3/4)\ D$, the maximum factored moments shall occur at all sections with full factored load ($D + L$) on entire slab system.
2. When **service** $L > (3/4)\ D$,
   - The ***max*** $M_u+$ near midspan occurs with factored ($D + 3/4\ L$) on the panel and on alternate panels; and
   - The ***max*** $M_u-$ in the slab at a support occurs with factored ($D + 3/4\ L$) on adjacent panels only.
3. Factored moments shall be taken not less than those occurring with full factored load ($D + L$) on all panels (case 1 above).



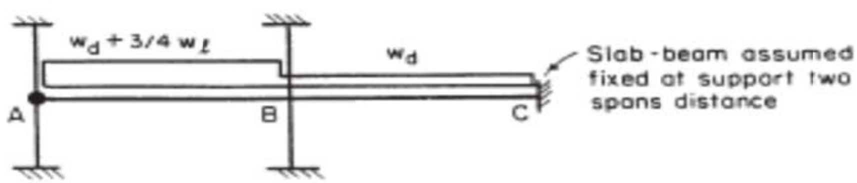
(1) Loading pattern for design moments in all spans with L ≤ 3/4 D



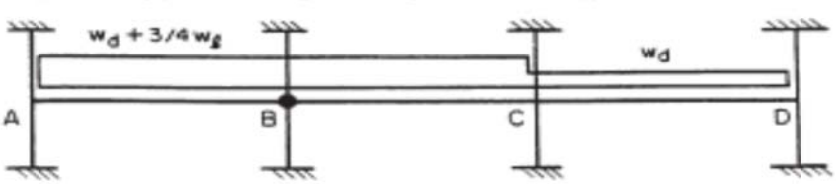
(2) Loading pattern for positive design moment in span AB*



(3) Loading pattern for positive design moment in span BC*



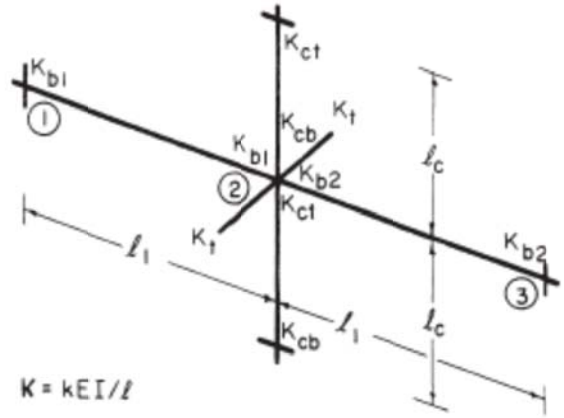
(4) Loading pattern for negative design moment at support A*



(5) Loading pattern for negative design moment at support B*

## Procedure (summary)

1. Determine slab-beam stiffness $\boldsymbol{K_{sb}}$
2. Determine equivalent column stiffness $\boldsymbol{K_{ec}}$
3. Determine slab-beam distribution factors $\boldsymbol{DF_{sb}}$




Slab-beam DF:

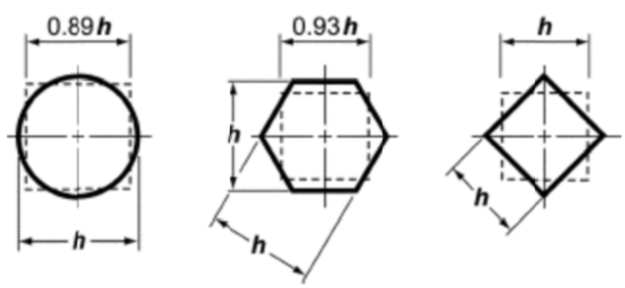
$$DF_{sb\ 2\text{-}1} = K_{b1} / [K_{b1} + K_{b2} + K_{ec}]$$

$$DF_{sb\ 2\text{-}3} = K_{b2} / [K_{b1} + K_{b2} + K_{ec}]$$

Equivalent column DF:

$$DF_{ec} = K_{ec} / [K_{b1} + K_{b2} + K_{ec}]$$

4. Determine ***COF*** and fixed-end moments ***FEM = (factorM)*** $w_u l_2 l_1^2$
5. Perform the ***moment distribution method*** to determine joint moments
6. Find design moments: $M^-$ at face of supports, and $M^+$ at midspans.




*Face of equivalent square is considered as the location of the critical design $M^-$*

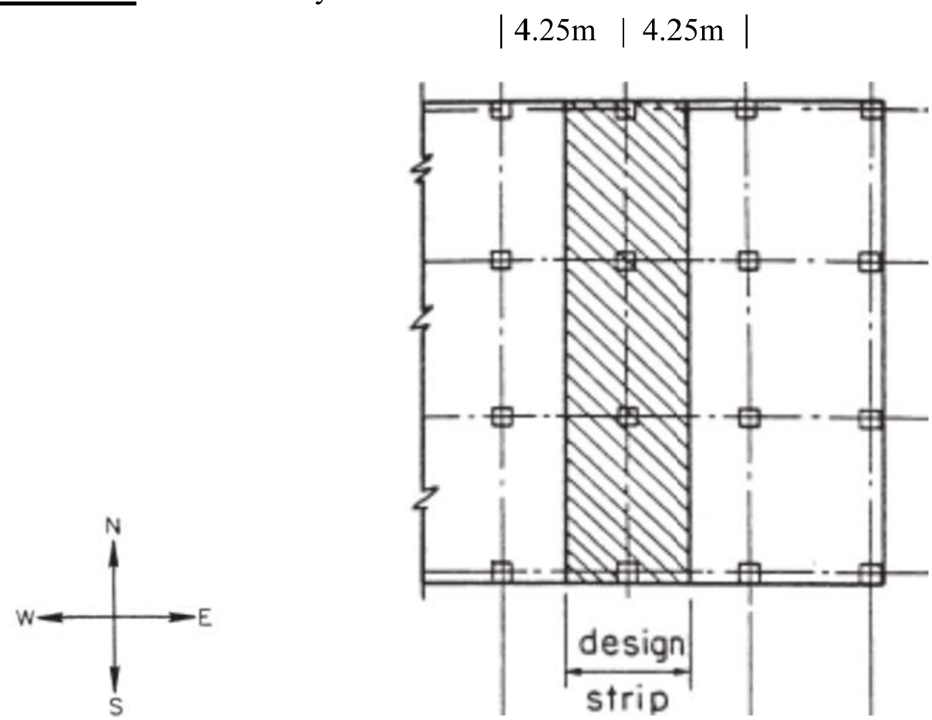
7. Distribute design moments to $M_{cs}$ and $M_{ms}$ , according to ***DDM***.
8. Design the reinforcement.

## Example: Equivalent Frame Method; Slab with beams

Use the EFM to determine the design moments for the flat plate system in the NS direction (*shaded design strip*) in an itermediate floor. Given;
No Edge beams, column dims: 500 × 500 mm
c.c. spans in N-S direction are 5.50 m
c.c. spans in E-W direction are 4.25 m
$f'_c$ = 28 MPa (slab), $f'_c$ = 42 MPa (columns), $f_y$ = 420 MPa
Service live load $L$ = 2 kN/m$^2$ , Partition weight = 1 kN/m$^2$
Story height = 3.5 m

## Solution:

***find slab thickness*** ***h*** satisfactory for deflection control.




For flat plate slab systems, the minimum overall thickness $h$ with Grade 60 reinforcement is (see ch.2):
Ext panel w/o edge beam $h = l_n / 30$ = 5100 / 30 = 170 mm > 125 mm
Try 170 mm slab for all panels (weight = 4.1 kN/m$^2$)

### *Check thickness for TW shear*

Use average effective depth d = 137 (20mm cover and No.13 bar)
Factored dead load, 1.2$D$ = 1.2 (4.1 + 1) = 6.12 kN/m$^2$
Factored live load, 1.6$L$ = 1.6 × 2 = 3.2 kN/m$^2$
Total factored load = 9.32 kN/m$^2$
Shear strength at d/2 distance around the support is computed as follows:
$V_u$ = 9.32 [(5.5×4.25) – (0.537)$^2$] = 215.2 kN
$V_c$ = (1/3)$\sqrt{f'c}\, b_o d$ (for square interior column) *Eq. (22.6.5.2 a)*
= (1/3)√28 (4×537)137 /1000 = 519 kN

$\varphi V_c$ = 0.75 ×519 = 389.3 kN > $V_u$ O.K. (Effects of moment transfer may be checked later after finding moments)

## ***Flexural stiffness of slab-beams at both ends, $K_{sb}$***

$K_{sb} = k\ EI_{sb} / l_1$

Table A13a; ext joint and int joint:

$c_{1A}/l_1$ = 500 / 5500 = 0.091 say 0.1 = $c_{1B} / l_1$

From Table: $M_{AB}$ = 0.085, $k_{AB}$ = 4.18, ***COF*** = 0.513

$K_{sb}$ **1-2 =** $K_{sb}$ **2-1** = 4.18 [4700√28 × 4250(170)$^3$/12] / 5500 = **3.29 × 10$^{10}$ Nmm**

$K_{sb}$ **2-3 =** $K_{sb}$ **3-2 = 3.29 × 10$^{10}$ Nmm**

## ***Flexural stiffness of column members at both ends, $K_c$***

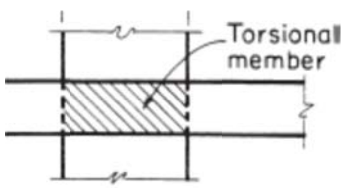
$K_c = k\ EI_c / l_c$

Table A13c; $l_c = l_1$ = 3.5 m, $c_{1A}$ = 0.17/2 = 0.085 m, $c_{1A}/l_1$ = 0.085 / 3.5 = 0.02

From Table: $k_{AB}$ = 4.43

$K_c$, **below =** $K_c$, **above** = 4.43 [4700√42 × 500(500)$^3$/12] / 3500 = 20.08 × 10$^{10}$ Nmm

## ***Torsional stiffness of torsional members, $K_t$***




$$K_t = \sum \frac{9E_{cs}C}{l_2(1 - c_2/l_2)^3}$$

| 500mm |

C = (1 – 170/500)(170)$^3$500/3 = 5.4× 10$^8$ mm$^4$

$K_t$ = 9(4700√28 C) / 4250(1 – 500/4250)$^3$ = 4.14 × 10$^{10}$ Nmm

## ***Equivalent column stiffness $K_{ec}$***

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t}$$

1/ $K_{ec}$ = 1 / 2×20.08 + 1 / 2×4.14 (**2** columns above and below, and **2** torsional members on each side)

$K_{ec}$ = **6.86 × 10$^{10}$ Nmm**

## ***Slab-beam joint distribution factors DF***.

At exterior joint,

***DF*** = 3.29 /(3.29 + 6.86) = **0.324**

At interior joint,

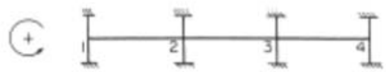
***DF*** = 3.29 /(3.29 + 3.29 + 6.86) = **0.245**

## ***Frame Analysis***

***L / D*** = 2 / 5.1 = 0.39 < 0.75 OK, design moments are assumed to occur at all critical sections with full factored load on all spans. (case 1 only)
Factored load and fixed-end moments.
Factored dead load 1.2$D$ = 1.2 (5.1) = 6.12 kN/m$^2$
Factored live load 1.6$L$ = 1.6 (2) = 3.2 kN/m$^2$
Factored load $q_u = D + L$ = 9.32 kN/m$^2$
FEM's for slab-beams = *(factorM)* $q_u l_2 l_1^2$
= 0.085 (9.32) (4.25)(5.5)$^2$ = **101.85 kNm**

## ***Moment distribution***

Counterclockwise moments acting on the member ends are taken as positive.



| Joint | 1 | 2 | | 3 | | 4 |
|---|---|---|---|---|---|---|
| Member | 1-2 | 2-1 | 2-3 | 3-2 | 3-4 | 4-3 |
| DF | 0.324 | 0.245 | 0.245 | 0.245 | 0.245 | 0.324 |
| COF | 0.513 | 0.513 | 0.513 | 0.513 | 0.513 | 0.513 |
| FEM<br>DM | 101.85<br>- 33.0 | - 101.85 | 101.85 | - 101.85 | 101.85 | -101.85<br>+ 33.0 |
| CO<br>DM | | -16.93<br>4.15 | <br>4.15 | <br>-4.15 | 16.93<br>-4.15 | |
| CO<br>DM | 2.13<br>- 0.69 | <br>0.52 | - 2.13<br>0.52 | 2.13<br>- 0.52 | <br>- 0.52 | - 2.13<br>0.69 |
| ***Σ M*** | **70.29** | **-114.11** | **104.39** | **-104.39** | **114.11** | **-70.29** |
| $M_{CL}$ | **57.78** | | **45.39** | | **57.78** | |

***Mid span moments*** ($M_{CL}$ ) are determined from the following equation:
$\mathbf{M_{CL}}$ ***(midspan)*** $\mathbf{= M_o - (M_{uL} + M_{uR})/2}$
where $M_o$ is the moment at midspan for a simple beam.
Positive moment in span 1-2:
$+M_{CL\ 1\text{-}2}$ = (9.32×4.25) 5.5$^2$/8 – (70.29 + 114.11) /2 = 57.78 kNm
Positive moment in span 2-3:
$+M_{CL\ 2\text{-}3}$ = (9.32×4.25) 5.5$^2$/8 – (104.39 + 104.39) / 2 = 45.39 kNm

***Shears***: End span 1-2
**V (*support*)** $= q_u l_2 l_1 / 2 \pm (M_R - M_L) / l_1$
V (*left support*) $= q_u l_2 l_1 / 2 - (M_R - M_L) / l_1$
= 9.32(4.25)(5.5)/2 – (114.11 – 70.29)/5.5
= 102.99 – 7.97 = 95.02 kN
V (*right support*) $= q_u l_2 l_1 / 2 + (M_R - M_L) / l_1$
= 102.99 + 7.97 = 110.96 kN
Shears: Interior span 2-3
V (*left support*) $= q_u l_2 l_1 / 2 - (M_R - M_L) / l_1$
= 9.32(4.25)(5.5)/2 – (104.39 – 104.39)/5.5

= 102.99 – 0 = 102.99 kN
V (*right support*) $= q_u l_2 l_1 / 2 + (M_R - M_L) / l_1$
= 102.99 + 0 = 102.99 kN

***Design –ve Moment = Joint moment – area of shear diagram bet support CL and face***
At joint 1, Design M− = 70.29 – (95.02 kN × 0.25 m) = 46.54 kNm
At joint 2 left, Design M− = 114.11 – (110.96 kN × 0.25 m) = 86.37 kNm
At joint 2 right, Design M− = 104.39 – (102.99 kN × 0.25 m) = 78.64 kNm
At joint 3 left, Design M− = 78.64 kNm

## Moments at critical sections (summary):

**Interior spans**
Interior $M^-$ = **78.64** kN, $M^+$ = **45.39** kNm
**End span**
Exterior $M^-$ = **46.54** kNm, $M^+$ = **57.78** kNm, Interior $M^-$ = **86.37** kNm

## Disribution to col strip, mid strip

$l_2 / l_1$ = 4.25 / 5.5 = 0.77, $(\alpha_{f1} l_2 / l_1) = 0$

| | $M_u$ (kNm) | % to cs | $M_{cs}$ (kNm) * | 2-half ms $M_{ms}$ (kNm) |
|---|---|---|---|---|
| *End Span* | | | | |
| Ext. negative | 46.54 | 100 | 46.54 | 0 |
| Positive | 57.78 | 60 | 34.67 | 23.11 |
| Int. negative | 86.37 | 75 | 64.78 | 21.59 |
| *Interior Span* | | | | |
| Negative | 78.64 | 75 | 58.98 | 19.66 |
| Positive | 45.39 | 60 | 27.23 | 17.81 |

## Slab Reinforcement

## Column strip:

| | Slab$M_{cs}$ (kNm) | $R = Mu / \varphi bd^2$ * | $\rho$ Table A.5 | $A_s$ mm²/m** | provide |
|---|---|---|---|---|---|
| *End Span* | | | | | |
| Ext. negative | 46.54 | 1.157 | 0.0028 | 406 | No.13@300 (430 mm²/m) |
| Positive | 34.67 | 0.863 | 0.0021 | 305 / use 306 | No.13@300 (430 mm²/m) |
| Int. negative | 64.78 | 1.612 | 0.0040 | 580 | No.13@200 (645 mm²/m) |
| *Interior Span* | | | | | |
| Negative | 58.98 | 1.467 | 0.0036 | 522 | No.13@200 645 mm²/m) |
| Positive | 27.23 | 0.678 | 0.0017 | 247 / use 306 | No.13@300 (430 mm²/m) |

* $b_{cs} = l_2/2$ = 2125 mm, $d$ = 145 mm
** $A_s = \rho bd$ , $A_{s\,min} = 0.0018bh$ =306 mm²/m, use No.13@300 (430 mm²/m)
max spacing = $2h$ = 2(150) = 300 mm OK

## *Middle strip:*

| | **2-half ms $M_{ms}$ (kNm)** | **$R = M_u / \varphi bd^2$ *** | **$\rho$ Table A.5** | **$A_s$ mm$^2$/m**** | **Provide** |
|---|---|---|---|---|---|
| *End Span* | | | | | |
| Ext. negative | 0 | 0 | 0 | 0 /use 306 | No.13@300 (430 mm$^2$/m) |
| Positive | 23.11 | 0.575 | 0.0014 | 203/use 306 | ditto |
| Int. negative | 21.22 | 0.537 | 0.0014 | 203/use 306 | ditto |
| *Interior Span* | | | | | |
| Negative | 19.31 | 0.489 | 0.0011 | 160/ use 306 | ditto |
| Positive | 17.81 | 0.443 | 0.0011 | 160/ use 306 | ditto |

* $b_{ms} = l_2 - b_{cs}$ = 4250 - 2125 = 2125 mm, $d$ = 145 mm

** $A_s = \rho bd$ , $A_{s\,min} = 0.0018\,bh$ =306 mm$^2$/m, use No.13@300 (430 mm$^2$/m)

max spacing = 2$h$ = 2(170) = 340 mm

## Effect of $\gamma_f M_u$ at exterior edge

$$\gamma_f = \frac{1}{1+\left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

$b_1 = c + d/2$ = 572.5 mm, $b_2 = c + d$ = 645 mm

$\gamma_f$ = 1 / [1 + (2/3)√(0.89] = 0.614

Add $A_s$ to be distributed over effective width = $c_2 + 3h$ = 1010 mm outside opposite faces of the column.

$R = \gamma_f M_u / \varphi bd^2$ = 0.614(46.54×10$^6$)/[0.9×1010×145$^2$] = 1.495

Table A5; ρ = 0.0038, $A_s$ = 0.0038×1010×145 = 557 mm$^2$ *(add top bars 5 No 13)*

## Effect of $\gamma_v M_u$ at exterior edge

$\gamma_v = 1 - \gamma_f$ = 1 – 0.614 = 0.386

$\gamma_v M_u$ = 0.386(46.54) = 17.96 kNm

$A_c = (2b_1 + b_2)d$ = 238525 mm2

$J/c$ = 4.496×10$^7$

$$\frac{2b_1^2 d(b_1+2b_2)+d^3(2b_1+b_2)}{6b_1}$$

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u}{J/C}$$

= 215.2×10$^3$/238525 + 17.96×10$^6$/4.496×10$^7$

= 0.902 + 0.399 = 1.301 MPa

$\varphi v_c = \varphi(1/3)\sqrt{f'_c}$ = 1.323 MPa > $v_u$ = 1.301 MPa. Shear strength is OK

## Moments to columns

***Interior columns, unbalanced M = $M_{col}$ =*** 114.11 – 104.39 = 9.72 kNm

***At exterior column***, $M_{col}$ = 70.29 kNm

Columns above and below slab shall resist moments in proportion of their stiffnesses;

$M_{col\,above} = [K_{col\,above} / (K_{col\,above} + K_{col\,below})]\,M_{col}$

$M_{col\,above} = M_{col\,below}$ = **(1/2)** $M_{col}$ (*columns above and below are identical*)

= (1/2)(9.72) = 4.86 kNm for interior columns

= (1/2)(70.29) = 35.15 kNm for exterior columns

## ***HW: EFM of TW Floor with Beams***

Use the EFM to determine the design moments for the TW slab system with beams in the NS direction (*shaded design strip*) in an itermediate floor. Given;

Edge beam dims: 350 × 650 mm
Interior beam dims: 350 × 500 mm
Column dims: 450 × 450 mm
Slab thickness $h$ = 150 mm
c.c. spans in N-S direction are 5.50 m
c.c. spans in E-W direction are 6.50 m
$f'_c$ = 28 MPa, $f_y$ = 420 MPa
Service live load $L$ = 7.5 kN/m$^2$, Story height = 3.5 m