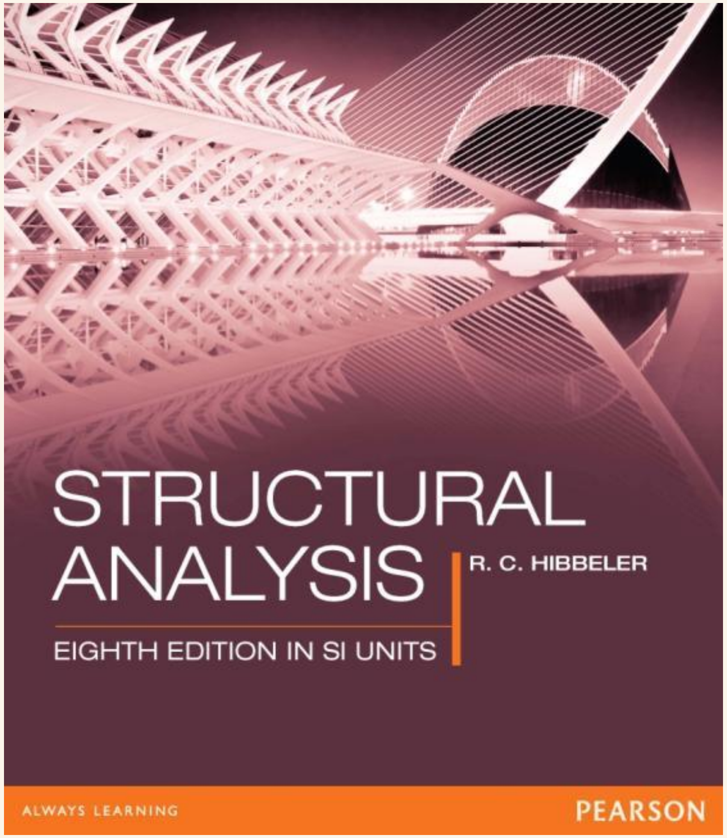
THEORY OF STRUCUTRE

STRUCTURAL ANALYSIS

EIGHTH EDITION IN SI UNITS

R. C. HIBBELER



CHAPTER 1:
TYPES OF STRUCTURES AND LOADS

1

Chapter Outline

1.1

INTRODUCTION

1.1

Introduction

- Structures refer to a system of connected parts used to support a load
- Factors to consider:
  - Safety
  - Esthetics
  - Serviceability
  - Economic & environmental constraints

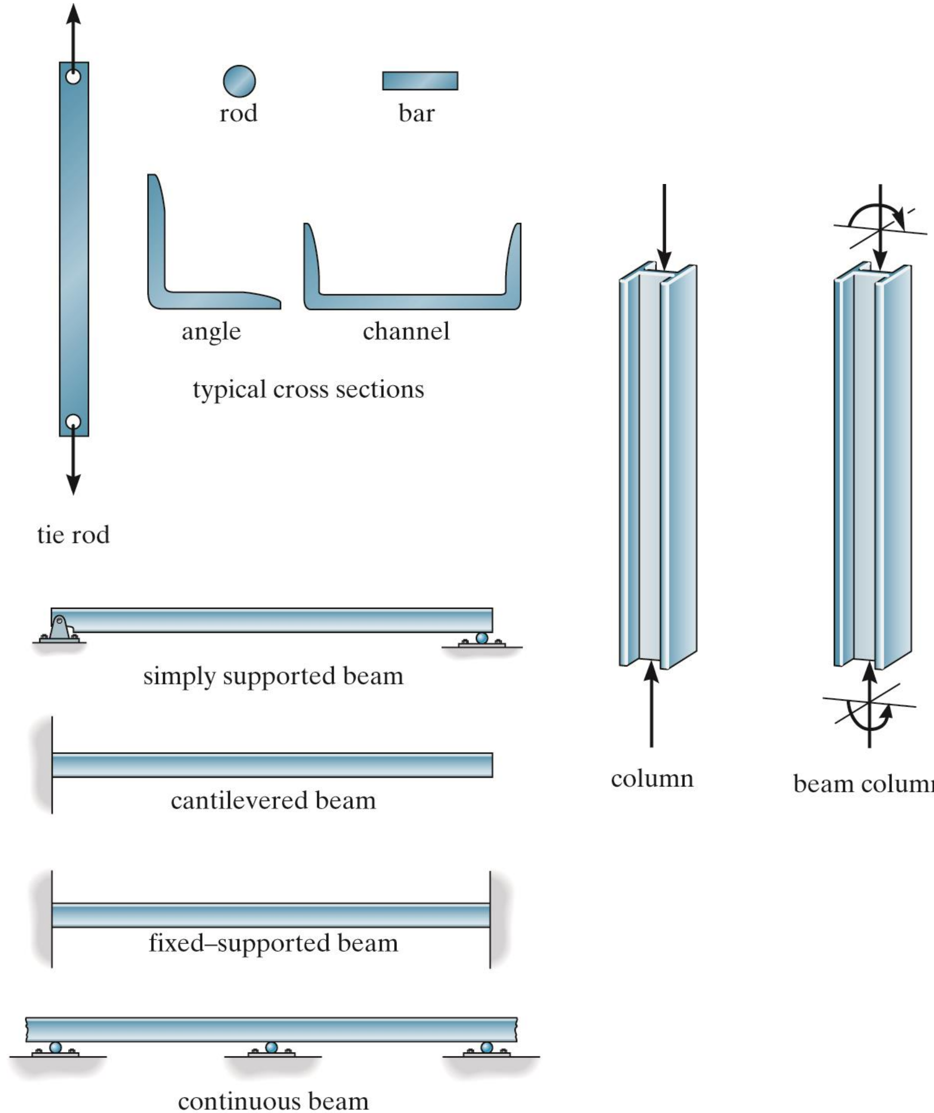
1.2
CLASSIFICATION OF STRUCTURES

1.2

Classification of Structures

- Structural elements
  - Tie rods
  - Beams
  - Columns

- Types of structures
  - Trusses
  - Cables & Arches
  - Surface Structures



1.3

LOADS

1.3

Loads

Loads

↓

Structural forms

↓

Elements carrying primary loads

↓

Various supporting members

↓

Foundation

Loads

- Design loading for a structure is often specified in codes
  - General building codes
  - Design codes

TABLE 1–1 Codes
General Building Codes
Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers
International Building Code
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI)
Manual of Steel Construction, American Institute of Steel Construction (AISC)
Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials (AASHTO)
National Design Specification for Wood Construction, American Forest and Paper Association (AFPA)
Manual for Railway Engineering, American Railway Engineering Association (AREA)

Loads

- Types of load
  - Dead loads
    - Weights of various structural members
    - Weights of any objects that are attached to the structure

TABLE 1–2 Minimum Densities for Design Loads from Materials*

	kN/m^3
Aluminum	26.7
Concrete, plain cinder	17.0
Concrete, plain stone	22.6
Concrete, reinforced cinder	17.4
Concrete, reinforced stone	23.6
Clay, dry	9.9
Clay, damp	17.3
Sand and gravel, dry, loose	15.7
Sand and gravel, wet	18.9
Masonry, lightweight solid concrete	16.5
Masonry, normal weight	21.2
Plywood	5.7
Steel, cold-drawn	77.3
Wood, Douglas Fir	5.3
Wood, Southern Pine	5.8
Wood, spruce	4.5

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures,* ASCE/SEI 7-10. Copies of this standard may be purchased from ASCE at www.pubs.asce.org.

TABLE 1–3 Minimum Design Dead Loads*

Walls	kN/m^2
100 mm clay brick	1.87
200 mm clay brick	3.78
300 mm clay brick	5.51
Frame Partitions and Walls	
Exterior stud walls with brick veneer	2.30
Windows, glass, frame and sash	0.38
Wood studs 50 × 100 mm unplastered	0.19
Wood studs 50 × 100 mm plastered one side	0.57
Wood studs 50 × 100 mm plastered two sides	0.96
Floor Fill	
Cinder concrete, per mm	0.017
Lightweight concrete, plain, per mm	0.015
Stone concrete, per mm	0.023
Ceilings	
Acoustical fiberboard	0.05
Plaster on tile or concrete	0.24
Suspended metal lath and gypsum plaster	0.48
Asphalt shingles	0.10
Fiberboard, 13 mm	0.04

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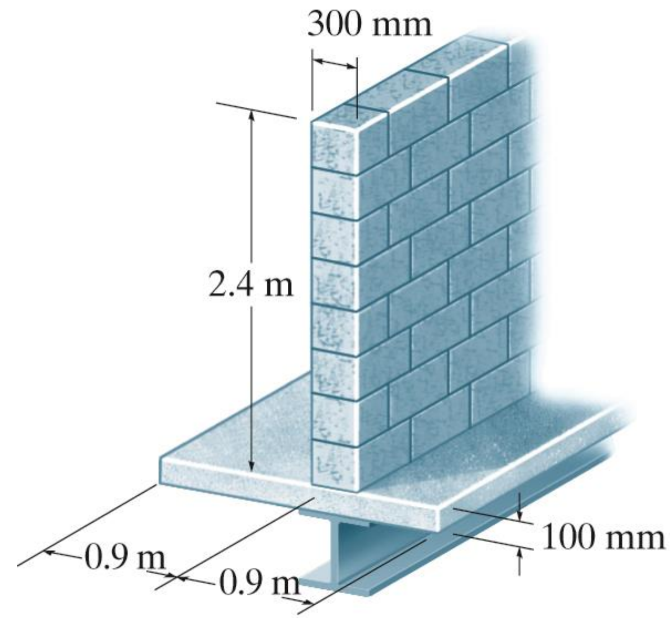
Loads

- Example 1.1
- The floor beam is used to support the 1.8 m width of a lightweight plain concrete slab having a thickness of 100 mm. The slab serves as a portion of the ceiling for the floor below & its bottom is coated with plaster. A 2.4 m high, 300 mm thick lightweight solid concrete block wall is directly over the top flange of the beam. Determine the loading on the beam measured per m length of the beam.



Loads

- Example 1.1 (Solution)

$$\text{concrete slab}: \quad (0.015\,\text{kN}/\text{m}^2 \bullet \text{mm})(100\,\text{mm})(1.8\,\text{m}) = 2.70\,\text{kN}/\text{m}$$

$$\text{plaster ceiling}: \quad (0.24\,\text{kN}/\text{m}^2)(1.8\,\text{m}) = 0.43\,\text{kN}/\text{m}$$

$$\text{block wall}: (16.5\,\text{kN}/\text{m}^3)(2.4\,\text{m})(0.3\,\text{m}) = 11.88\,\text{kN/m}$$

$$\text{Total} = 2.70 + 0.43 + 11.88 = 15.01\,\text{kN/m}$$

Loads

- Types of load
  - Live loads
    - Varies in magnitude & location
  - Building loads
    - Depends on the purpose for which the building is designed
    - These loadings are generally tabulated in local, state or national code

TABLE 1–4 Minimum Live Loads*

Occupancy or Use	Live Load kN/m^2	Occupancy or Use	Live Load kN/m^2
Assembly areas and theaters		Residential	
Fixed seats	2.87	Dwellings (one- and two-family)	1.92
Movable seats	4.79	Hotels and multifamily houses	
Garages (passenger cars only)	2.40	Private rooms and corridors	1.92
Office buildings		Public rooms and corridors	4.79
Lobbies	4.79	Schools	
Offices	2.40	Classrooms	1.92
Storage warehouse		Corridors above first floor	3.83
Light	6.00		
Heavy	11.97		

*Reproduced with permission from *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10.

Loads

- Types of load
  - Building loads
    - Uniform, concentrated loads

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right)$$

where

$L =$ reduced design live load/m^2 of area supported by the member

$L_o =$ unreduced design live load/m^2 of area supported by the member

$K_{LL} =$ live load element factor. For interior column $K_{LL} = 4$

$A_T =$ tributary area in m^2

Loads

- Types of load
  - Building loads
    - Uniform, concentrated loads

$L \geq 0.5L_o$ for members supporting one floor

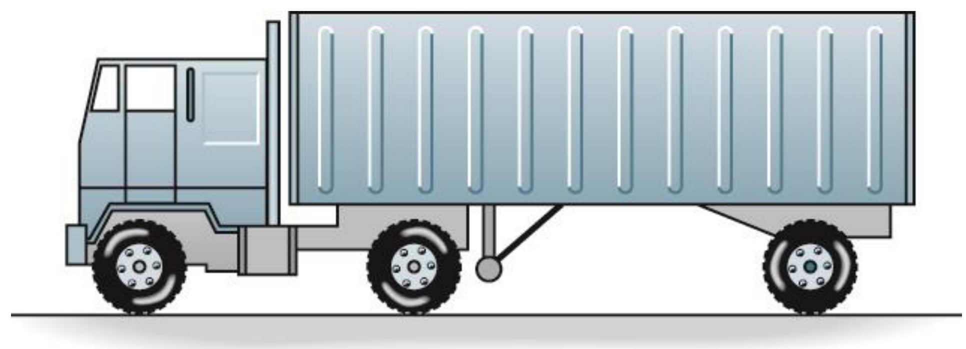
$L \geq 0.4L_o$ for members supporting more than one floor

No reduction is allowed for loads $\geq 4.79\ \text{kN/m}^2$

or for structures used for public assembly, garage or roof.

Loads

- Types of load
  - Highway Bridge loads
    - Primary live loads are those due to traffic
    - Specifications for truck loadings are reported in AASHTO
    - For 2-axle truck, these loads are designated with H followed by the weight of truck in tons and another no. gives the year of the specifications that the load was reported



Loads

- Types of load
  - Railway Bridge loads
    - Loadings are specified in AREA
    - A modern train having a 320 kN loading on the driving axle of the engine is designated as an E-72 loading

Loads

- Types of load
  - Impact loads
    - Due to moving vehicles

Loads

- Types of load
  - Wind loads
    - Kinetic energy of the wind is converted into potential energy of pressure when structures block the flow of wind
    - Effect of wind depends on density & flow of air, angle of incidence, shape & stiffness of the structure & roughness of surface
    - For design, wind loadings can be treated using static or dynamic approach

Loads

- Types of load
  - Snow loads
    - Design loadings depend on building's general shape & roof geometry, wind exposure, location, its importance and whether or not it is heated
    - Snow loads are determined from a zone map reporting 50-year recurrence intervals of an extreme snow depth

Loads

- Types of load
  - Earthquake loads
    - Earthquake produce loadings through its interaction with the ground & its response characteristics
    - Their magnitude depends on amount & type of ground acceleration, mass & stiffness of structure

Loads

- Types of load
  - Hydrostatic & Soil Pressure
    - The pressure developed by these loadings when the structures are used to retain water or soil or granular materials
    - E.g. tanks, dams, ships, bulkheads & retaining walls
  - Other natural loads
    - Effect of blast
    - Temperature changes
    - Differential settlement of foundation

1.4

STRUCTURE DESIGN

1.4

Structure Design

- Material uncertainties occur due to
  - variability in material properties
  - residual stress in materials
  - intended measurements being different from fabricated sizes
  - material corrosion or decay

- Many types of loads can occur simultaneously on a structure

Structure Design

- In working-stress design, the computed elastic stress in the material must not exceed the allowable stress along with the following typical load combinations as specified by the ASCE 7-10 Standard

Structure Design

- In strength design is material uncertainty and load uncertainty are separately determined

- This method uses load factors applied to the loads or combination of loads
  - 1.4 (Dead load)
  - 1.2 (dead load) + 1.6 (live load) + 0.5 (snow load)
  - 0.9 (dead load) + 1.0(wind load)
  - 0.9 (dead load) + 1.0 (earthquake load)

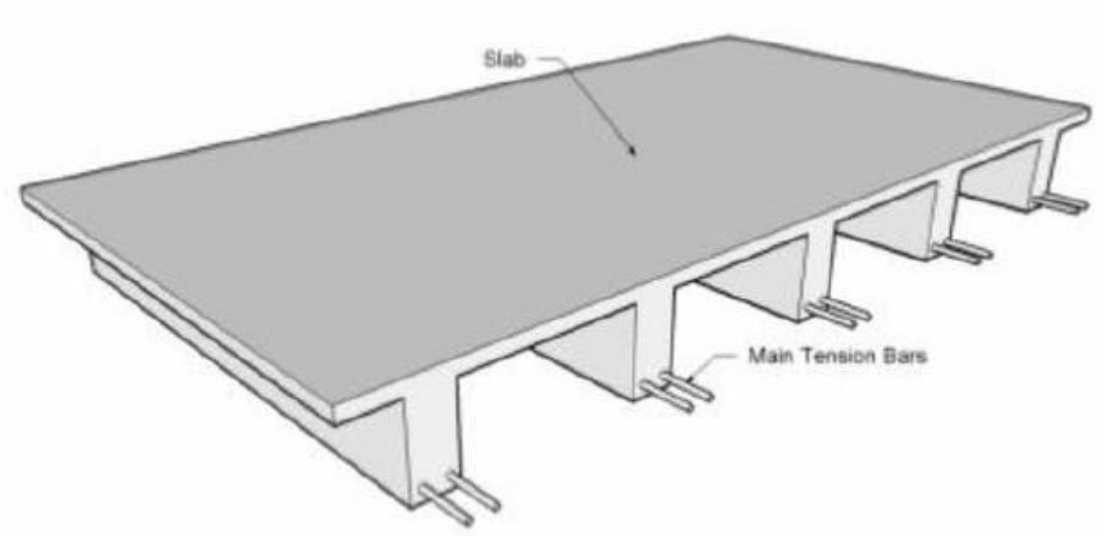
HW 1-1

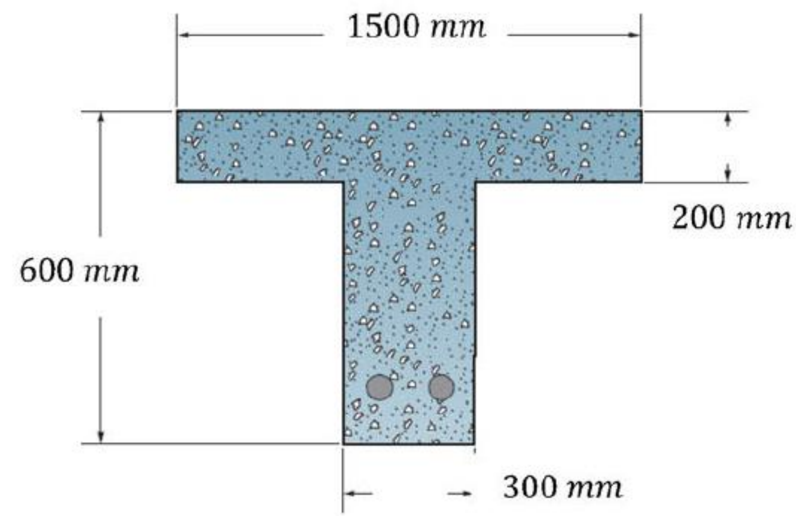
The floor of the office building is made of 150 mm thick normal-weight concrete. If the office floor is a slab having a length of 6 m and width of 4.5 m, determine the resultant force caused by the dead load and the live load.

Ans=162 KN

HW 1-2

The T-beam is made from concrete having a specific weight of 24 kN/m^3. Determine the dead load per meter length of beam and the total weight of T-beam if the length is 6 m. Neglect the weight of the steel reinforcement.

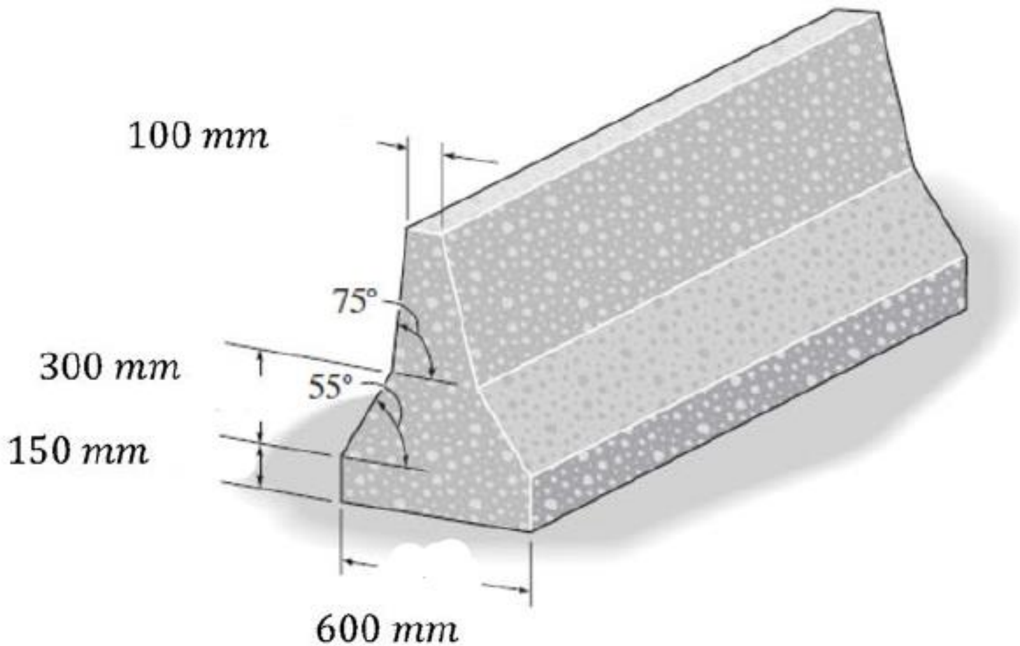




Ans=60.48 KN

HW 1-3

The "New Jersey" barrier is commonly used during highway construction. Determine its weight per meter of length if it is made from plain stone concrete.



Ans=5.45 KN/m

CHAPTER 2:
ANALYSIS OF STATISCALLY DETERMINATE STRUCTURES

2

Chapter Outline

2.1
IDEALIZED STRUCTURE

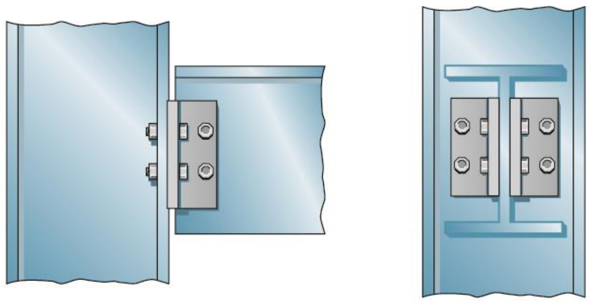
2.1

Idealized Structure

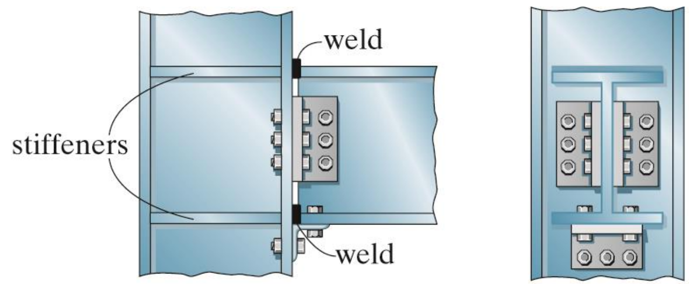
- To develop the ability to model or idealize a structure so that the structural engineer can perform a practical force analysis of the members

- Support Connections
  - Pin connection (allows some freedom for slight rotation)
  - Roller support (allows some freedom for slight rotation)
  - Fixed joint (allows no relative rotation)

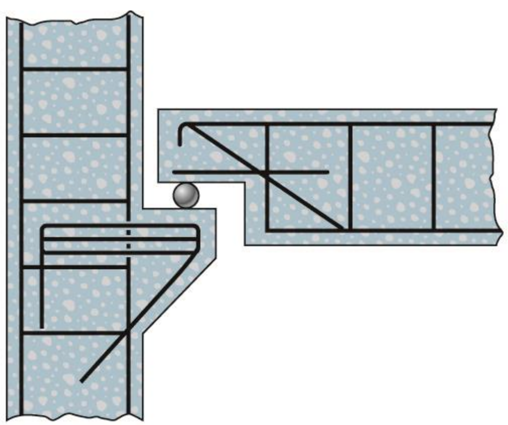
Idealized Structure



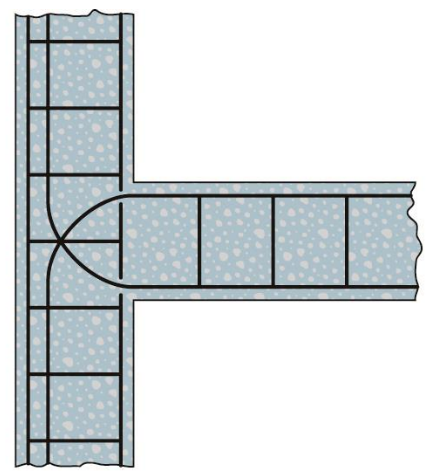
typical “pin-supported” connection (metal)



typical “fixed-supported” connection (metal)

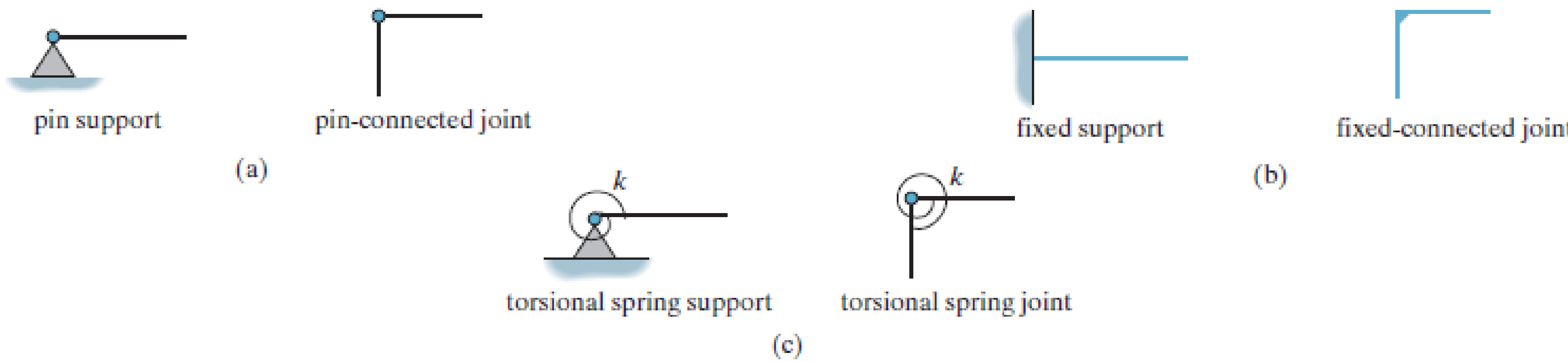


typical “roller-supported” connection (concrete)



typical “fixed-supported” connection (concrete)

Idealized Structure



Idealized Structure

- Support Connections
  - In reality, all connections exhibit some stiffness toward joint rotations owing to friction & material behavior
  - If $k = 0$, the joint is pin, and if $k \to \infty$, the joint is fixed
  - When selecting the model for each support, the engineer must be aware of how the assumptions will affect the actual performance
  - The analysis of the loadings should give results that closely approximate the actual loadings

Idealized Structure

- Support Connections
  - In reality, all supports actually exert distributed surface loads on their contacting members

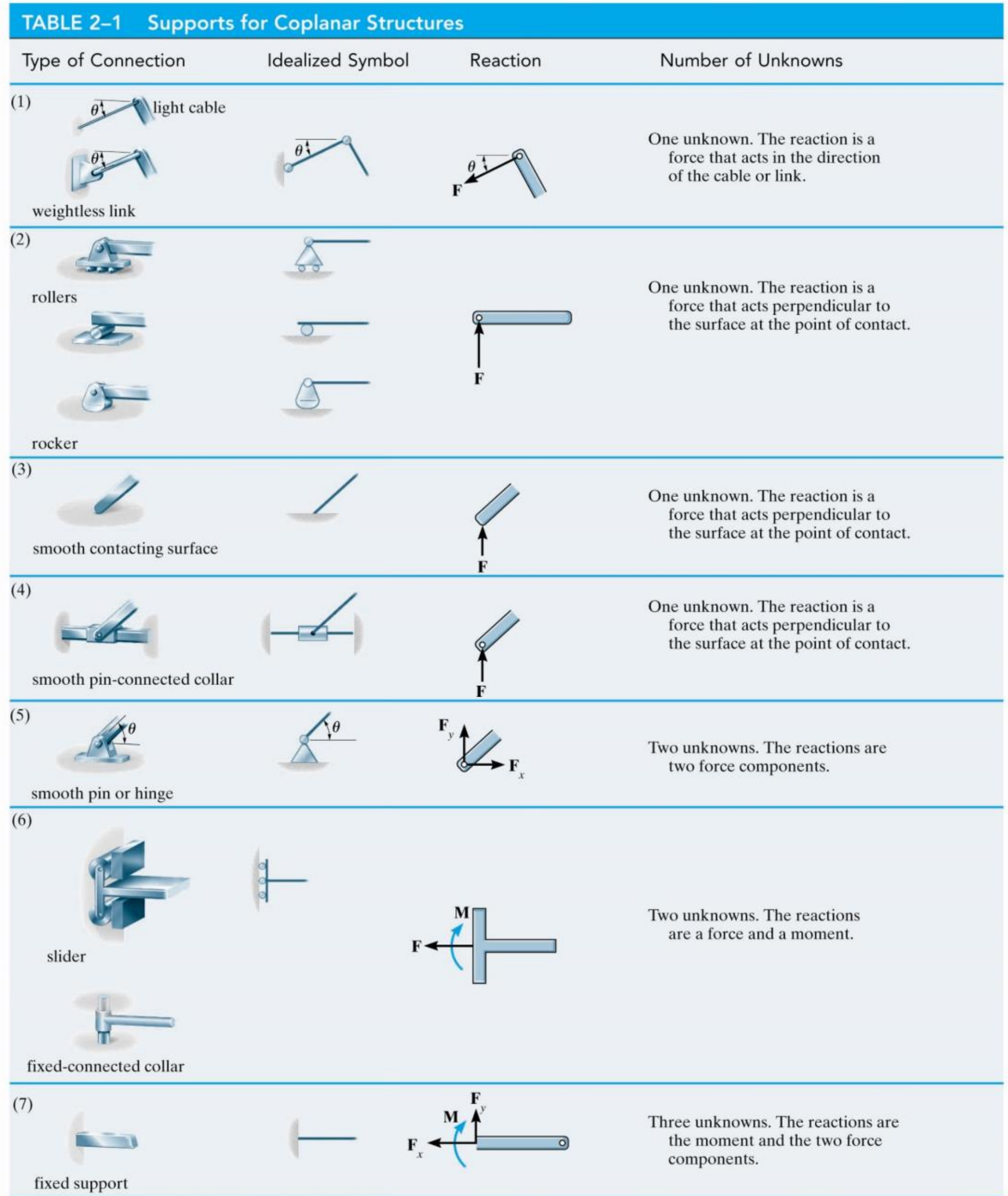
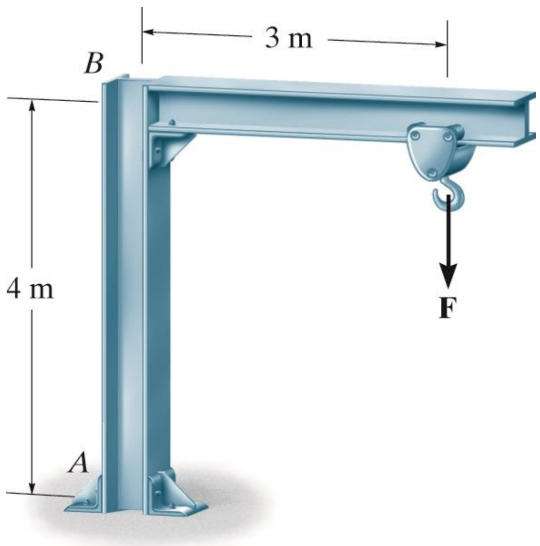


TABLE 2–1 Supports for Coplanar Structures

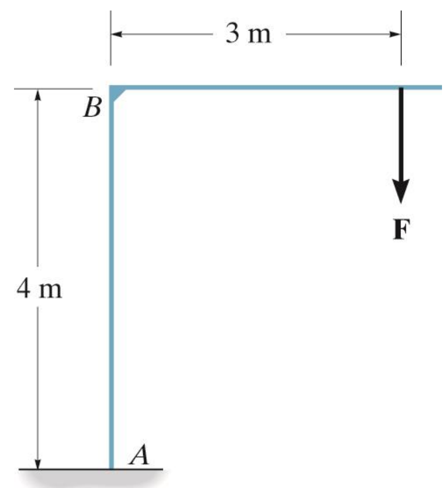
Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) light cable, weightless link		θ, F	One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) rollers, rocker		F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3) smooth contacting surface		F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) smooth pin-connected collar		F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5) smooth pin or hinge		F_y, F_x	Two unknowns. The reactions are two force components.
(6) slider, fixed-connected collar		M, F	Two unknowns. The reactions are a force and a moment.
(7) fixed support		F_y, M, F_x	Three unknowns. The reactions are the moment and the two force components.

Idealized Structure

- Idealized Structure
  - Consider the jib crane & trolley, we neglect the thickness of the 2 main member & will assume that the joint at B is fabricated to be rigid
  - The support at *A* can be modeled as a fixed support



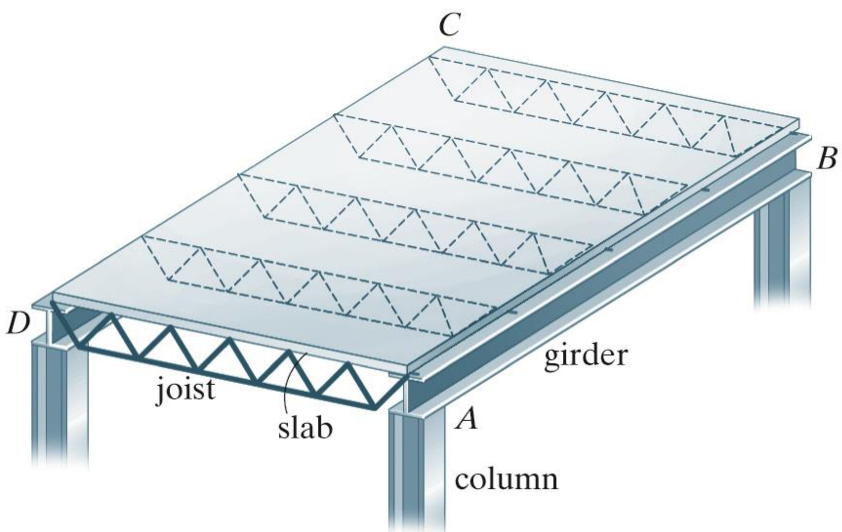
actual structure



idealized structure

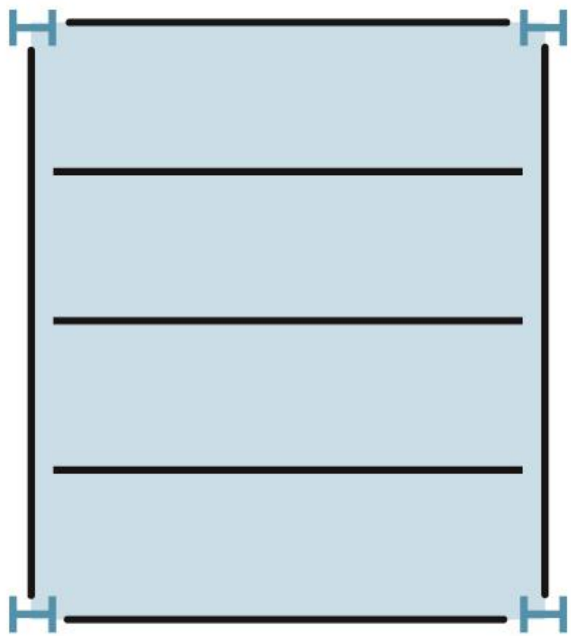
Idealized Structure

- Idealized Structure
  - Consider the framing used to support a typical floor slab in a building
  - The slab is supported by floor joists located at even intervals
  - These are in turn supported by 2 side girders *AB* & *CD*



Idealized Structure

- Idealized Structure
  - For analysis, it is reasonable to assume that the joints are pin and/or roller connected to girders & the girders are pin and/or roller connected to columns



idealized framing plan

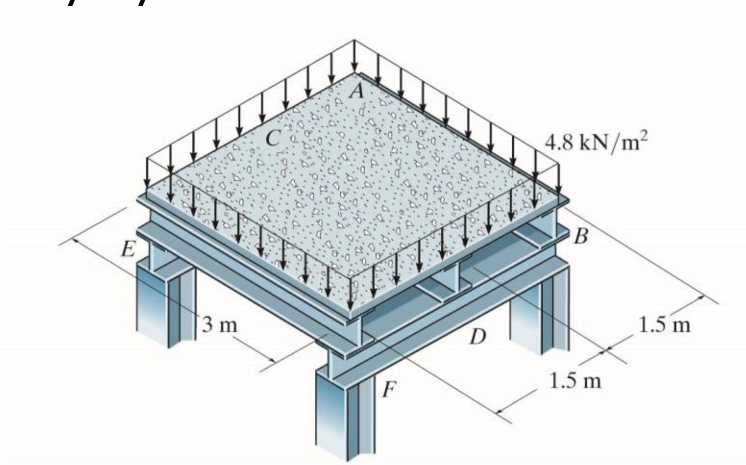
Idealized Structure

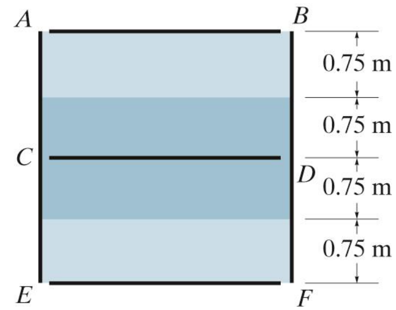
- Tributary Loadings
  - There are 2 ways in which the load on surfaces is transmitted to various structural elements

  1. 1-way system
  2. 2-way system

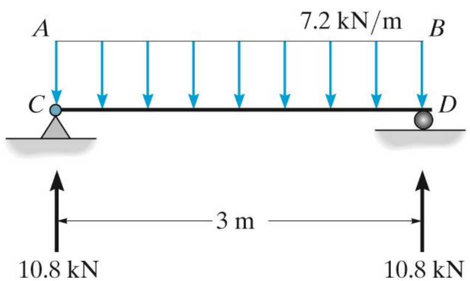
Idealized Structure

- Tributary Loadings
  1. 1-way system

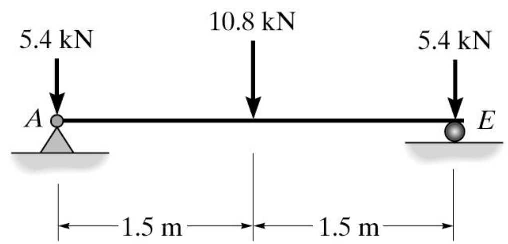




idealized framing plan



idealized beam



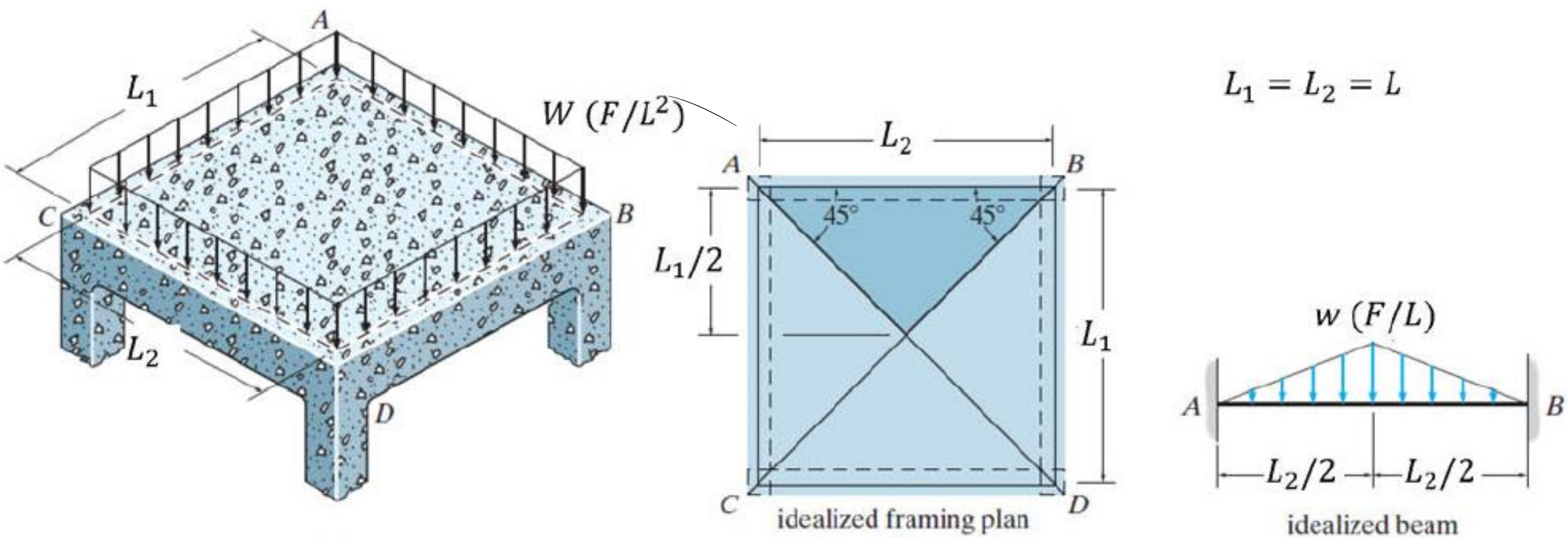
idealized girder

Idealized Structure

- Tributary Loadings

  2-way system

  Case 1: $L_1 = L_2$



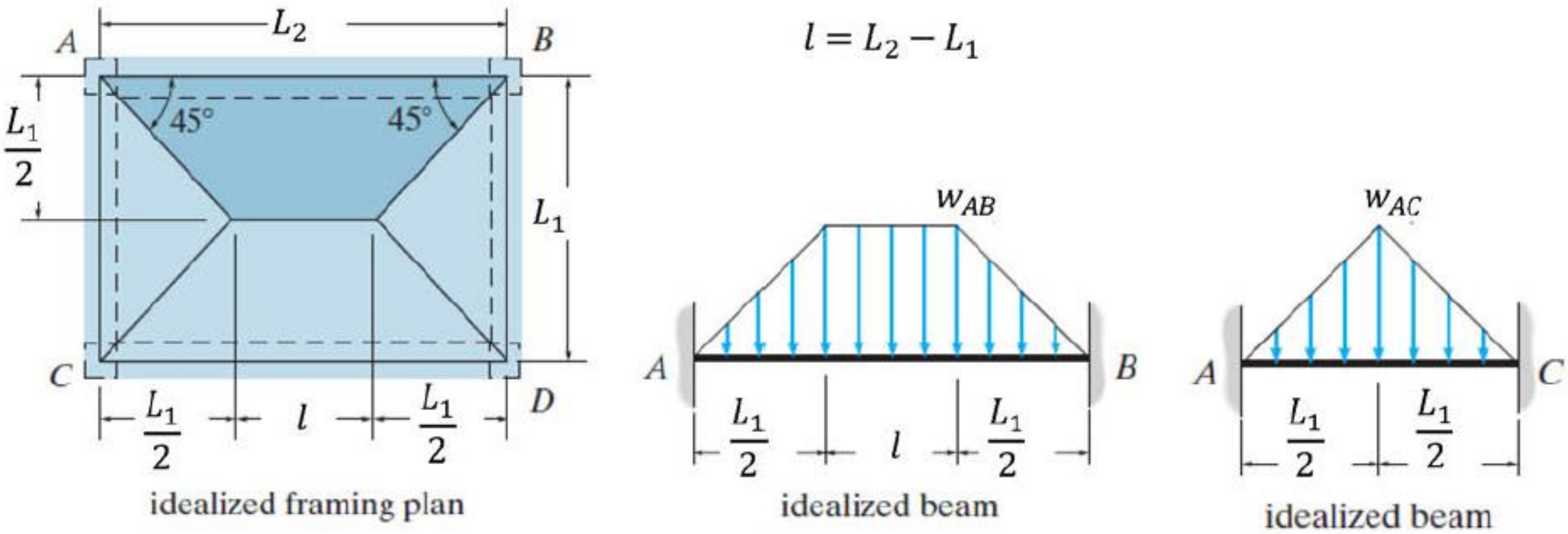
(a) (b) (c)

$$w\left(\frac{F}{L}\right) = W\left(\frac{F}{L^2}\right) \times \frac{L}{2} = \frac{WL}{2}$$

Idealized Structure

Case 2: L1 $\neq L_2$

$$l = L_2 - L_1$$



(a)

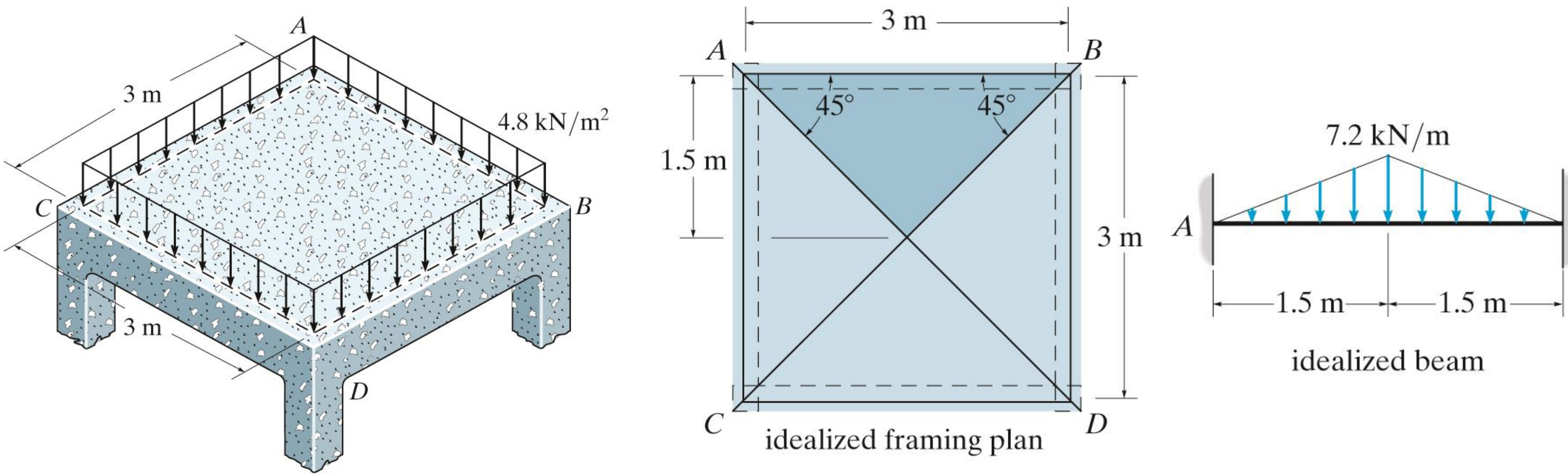
(c)

$$W_{AB} \quad \left(\frac{F}{L}\right) = W\left(\frac{F}{L^2}\right) \times \frac{L_1}{2} = \frac{WL_1}{2}$$

$$W_{AC} \quad \left(\frac{F}{L}\right) = W\left(\frac{F}{L^2}\right) \times \frac{L_1}{2} = \frac{WL_1}{2}$$

Idealized Structure

- Tributary Loadings
  1. 2-way system



Idealized Structure

Example 2.1

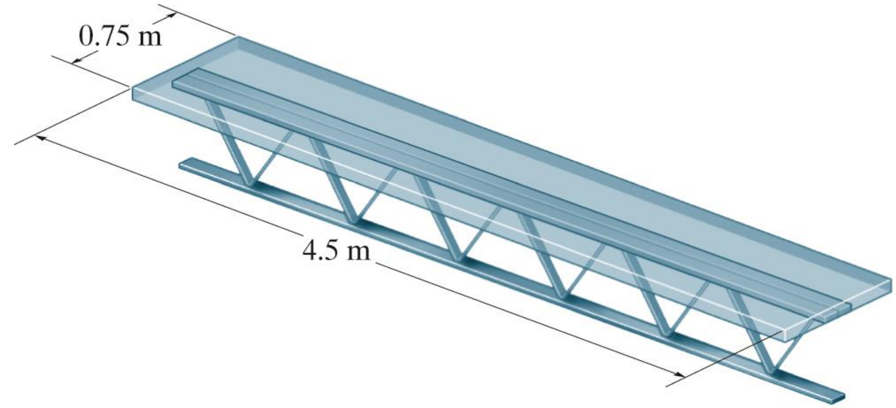
The floor of a classroom is to be supported by the bar joists as shown. Each joist is 4.5 m long and they are spaced 0.75 m on centers.

The floor itself is to be made from lightweight concrete that is 100 mm thick.

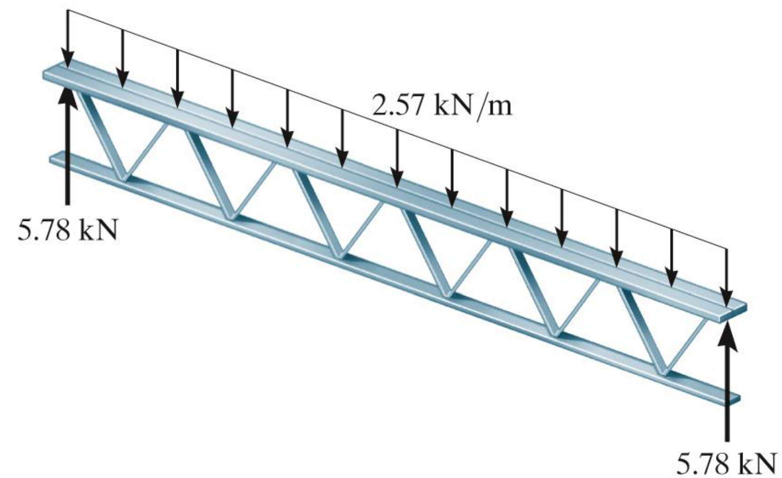
Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

Idealized Structure

Example 2.1 (Solution)

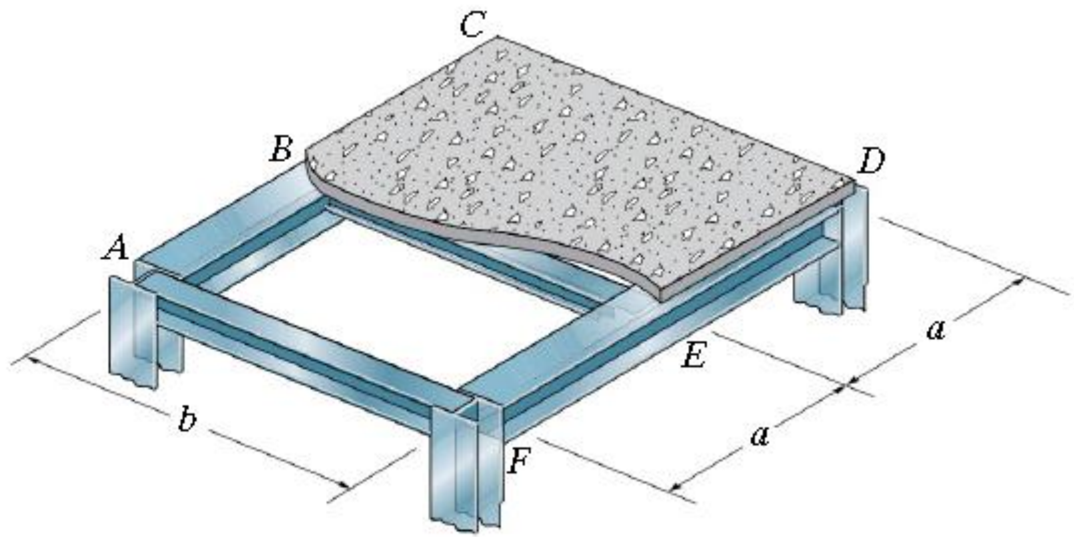


$$\text{Dead load, weight of concrete slab}$$
$$= (100)(0.015)$$
$$= 1.50\,\text{kN/m}^2$$
$$\text{Live load} = 1.92\,\text{kN/m}^2$$
$$\text{Total load} = 1.50 + 1.92 = 3.42\,\text{kN/m}^2$$
$$L_1 = 0.75\,\text{m}, L_2 = 4.5\,\text{m}$$
$$L_1 / L_2 > 2 \Rightarrow 1\text{-way slab}$$
$$\text{Uniform load along its length, } w$$
$$= 3.42\,\text{kN/m}^2(0.75\,\text{m}) = 2.57\,\text{kN/m}$$



HW 2-1

The steel framework is used to support the reinforced stone concrete slab that is used for an office. The slab is 200 mm thick. Sketch the loading that acts along members *BE* and *FED*. Take $a = 2$ m, $b = 5$ m. *Hint:* See Tables 1.2 and 1.4.



Ans. ***BE*: 14.2 kN/m**

FED*: 35.6 kN at *E

2.2

PRINCIPLE OF SUPERPOSITION

2.2

Principle of Superposition

- The total disp. or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately
- Linear relationship exist among loads, stresses & displacements
- 2 requirements for the principle to apply:
  - Material must behave in a linear-elastic manner, Hooke's Law is valid
  - The geometry of the structure must not undergo significant change when the loads are applied, small displacement theory

2.3

EQUATIONS OF EQUILIBRIUM

2.3

Equations of Equilibrium

- For equilibrium:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- For most structures, it can be reduced to:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_o = 0$$

2.4
DETERMINACY AND STABILITY

2.4

Determinacy and Stability

- Determinacy
  - Equilibrium equations provide both the necessary and sufficient conditions for equilibrium
  - All forces can be determined strictly from these equations
  - No. of unknown forces > equilibrium equation => statically indeterminate
  - This can be determined using free body diagrams

Determinacy and Stability

- Determinacy
  - For a coplanar structure

$$r = 3n, \quad \text{statically determinate}$$

$$r > 3n, \quad \text{statically indeterminate}$$

  - The additional equations needed to solve for the unknown equations are obtained as compatibility equations

Determinacy and Stability

Example 2.4

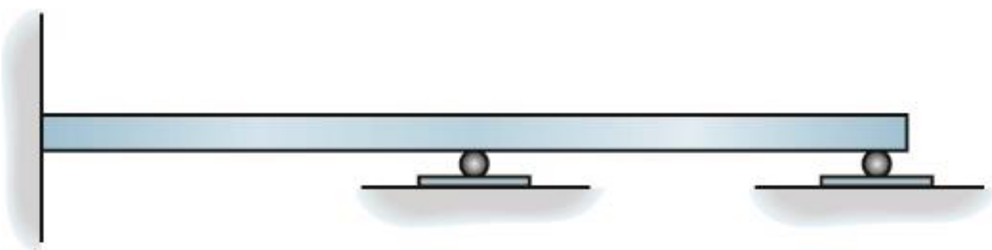
Classify each of the beams as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

Determinacy and Stability

Example 2.4 (Solution)

$$r = 3, n = 1, 3 = 3(1)$$

Statically determinate

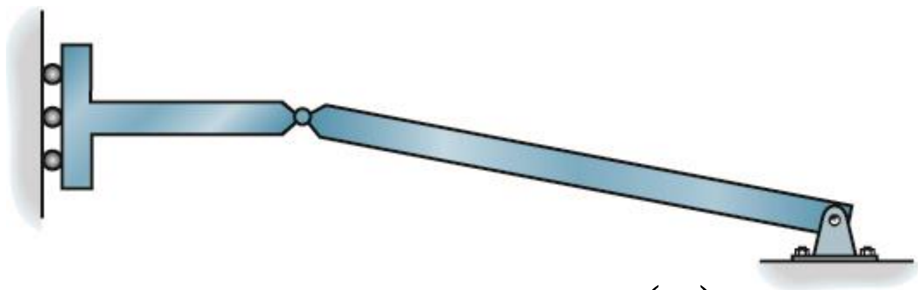


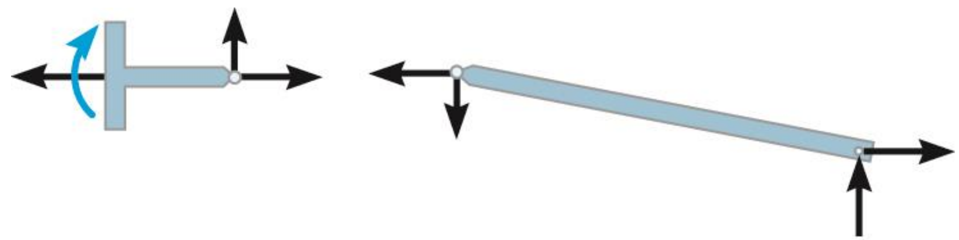
$$r = 5, n = 1, 5 > 3(1)$$

Statically indeterminate to the second degree

Determinacy and Stability

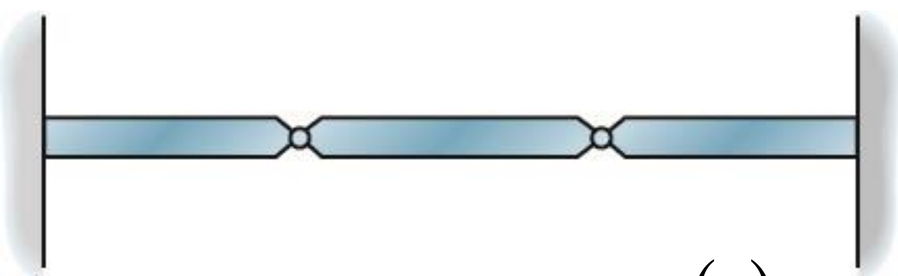
Example 2.4 (Solution)





$$r = 6, n = 2, 6 = 3(2)$$

Statically determinate



$$r = 10, n = 3, 10 > 3(3)$$

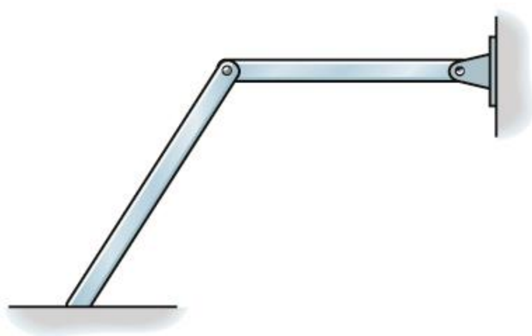
Statically indeterminate to the first degree

Determinacy and Stability

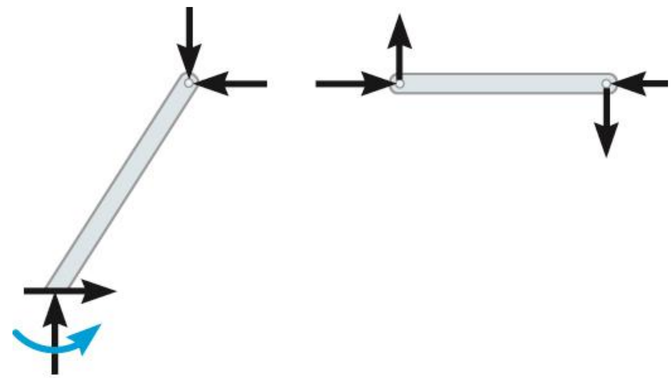
Example 2.5
Classify each of the pin-connected structures as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The structures are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.

Determinacy and Stability

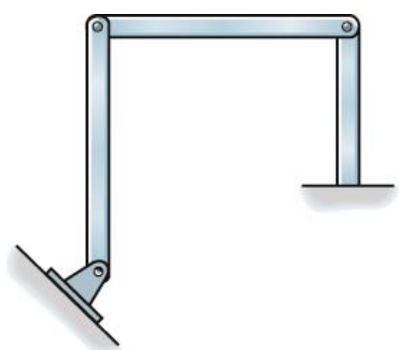
Example 2.5 (Solution)



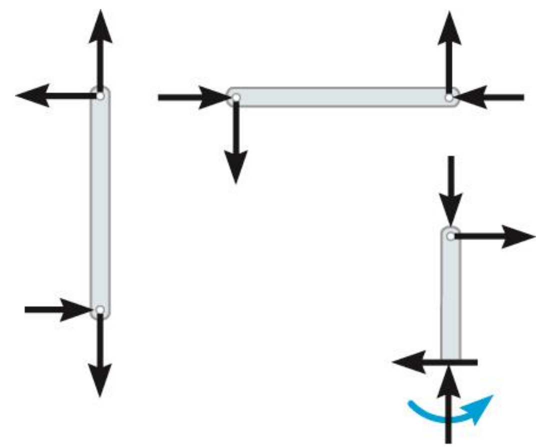
$$r = 7, n = 2, 7 > 6$$



Statically indeterminate to the first degree



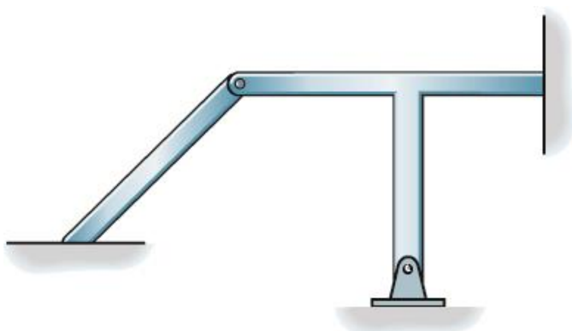
$$r = 9, n = 3, 9 = 9$$

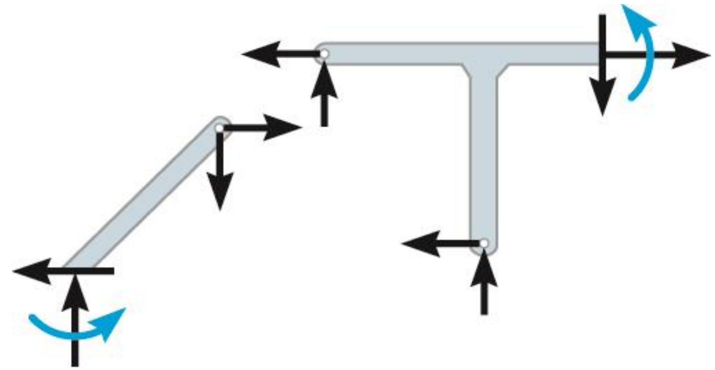


Statically determinate

Determinacy and Stability

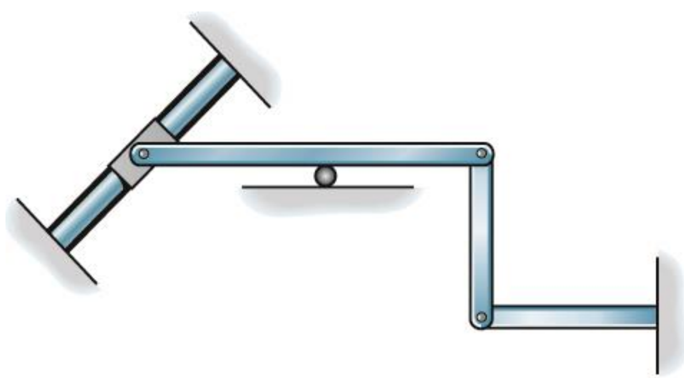
Example 2.5 (Solution)

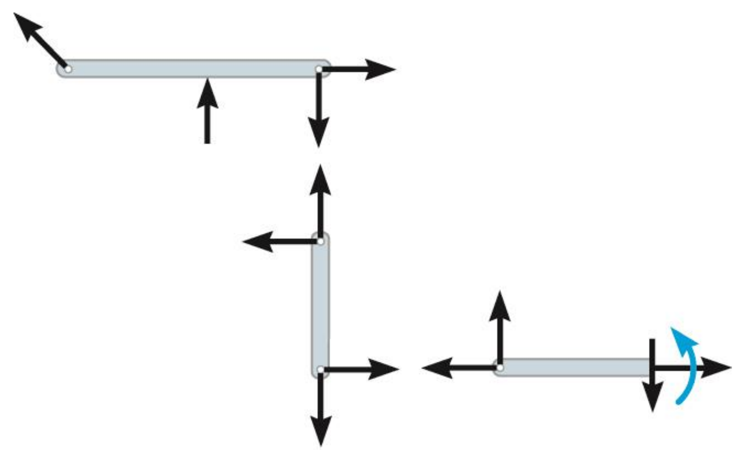




$$r = 10, n = 2, 10 > 6$$

Statically indeterminate to the fourth degree





$$r = 9, n = 3, 9 = 9$$

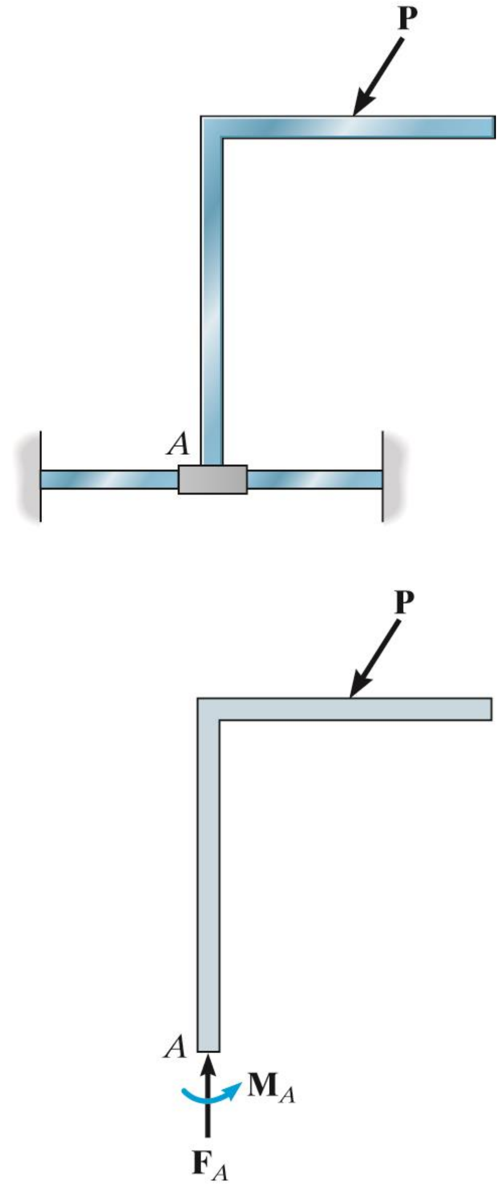
Statically determinate

Determinacy and Stability

- Stability
  - To ensure equilibrium of a structure or its members:
    - Must satisfy equations of equilibrium
    - Members must be properly held or constrained by their supports

Determinacy and Stability

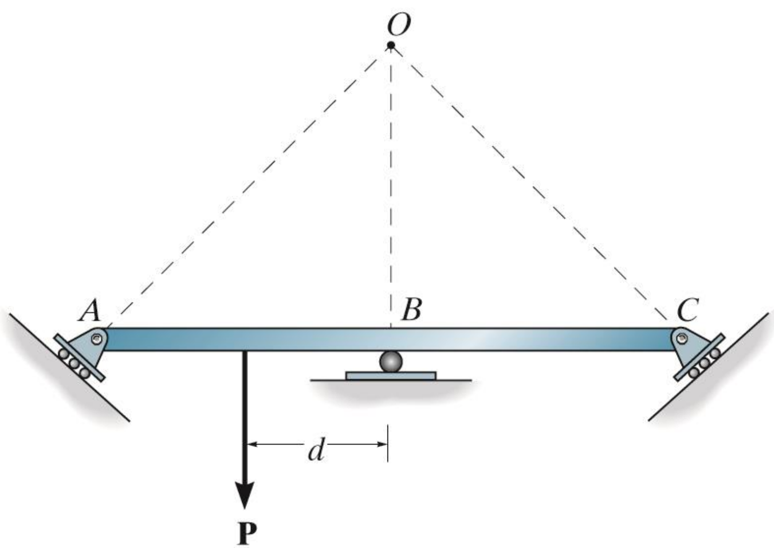
- Partial constraints
  - Fewer reactive forces than equations of equilibrium
  - $\sum F_x = 0$ will not be satisfied
  - Member will be unstable

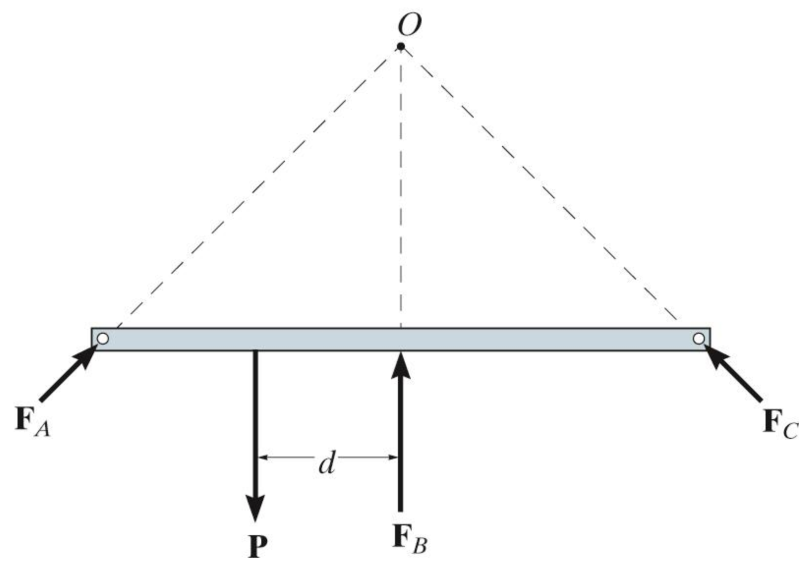


partial constraints

Determinacy and Stability

- Improper constraints
  - In some cases, unknown forces may equal equations of equilibrium in number
  - However, instability or movement of structure could still occur if support reactions are concurrent at a point





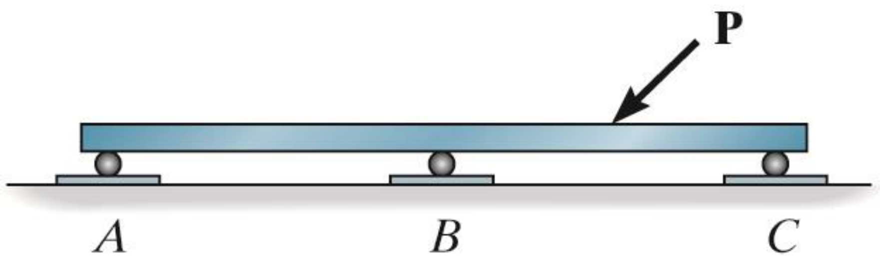
concurrent reactions

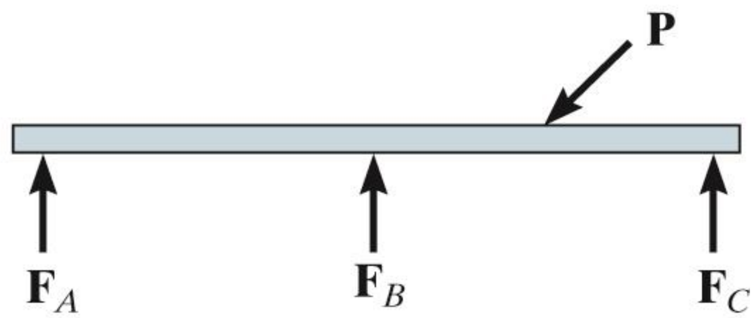
Determinacy and Stability

- Improper constraints

$$Pd \neq 0$$

- Rotation about *O* will take place
- Similarly instability can occur if all reactive forces are parallel





parallel reactions

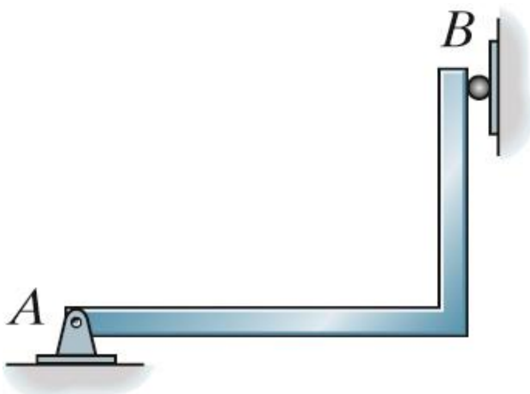
Determinacy and Stability

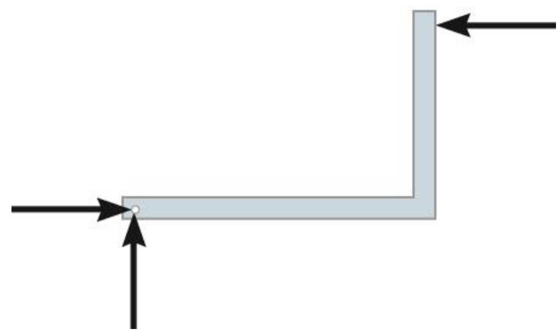
Example 2.7

Classify each of the structures as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.

Idealise Structure

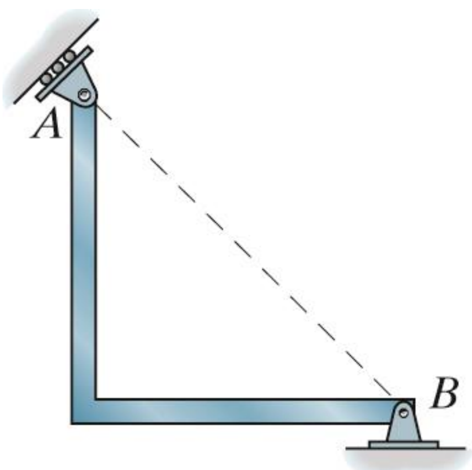
Example 2.7 (Solution)

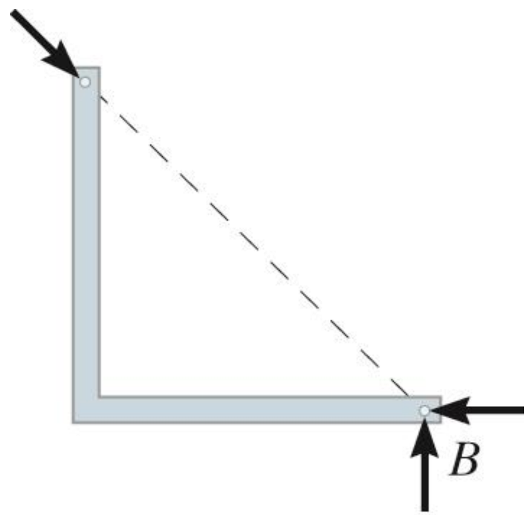




Stable,

statically determine



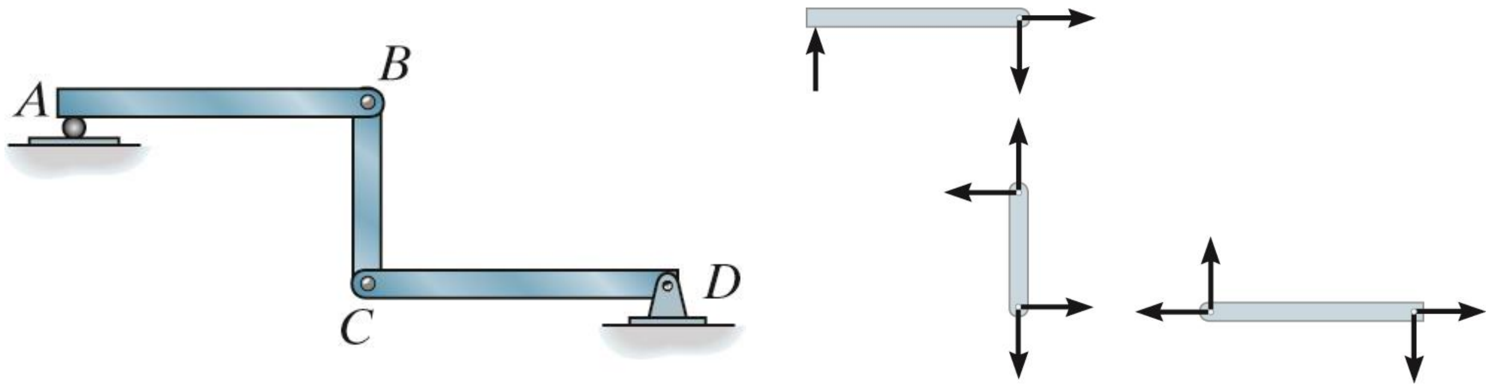


Unstable

Idealise Structure

Example 2.7 (Solution)

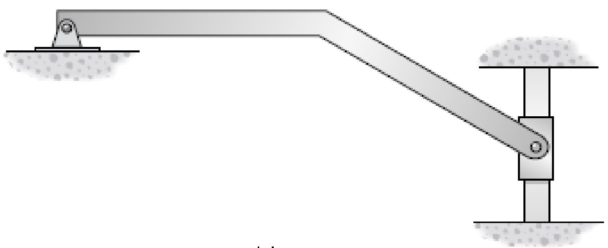
Unstable



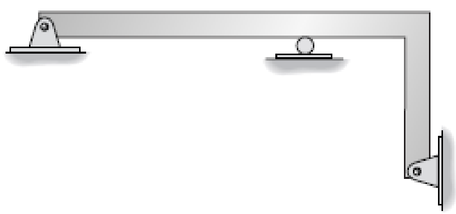
Unstable

HW 2-2

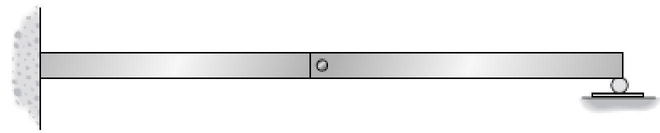
Classify each of the structures as statically determinate or indeterminate. If indeterminate, specify the degree of indeterminacy



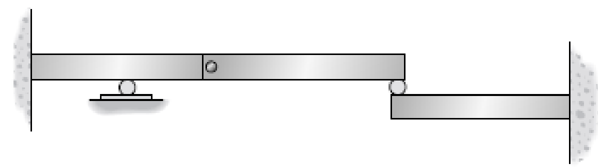
(a)



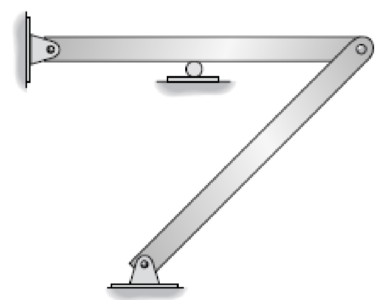
(b)



(c)



(d)



(e)

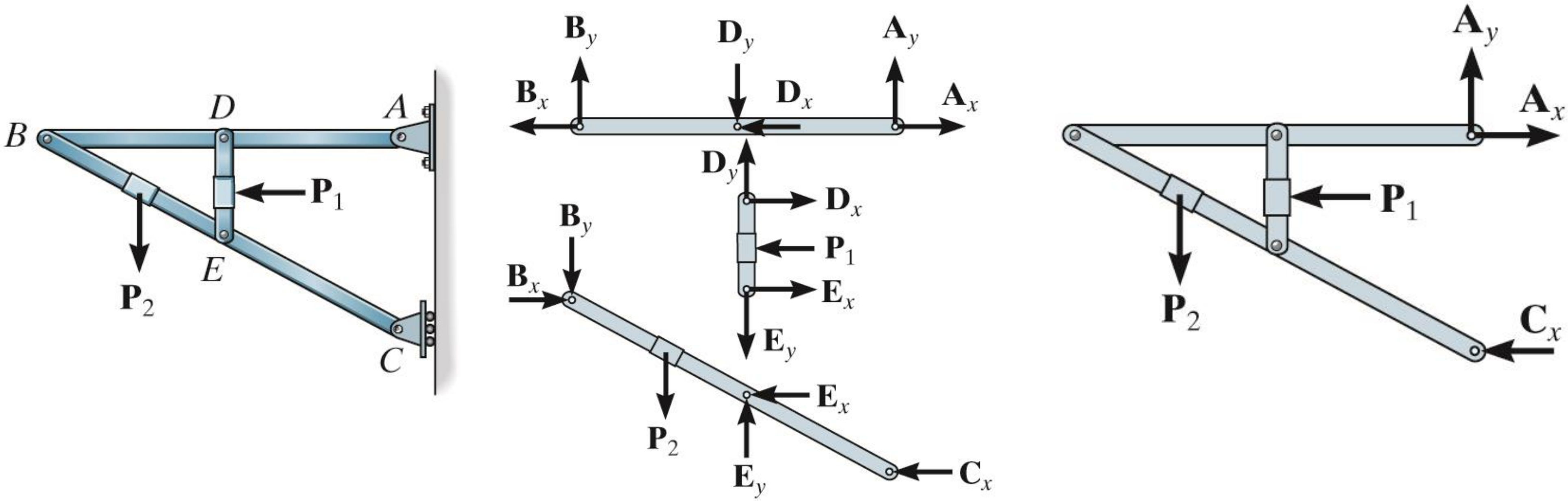
2.5

APPLICATION OF THE EQUATIONS OF EQUILIBRIUM

2.5

Application of the Equations of Equilibrium

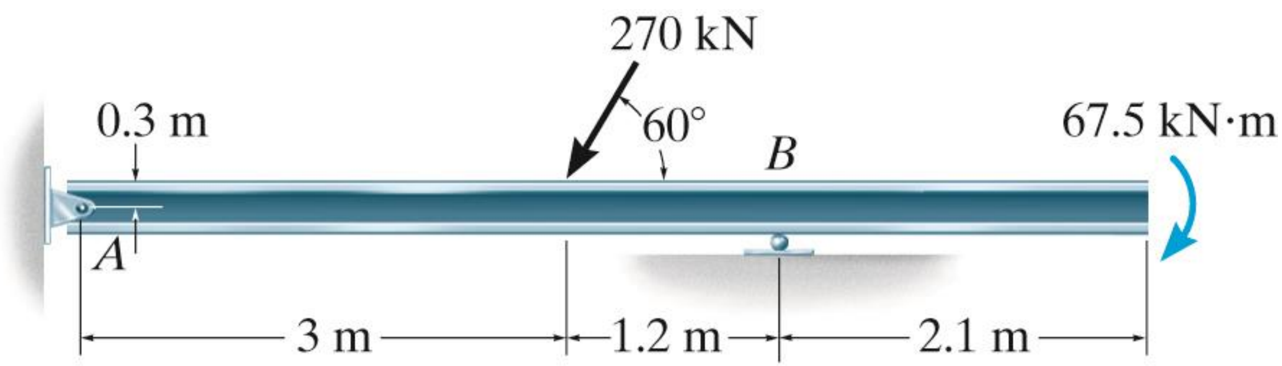
- Consider 3-member frame subjected to loads $\mathbf{P}_1$ & $\mathbf{P}_2$
- There are 9 unknowns in total
- 9 equations of equilibrium can be written, 3 for each member
- It is statically determinate

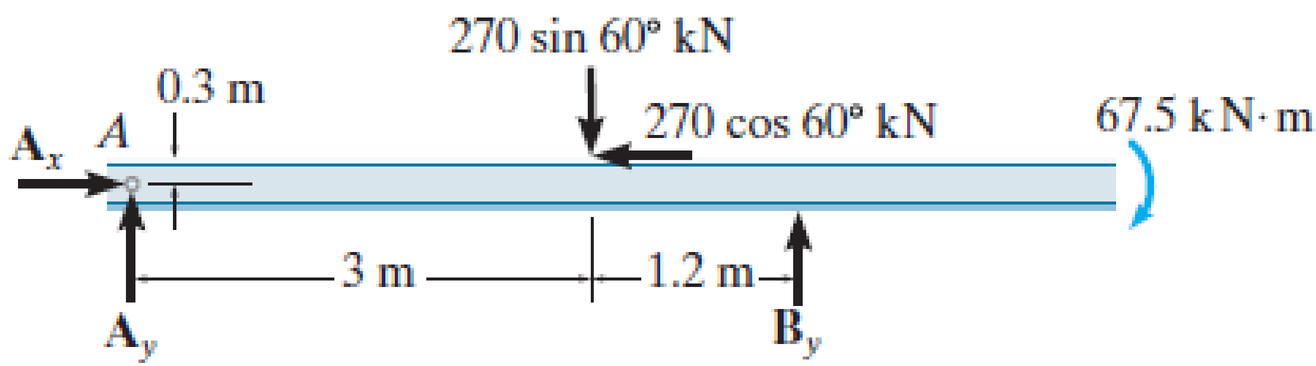


Application of the Equations of Equilibrium

Example 2.8

Determine the reactions on the beam as shown.





Application of the Equations of Equilibrium

Example 2.8 (Solution)

$$\underset{\rightarrow}{+}\sum F_x = 0;\ A_x - 270\cos 60^0 = 0$$

$$A_x = 135\,\text{kN}$$

With anti-clockwise moments in the + direction,

$$\sum M_A = 0;\ -270\sin 60^0(3) + 270\cos 60^0(0.3) + B_y(4.2) - 67.5 = 0$$

$$B_y = 173.4\,\text{kN}$$

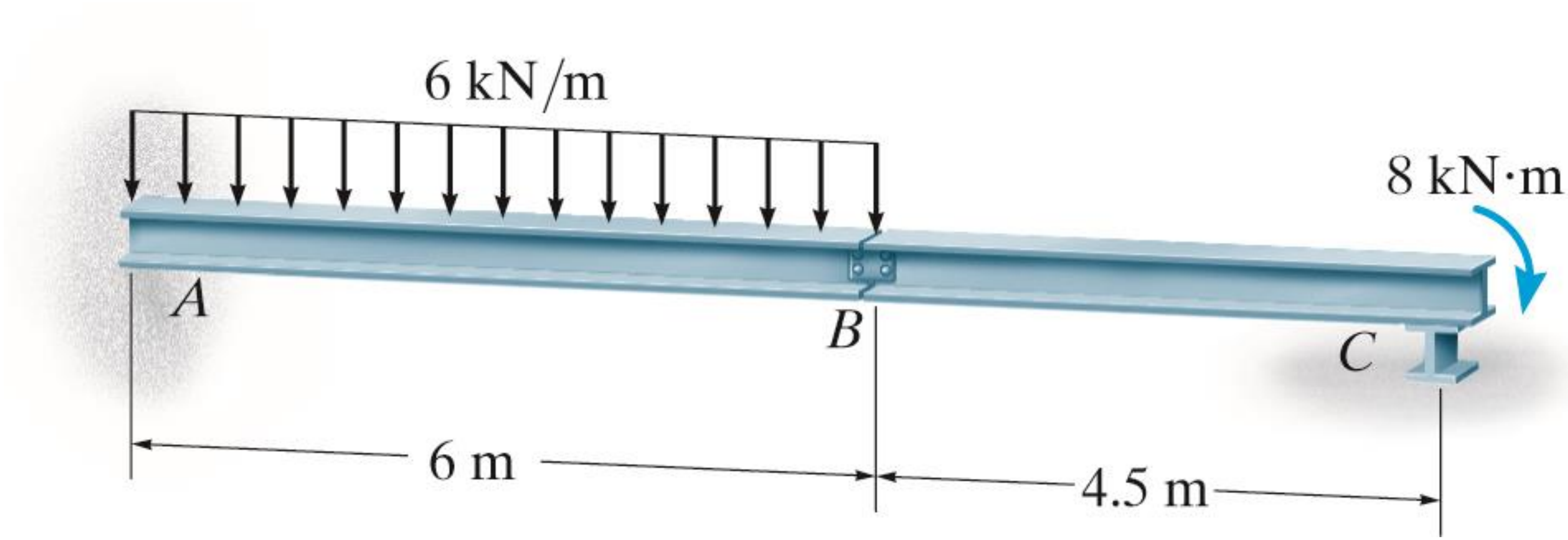
$$+\uparrow\sum F_y = 0;\ -270\sin 60^0 + 173.4 + A_y = 0$$

$$A_y = 60.4\,\text{kN}$$

Application of the Equations of Equilibrium

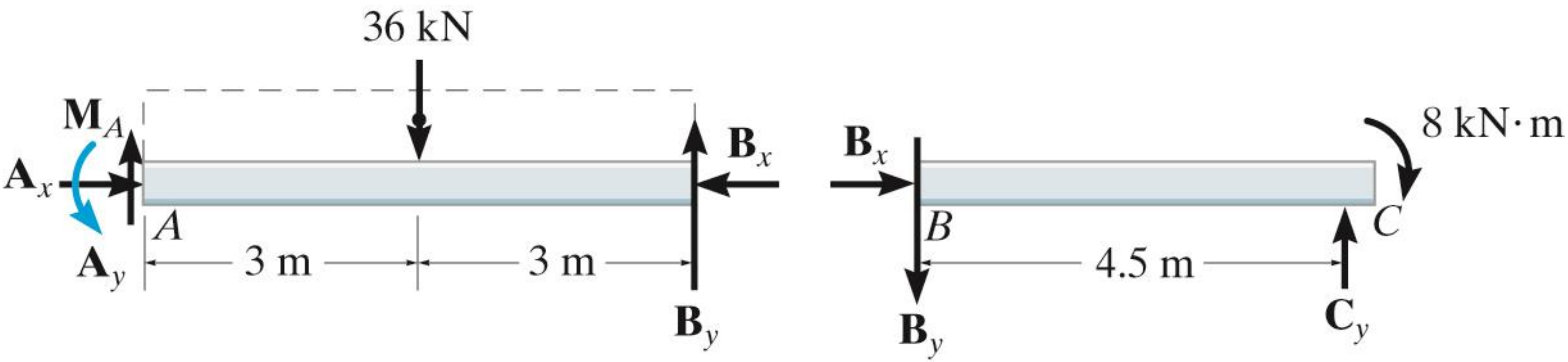
Example 2.11

The compound beam shown is fixed at *A*. Determine the reactions at *A*, *B*, and *C*. Assume that the connection at *B* is a pin and *C* is a roller.



Application of the Equations of Equilibrium

Example 2.11 (Solution)



Application of the Equations of Equilibrium

Example 2.11 (Solution)

Segment BC:

With anti-clockwise moments in the + direction,

$$\sum M_c = 0;\ -8 + B_y(4.5) = 0 \quad \Rightarrow B_y = 1.78\ \text{kN}$$

$$+\uparrow \sum F_y = 0;\ -1.78 + C_y = 0 \quad \Rightarrow C_y = 1.78\ \text{kN}$$

$$\underset{\rightarrow}{+} \sum F_x = 0;\ B_x = 0$$

Segment AB:

With anti-clockwise moments in the + direction,

$$\sum M_A = 0;\ M_A - 36(3) + (1.78)(6) = 0 \Rightarrow M_A = 97.3\ \text{kN} \bullet \text{m}$$

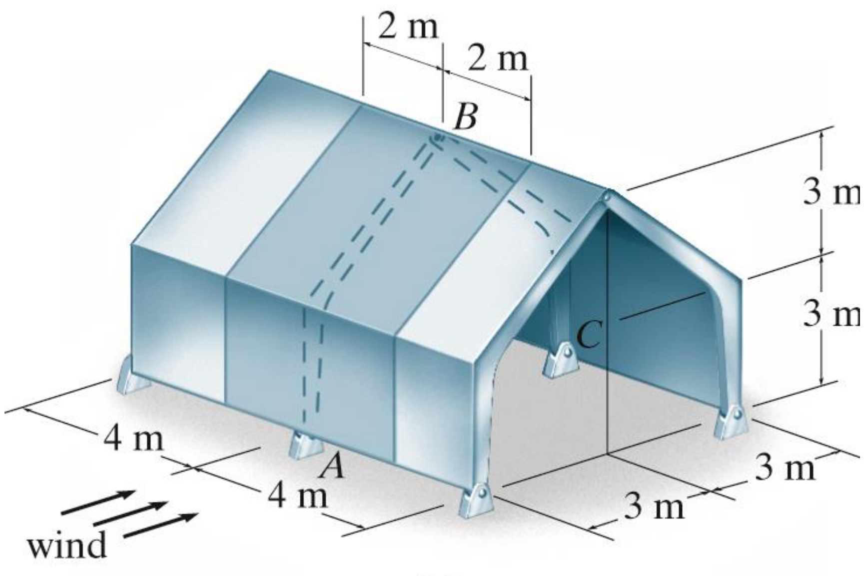
$$+\uparrow \sum F_y = 0;\ A_y - 36 + 1.78 = 0 \Rightarrow A_y = 34.2\ \text{kN}$$

$$\underset{\rightarrow}{+} \sum F_x = 0;\ A_x = 0$$

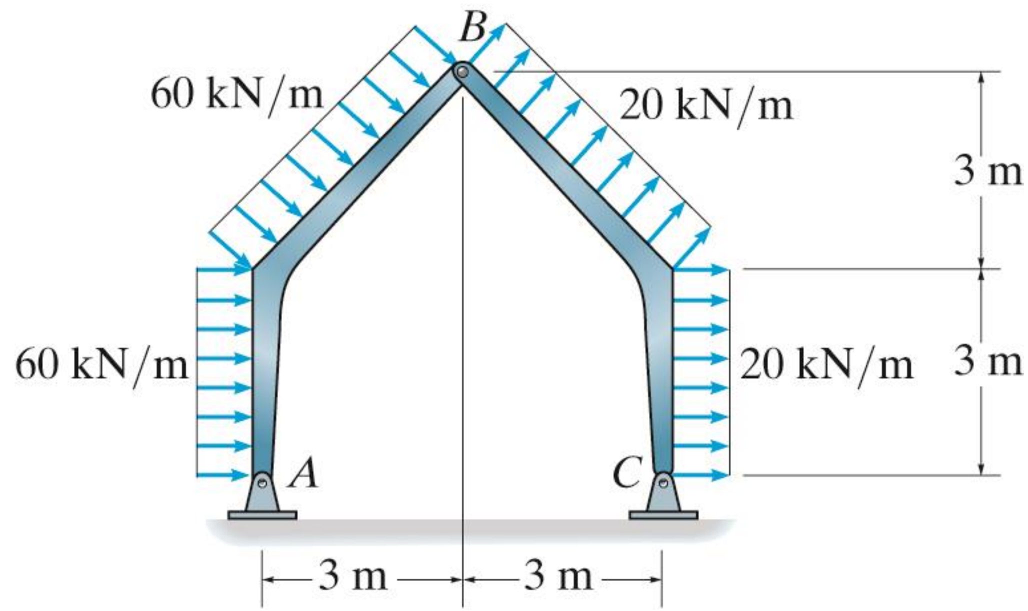
Application of the Equations of Equilibrium

Example 2.13

The side of the building is subjected to a wind loading that creates a uniform normal pressure of 15 kPa on the windward side and a suction pressure of 5 kPa on the leeward side. Determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.



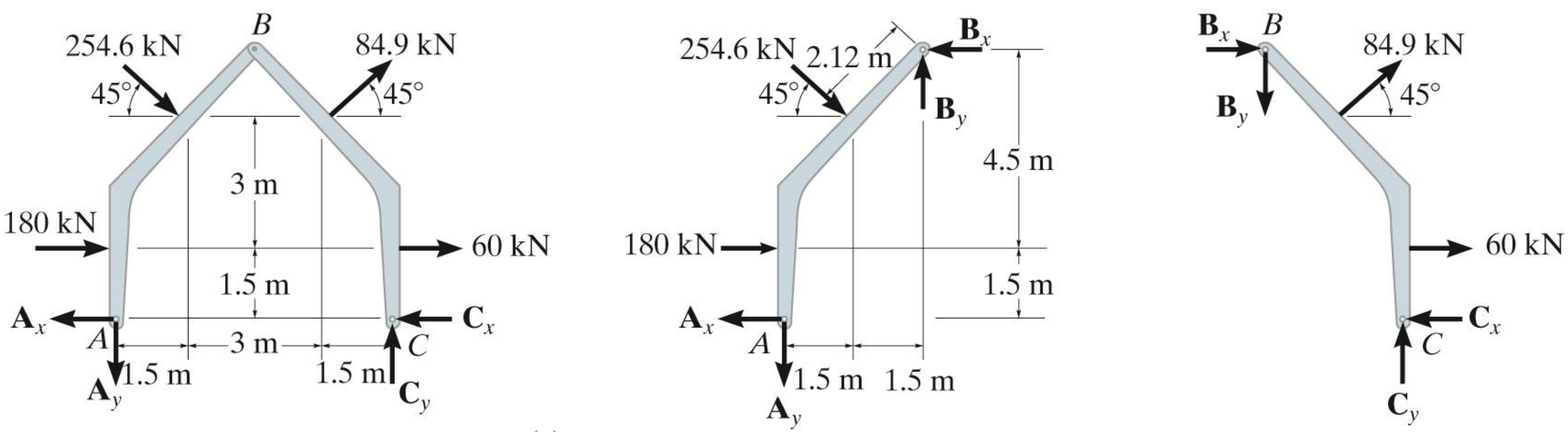
(a)



Application of the Equations of Equilibrium

Example 2.13 (Solution)

Since the loading is evenly distributed, the central gable arch supports a loading acting on the walls & roof of the dark-shaded tributary area. This represents a uniform distributed load of (15 kN/m^2)(4 m)=60 kN/m on the windward side and (5 kN/m^2)(4 m)=20 kN/m on the suction side.



Application of the Equations of Equilibrium

Example 2.13 (Solution)

By applying equilibrium equations in the following sequence,

Entire Frame:

With anti-clockwise moments in the + direction,

$$\sum M_A = 0;\ -(180+60)(1.5)-(254.6+84.9)\cos 45^{o}(4.5)$$

$$-(254.6\sin 45^{o})(1.5)+(84.9\sin 45^{o})(4.5)+C_y(6)=0$$

$$C_y = 240.0\,\text{kN}$$

$$+\uparrow \sum F_y = 0;\ -A_y - 254.6\sin 45^{o} + 84.9\sin 45^{o} + 240.0 = 0$$

$$A_y = 120.0\,\text{kN}$$

Application of the Equations of Equilibrium

Example 2.13 (Solution)

Member AB:

With anti-clockwise moments in the + direction,

$$\sum M_B = 0;\ -A_x(6) + 120.0(3) + 180(4.5) + (254.6)(2.12) = 0$$

$$A_x = 285.0\,\text{kN}$$

$$\underset{\rightarrow}{+}\sum F_x = 0;\ -285.0 + 180 + 254.6\cos 45^\circ - B_x = 0$$

$$B_x = 75.0\,\text{kN}$$

$$+\uparrow \sum F_y = 0;\ -120.0 - 254.6\sin 45^\circ + B_y = 0$$

$$B_y = 300.0\,\text{kN}$$

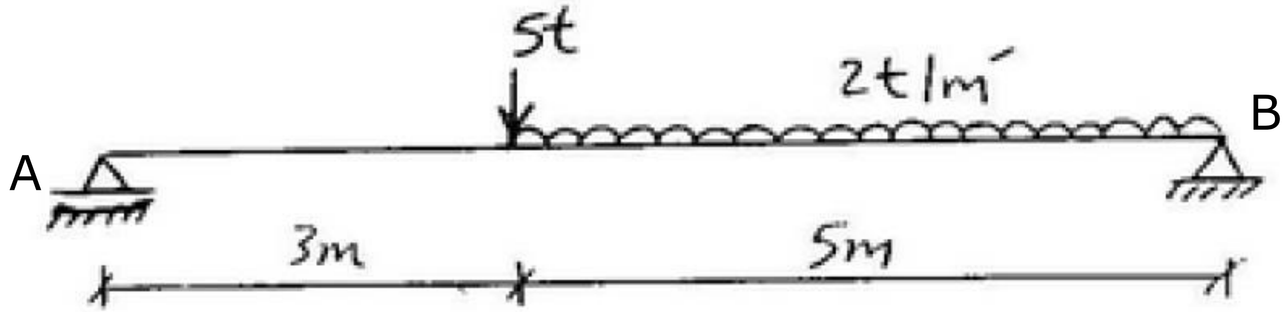
Member AB:

$$\underset{\rightarrow}{+}\sum F_x = 0;\ -C_x + 60 + 84.9\cos 45^\circ + 75.0 = 0$$

$$C_x = 195.0\,\text{kN}$$

HW 2-3

Find the reactions in the supports



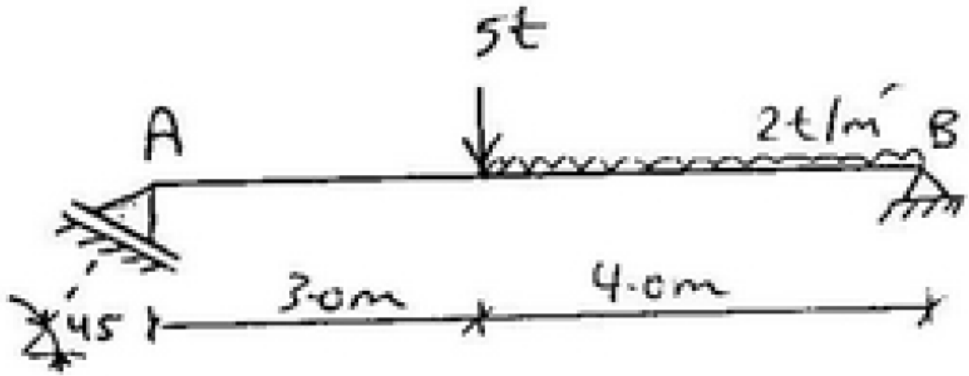
Ans. Bx=0

By=8.75 ton

Ay=6.25 ton

HW 2-4

Find the reactions in the supports



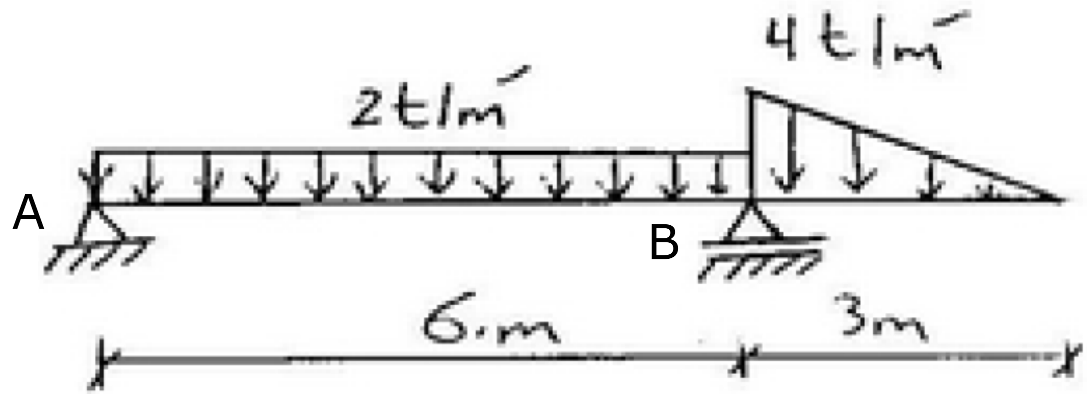
Ans. Bx=5.14 ton

By=7.86 ton

RA= 7.27 ton

HW 2-5

Find the reactions in the supports



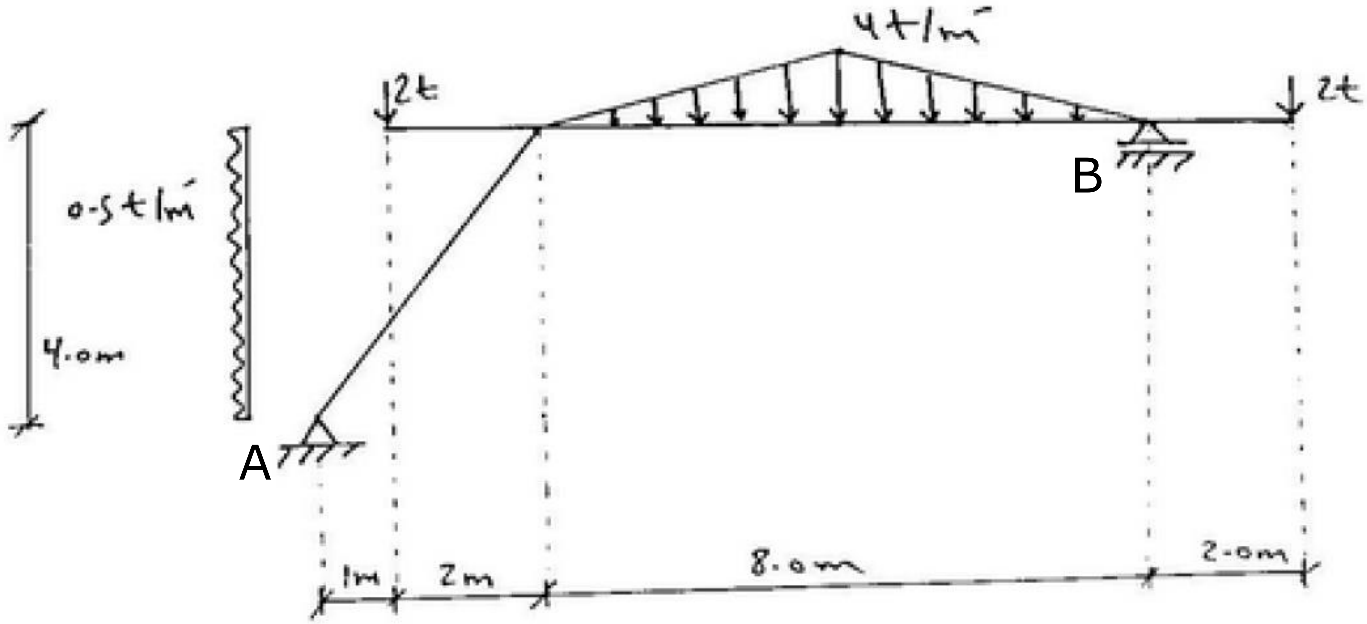
Ans. Ay=5 ton

Ax= 0

By=13 ton

HW 2-6

Find the reactions in the supports



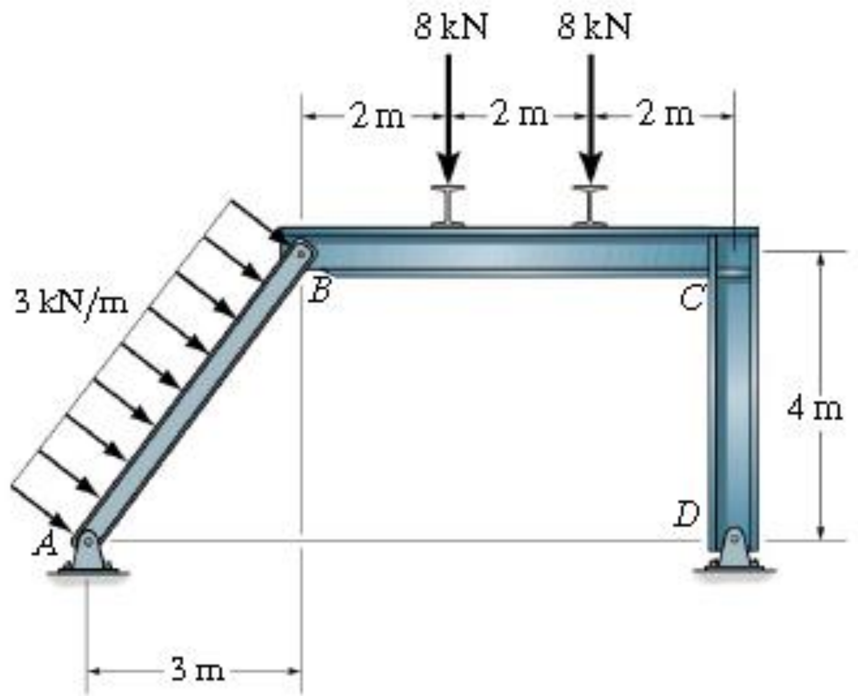
Ans. Ay=6.9 ton

Ax= 2 ton

By=13.1 ton

HW 2-7

Determine the horizontal and vertical components of reaction at the pins *A, B,* and *D* of the three-member frame. The joint at *C* is fixed connected.



Ans. Ay=10.167 KN

Ax= 1.75 KN

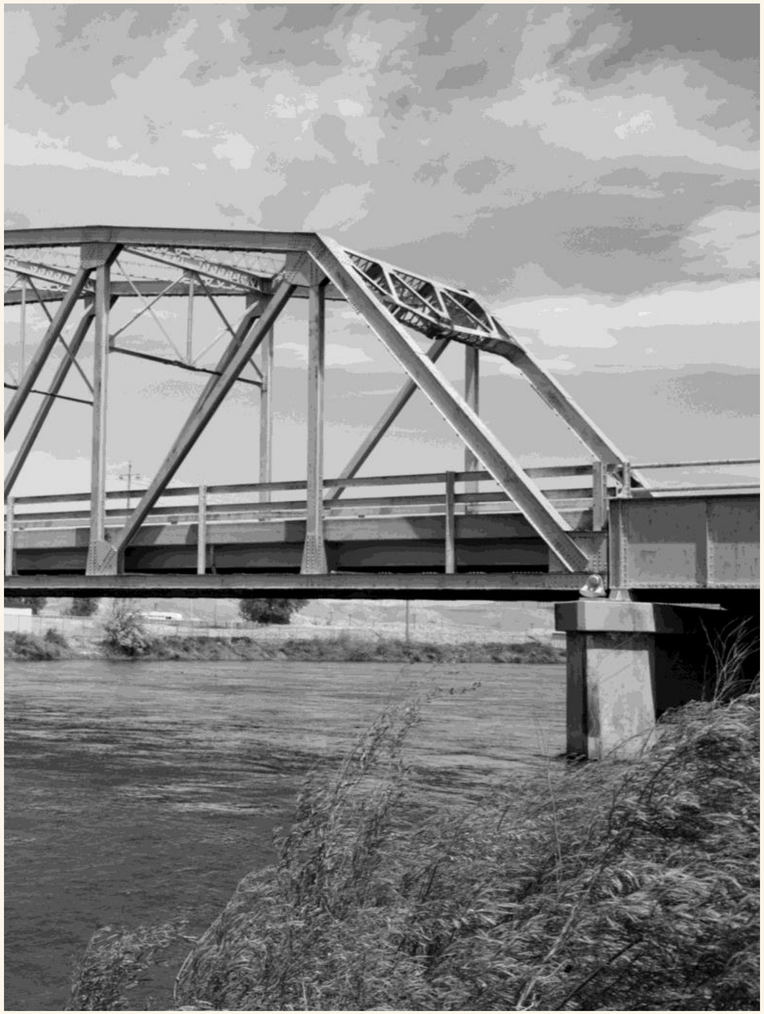
Bx=10.25 KN

By=1.167 KN

Dx=10.25 KN

Dy=14.8 KN

CHAPTER 3:
ANALYSIS OF STATISCALLY DETERMINATE TRUSSES



3

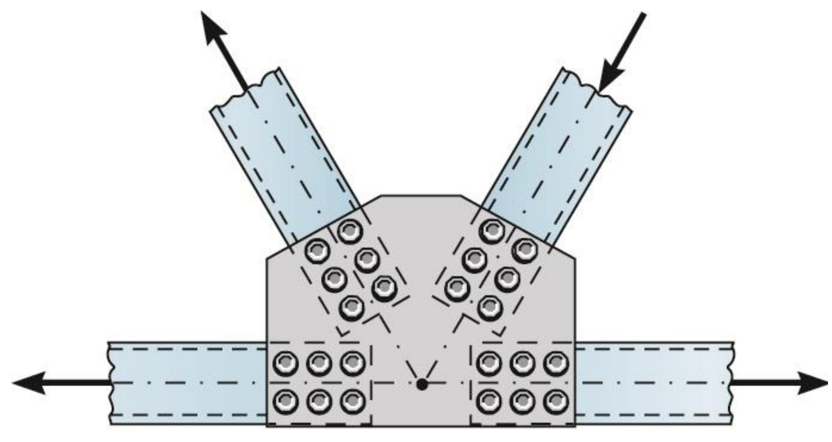
Chapter Outline

3.1
COMMON TYPES OF TRUSSES

3.1

Common Types of Trusses

- A truss is a structure composed of slender members joined together at their end points
- The joint connections are usually formed by bolting or welding the ends of the members to a common plate called a gusset plate
- Planar trusses lie in a single plane and is often used to support roofs and bridges



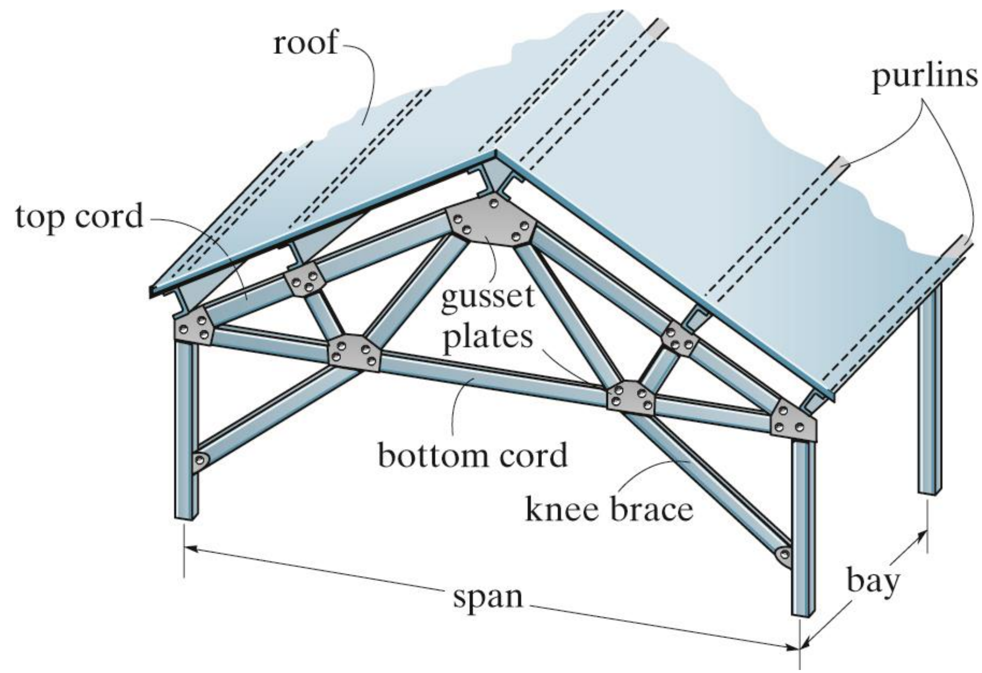
gusset plate

Common Types of Trusses

- Roof Trusses
  - They are often used as part of an industrial building frame

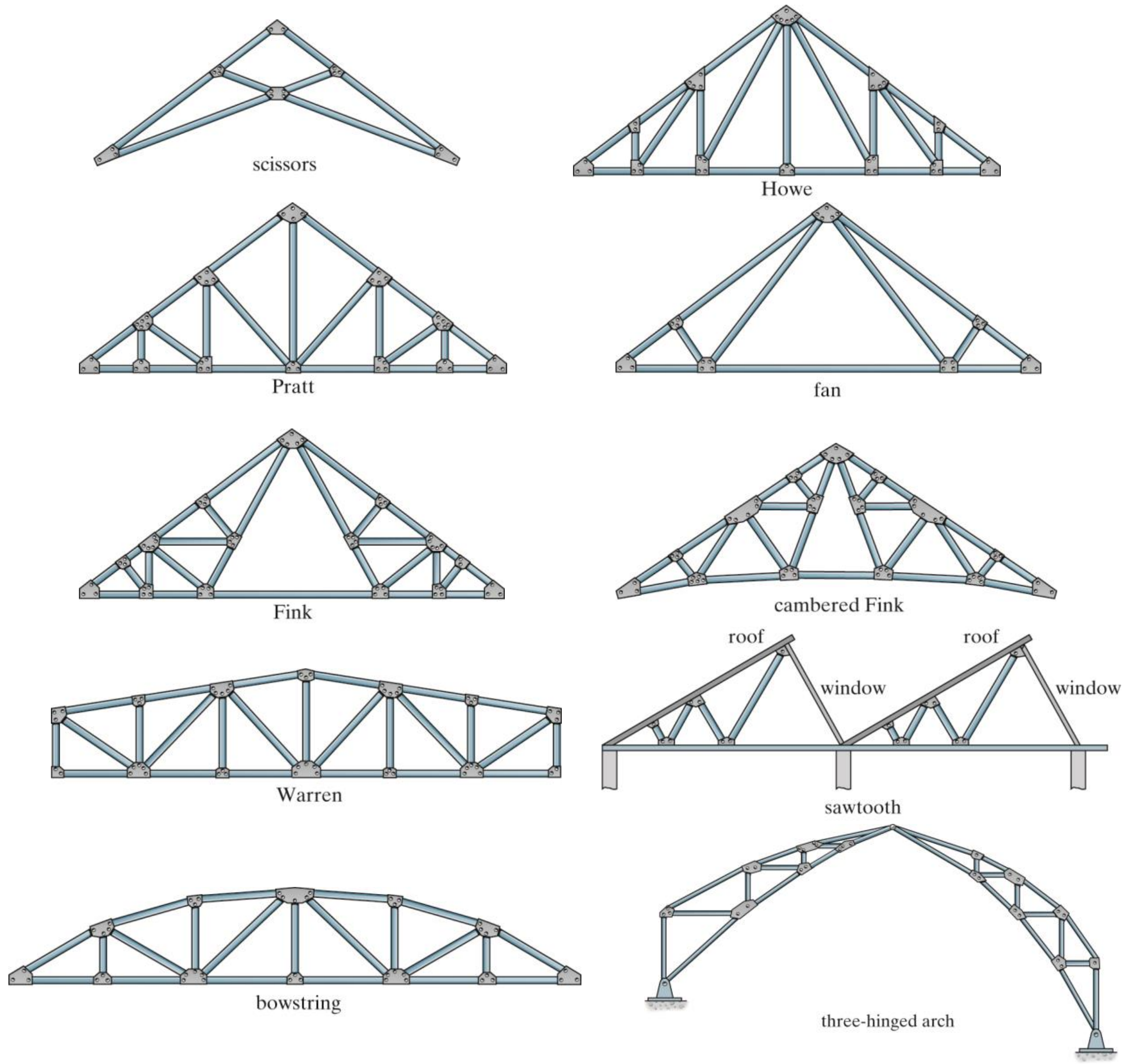
  - Roof load is transmitted to the truss at the joints by means of a series of purlins

  - To keep the frame rigid & thereby capable of resisting horizontal wind forces, knee braces are sometimes used at the supporting column



Common Types of Trusses

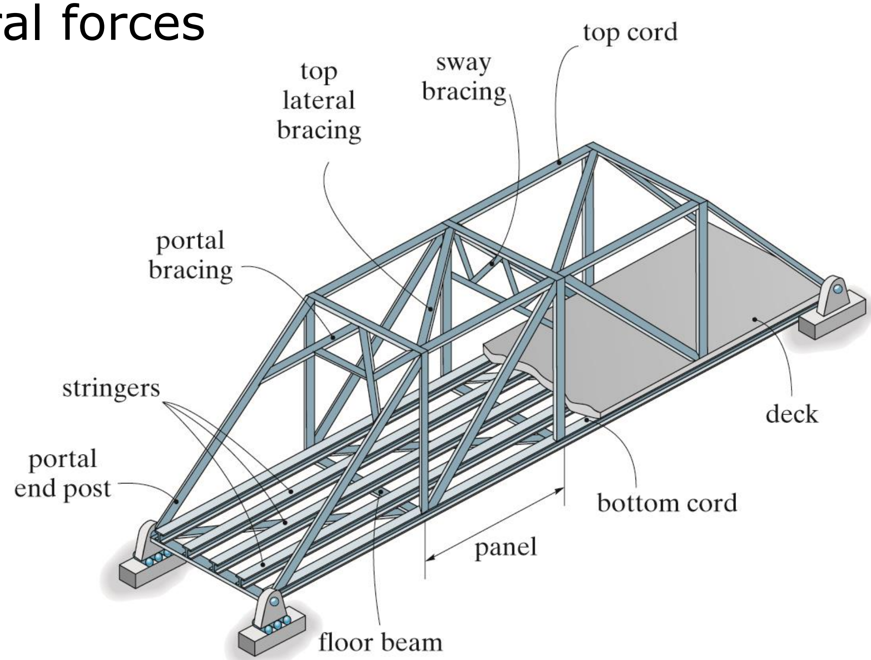
- Roof Trusses



Common Types of Trusses

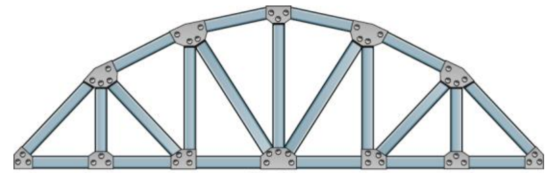
- Bridge Trusses
  - The load on the deck is first transmitted to stringers -> floor beams -> joints of supporting side truss

  - The top & bottom cords of these side trusses are connected by top & bottom lateral bracing which resists lateral forces

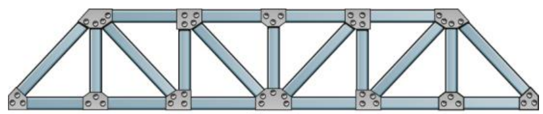


Common Types of Trusses

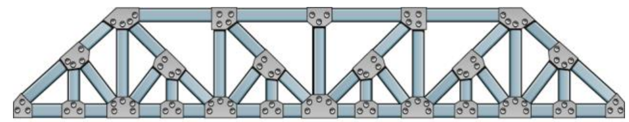
- Bridge Trusses
  - Additional stability is provided by the portal & sway bracing
  - In the case of a long span truss, a roller is provided at one end for thermal expansion



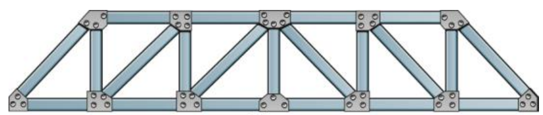
Parker



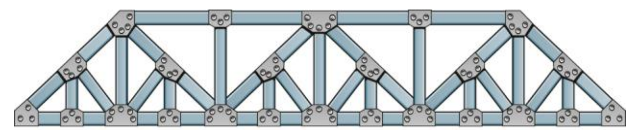
Pratt



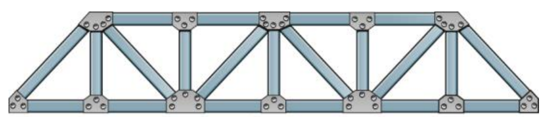
Baltimore



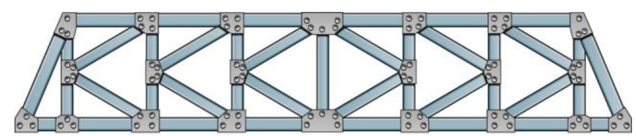
Howe



subdivided Warren



Warren (with verticals)



K-truss

Common Types of Trusses

- Assumptions for Design
  - The members are joined together by smooth pins
  - All loadings are applied at the joints

- Due to the 2 assumptions, each truss member acts as an axial force member

3.2

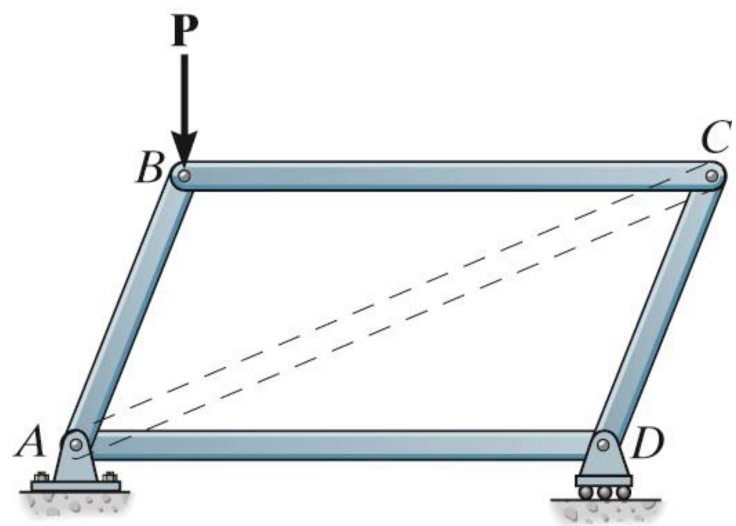
CLASSIFICATION OF COPLANAR TRUSSES

3.2

Classification of Coplanar Trusses

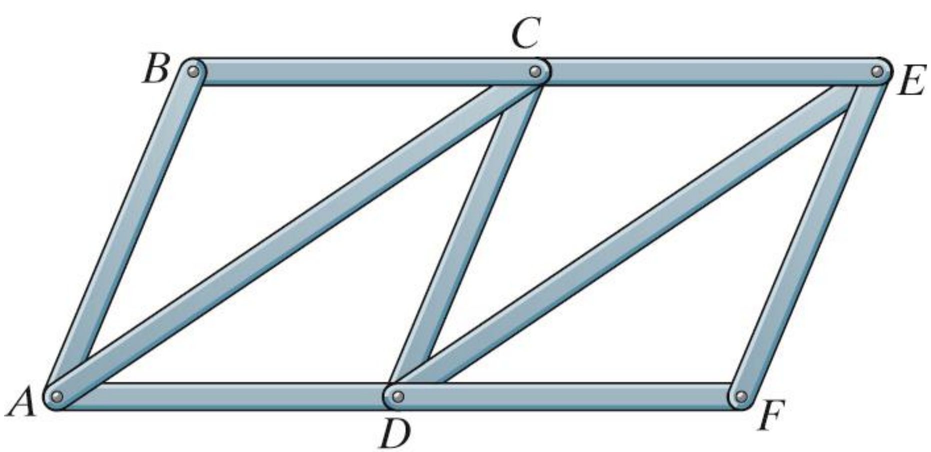
- Simple , Compound or Complex Truss

- Simple Truss
  - To prevent collapse, the framework of a truss must be rigid
  - The simplest framework that is rigid or stable is a triangle

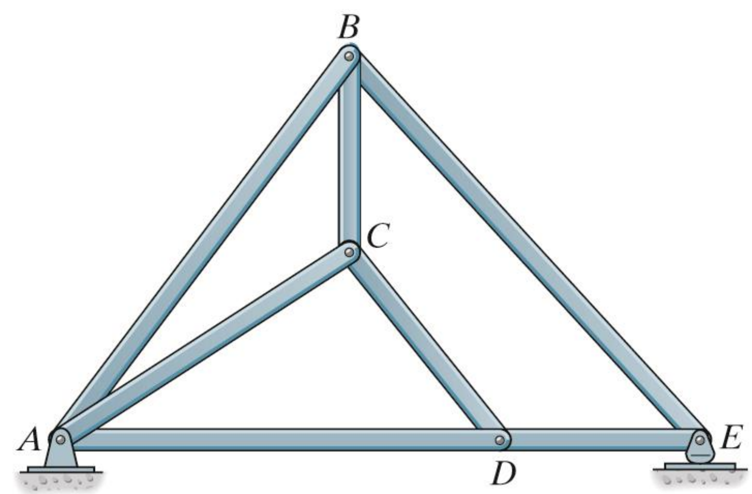


Classification of Coplanar Trusses

- Simple Truss
  - A simple truss is the basic "stable" triangle element is *ABC*
  - The remainder of the joints *D*, *E* & *F* are established in alphabetical sequence
  - Simple trusses do not have to consist entirely of triangles



simple truss



simple truss

Classification of Coplanar Trusses

- Compound Truss
  - It is formed by connecting 2 or more simple trusses together

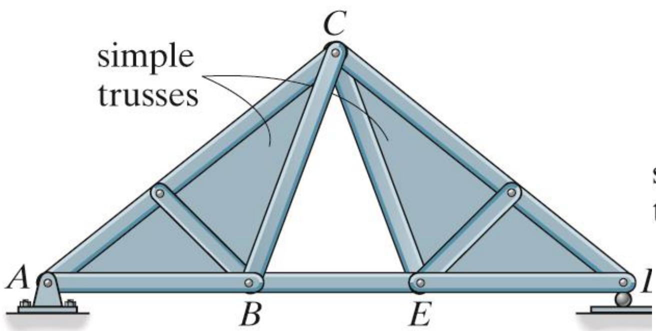
  - Often, this type of truss is used to support loads acting over a larger span as it is cheaper to construct a lighter compound truss than a heavier simple truss

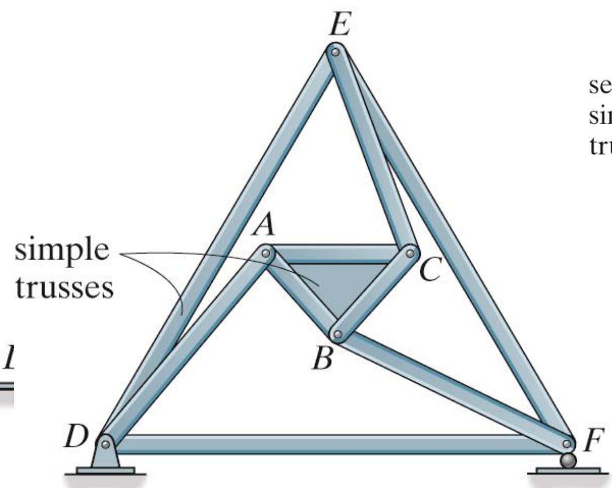
Classification of Coplanar Trusses

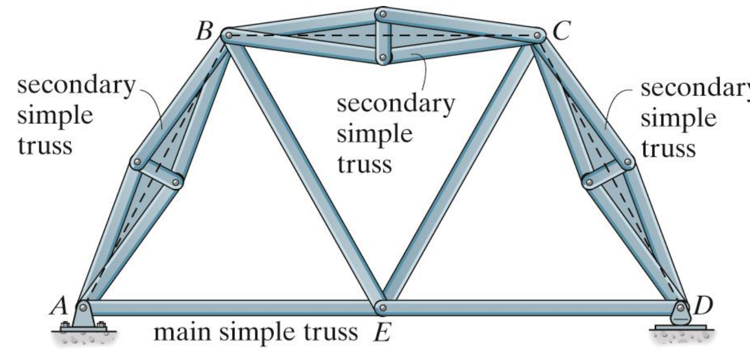
- Compound Truss
  - Type 1
    - The trusses may be connected by a common joint & bar
  - Type 2
    - The trusses may be joined by 3 bars
  - Type 3
    - The trusses may be joined where bars of a large simple truss, called the main truss, have been substituted by simple truss, called secondary trusses

Classification of Coplanar Trusses

- Compound Truss

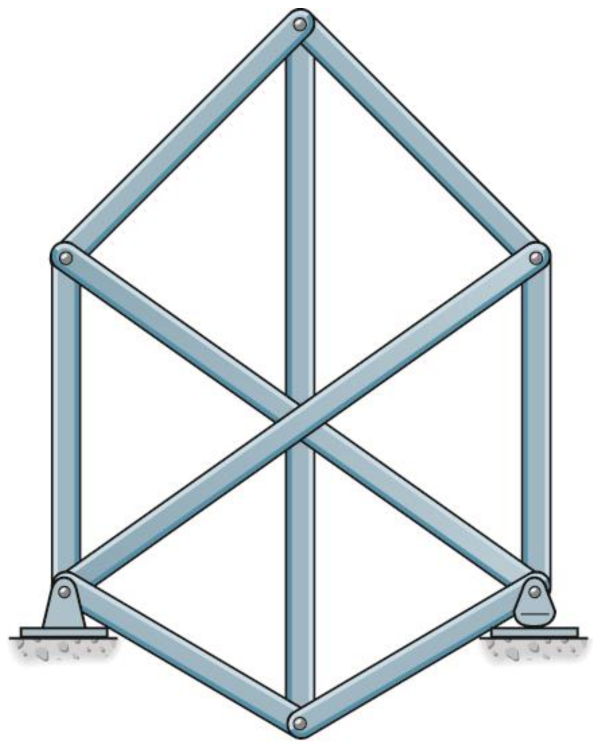






Classification of Coplanar Trusses

- Complex Truss
  - A complex truss is one that cannot be classified as being either simple or compound



Complex truss

Classification of Coplanar Trusses

- Determinacy
  - Total unknowns = forces in *b* no. of bars of the truss + total no. of external support reactions
  - Force system at each joint is coplanar & concurrent
  - Rotational or moment equilibrium is automatically satisfied

Classification of Coplanar Trusses

- Determinacy
  - Therefore only

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

  - By comparing the total unknowns with the total no. of available equilibrium equations, we have:

$$b + r = 2j \quad \text{statically determinate}$$

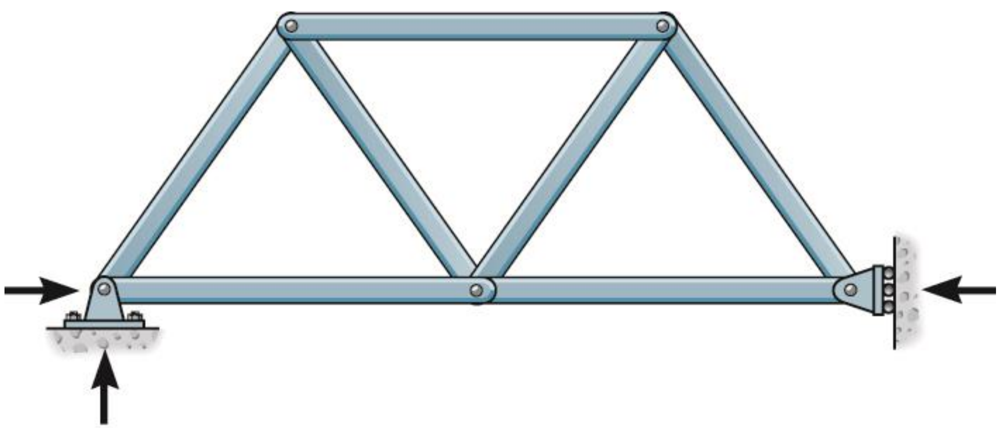
$$b + r > 2j \quad \text{statically indeterminate}$$

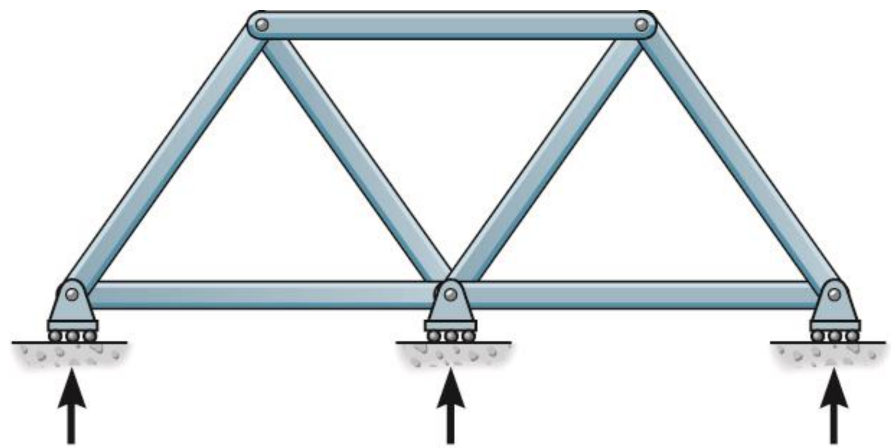
Classification of Coplanar Trusses

- Stability
  - If $b + r < 2j$ => collapse
  - A truss can be unstable if it is statically determinate or statically indeterminate
  - Stability will have to be determined either through inspection or by force analysis

Classification of Coplanar Trusses

- Stability
  - External Stability
    - A structure is externally unstable if all of its reactions are concurrent or parallel
    - These trusses are externally unstable since the support reactions have lines of action that are either concurrent or parallel

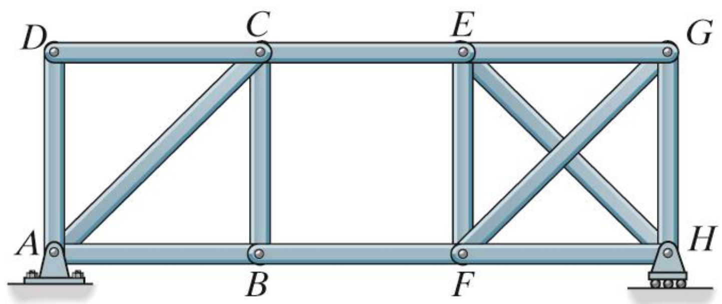




unstable parallel reactions

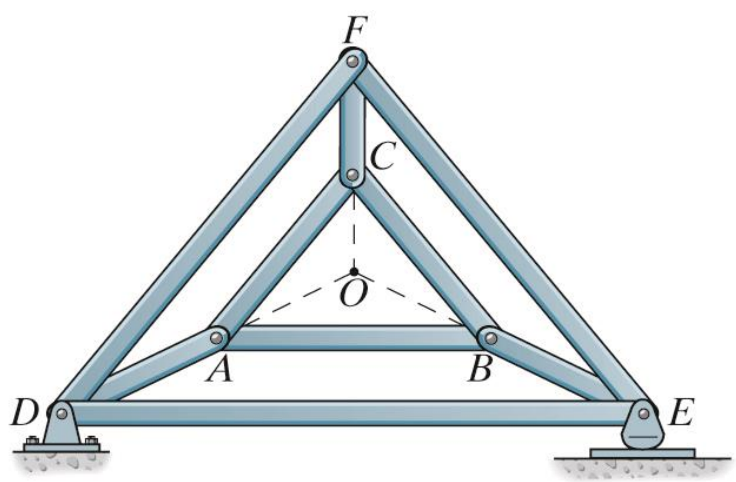
Classification of Coplanar Trusses

- Stability
  - Internal Stability
    - The internal stability can be checked by careful inspection of the arrangement of truss members
    - If it can be determined that each joint is held fixed so that it cannot move in a "rigid body" sense with respect to the other joints, then the truss will be stable
    - A simple truss will always be internally stable
    - If a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a "critical form"



Classification of Coplanar Trusses

- Stability
  - Internal Stability
    - To determine the internal stability of a compound truss, it is necessary to identify the way in which the simple truss are connected together
    - The truss shown is unstable since the inner simple truss *ABC* is connected to *DEF* using 3 bars which are concurrent at point *O*



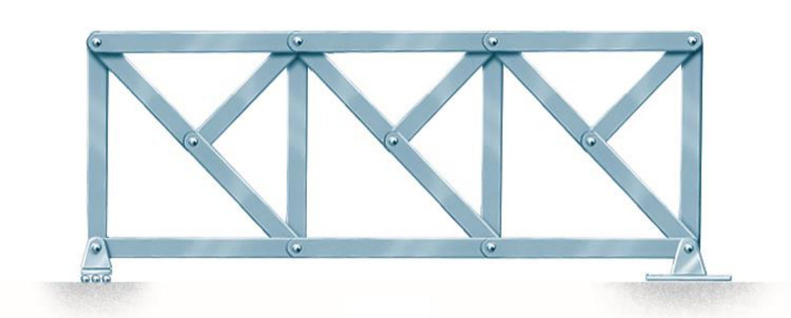
Classification of Coplanar Trusses

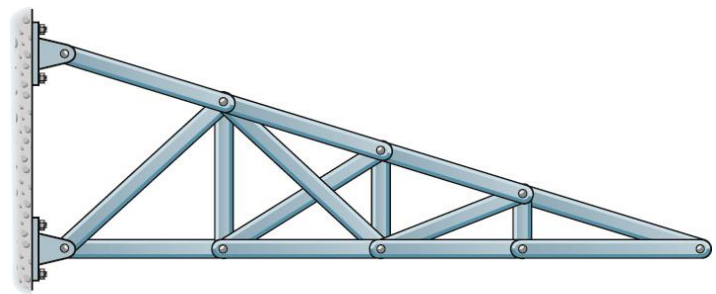
- Stability
  - Internal Stability
    - Thus an external load can be applied at *A*, *B* or *C* & cause the truss to rotate slightly
    - For complex truss, it may not be possible to tell by inspection if it is stable
    - The instability of any form of truss may also be noticed by using a computer to solve the $2j$ simultaneous eqns for the joints of the truss
    - If inconsistent results are obtained, the truss is unstable or have a critical form

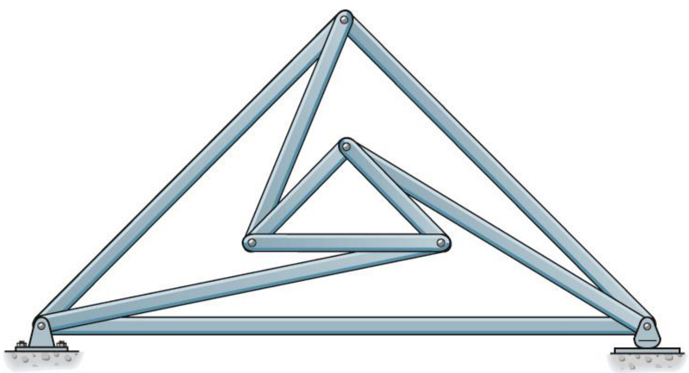
Classification of Coplanar Trusses

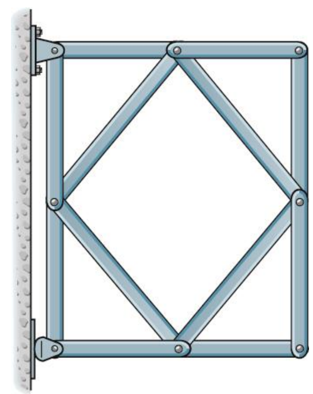
Example 3.1

Classify each of the trusses as stable, unstable, statically determinate, or statically indeterminate. The trusses are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the trusses.







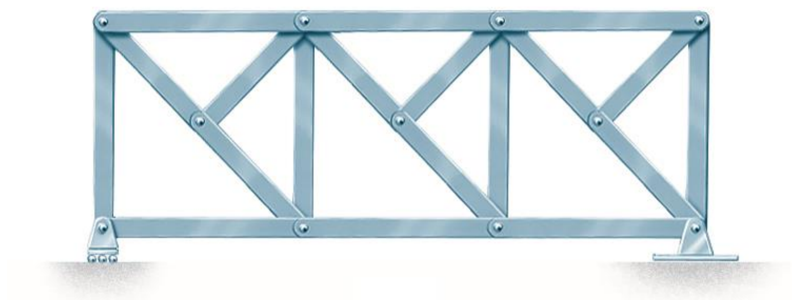


Classification of Coplanar Trusses

Example 3.1 (Solution)

For (a),

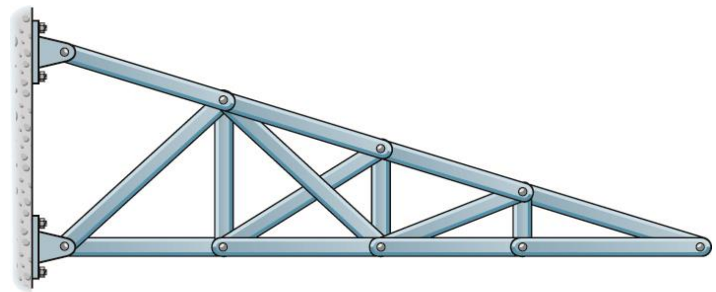
- Externally stable
- Reactions are not concurrent or parallel
- $b = 19$, $r = 3$, $j = 11$
- $b + r = 2j = 22$
- Truss is statically determinate
- By inspection, the truss is internally stable



Classification of Coplanar Trusses

Example 3.1 (Solution)

For (b),



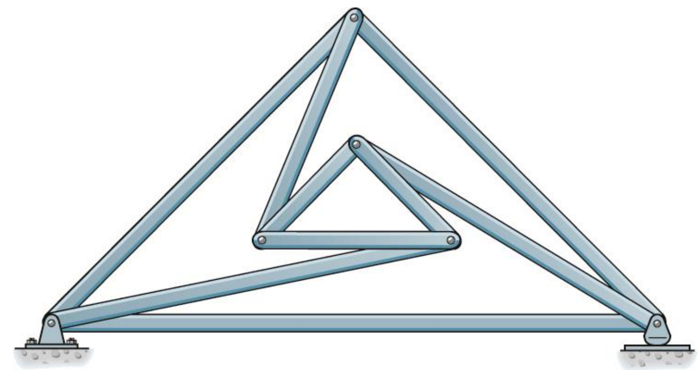
- Externally stable
- b = 15, r = 4, j = 9
- $b + r = 19 > 2j = 18$
- Truss is statically indeterminate
- By inspection, the truss is internally stable

Classification of Coplanar Trusses

Example 3.1 (Solution)

For (c),

- Externally stable
- $b = 9$, $r = 3$, $j = 6$
- $b + r = 12 = 2j$
- Truss is statically determinate
- By inspection, the truss is internally stable

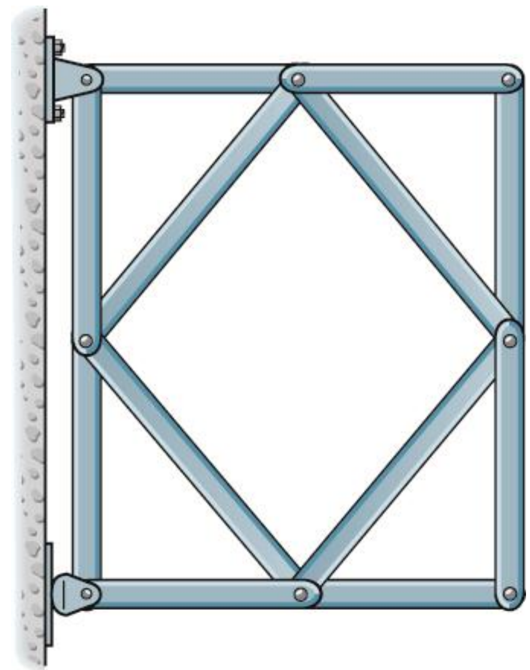


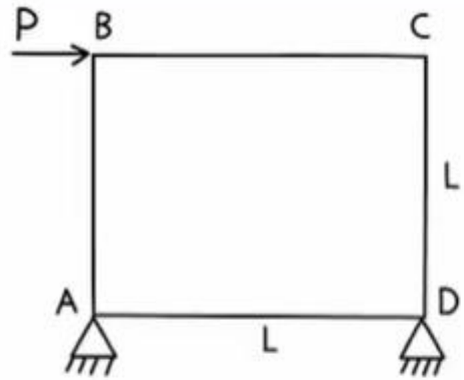
Classification of Coplanar Trusses

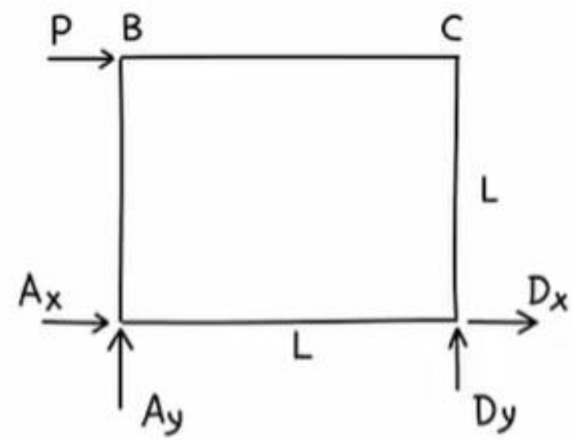
Example 3.1 (Solution)

For (d),

- Externally stable
- $b = 12$, $r = 3$, $j = 8$
- $b + r = 15 < 2j = 16$
- The truss is internally unstable



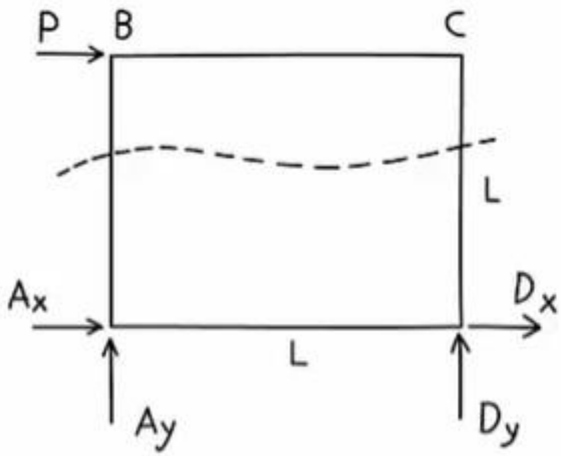




$$\Sigma F_x = 0 \Rightarrow A_x + D_x + P = 0 \quad \Rightarrow A_x + D_x = -P$$

$$\Sigma F_y = 0 \Rightarrow A_y + D_y = 0 \quad \Rightarrow A_y + P = 0 \Rightarrow A_y = -P$$

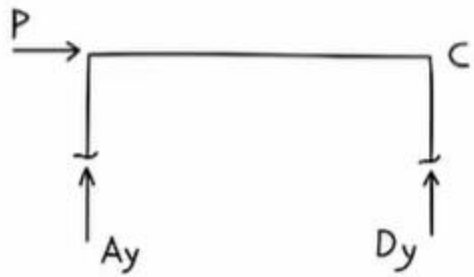
$$\Sigma M_A = 0 \Rightarrow P(L) - D_y L = 0 \quad \Rightarrow D_y = P$$



Truss is externally stable

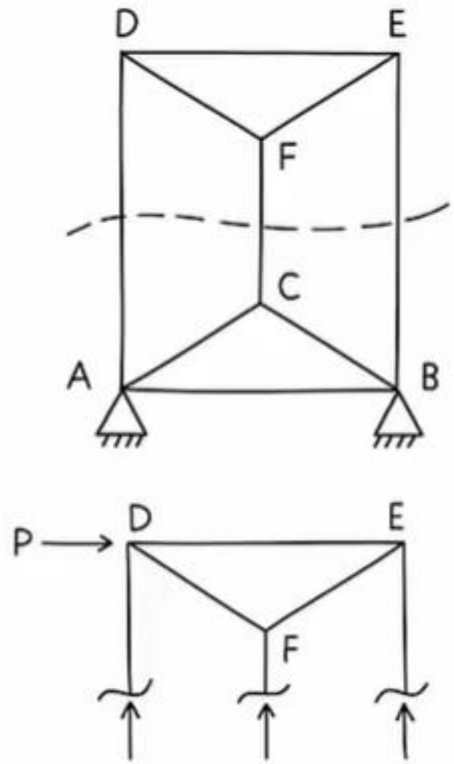
Internal Stability differs from External Stability

Is the truss internally stable?



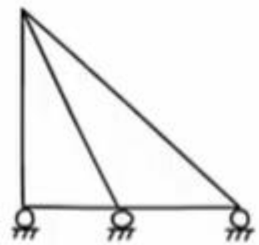
$$\Sigma F_x = 0 \Rightarrow P \neq 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

The truss is internally unstable



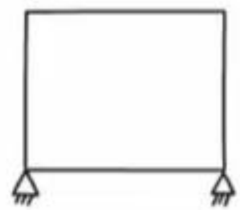
$$\Sigma F_x = 0 \Rightarrow P \neq 0$$

Internally Unstable



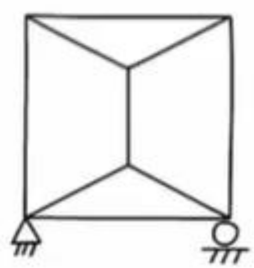
Number of Equations: 4*2 = 8
Number of Unknowns: 5+3 = 8

Unstable? Not according to the rule



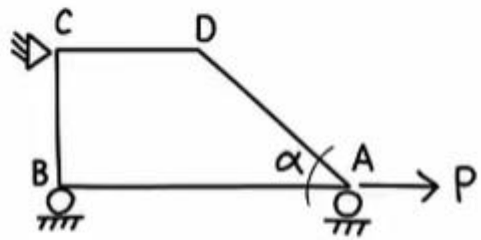
Number of Equations: 4*2 = 8
Number of Unknowns: 4+4 = 8

Unstable? Not according to the rule

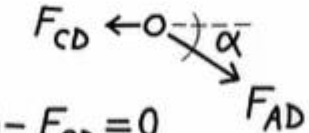


Number of Equations: 6*2 = 12
Number of Unknowns: 9+3 = 12

Unstable? Not according to the rule



Joint D:



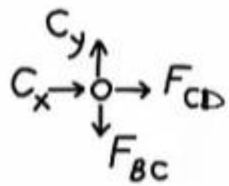
$$\Sigma F_x = 0 \Rightarrow F_{AD}\cos(\alpha) - F_{CD} = 0$$

$$\Sigma F_y = 0 \Rightarrow -F_{AD}\sin(\alpha) = 0$$

$$F_{AD} = 0$$

$$F_{CD} = 0$$

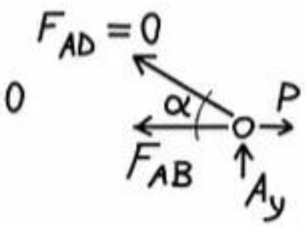
Joint C:



$$\Sigma F_y = 0 \Rightarrow C_y - F_{BC} = 0$$

$$\Sigma F_x = 0 \Rightarrow C_x + F_{CD} = 0$$

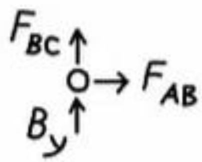
Joint A:



$$\Sigma F_x = 0 \Rightarrow P - F_{AB} = 0$$

$$\Sigma F_y = 0 \Rightarrow A_y = 0$$

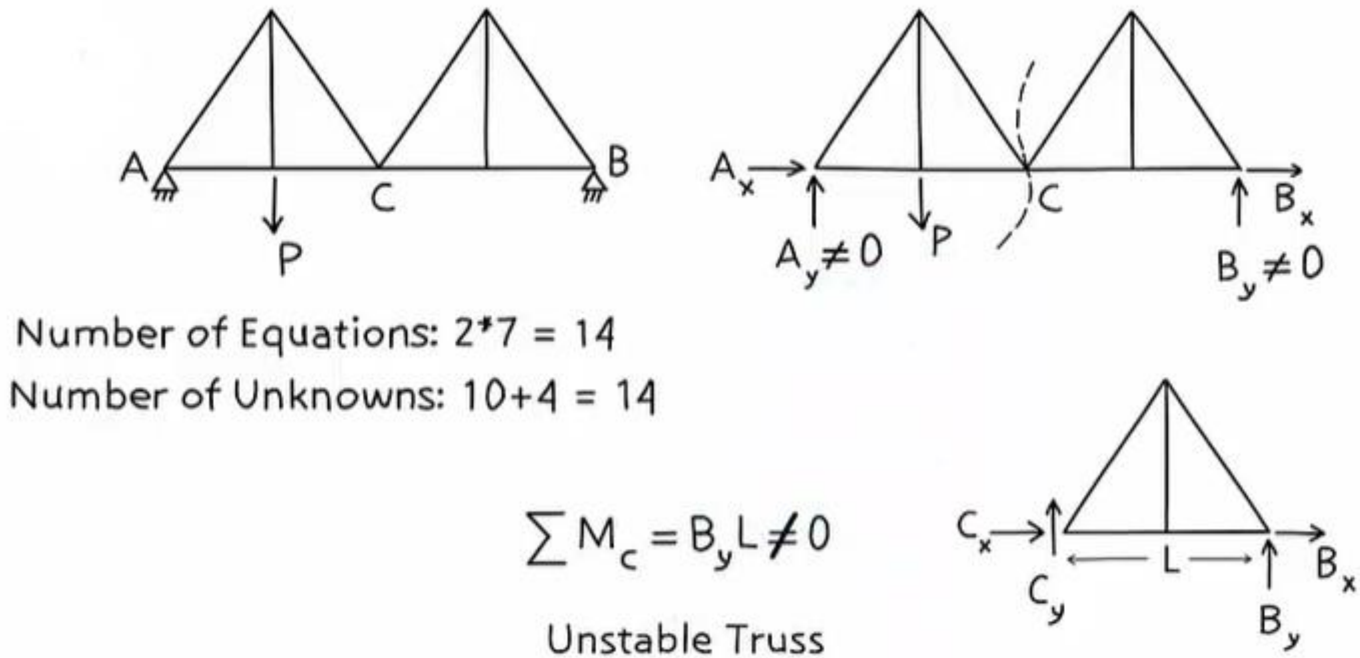
Joint B:



$$\Sigma F_x = 0 \Rightarrow F_{AB} = 0$$

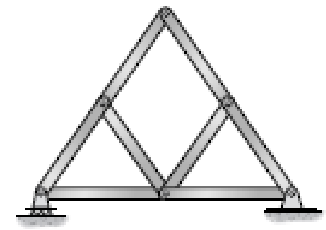
$$\Sigma F_y = 0 \Rightarrow B_y + F_{BC} = 0$$

Inconsistent Equations $\Rightarrow$ Unstable Truss

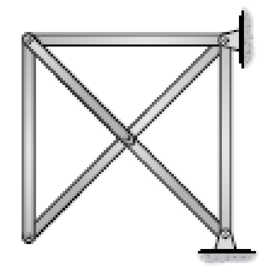

A
B
C
P
A_x
$A_y \neq 0$
P
C
B_x
$B_y \neq 0$
Number of Equations: 2*7 = 14
Number of Unknowns: 10+4 = 14
$\sum M_c = B_y L \neq 0$
Unstable Truss
C_x
C_y
L
B_x
B_y

HW 3-1

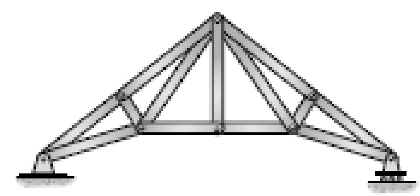
Classify each of the following trusses as statically determinate, statically indeterminate, or unstable. If indeterminate, state its degree.



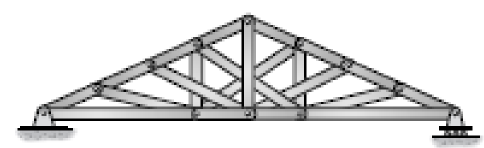
(a)



(b)



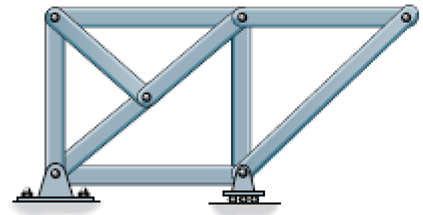
(c)



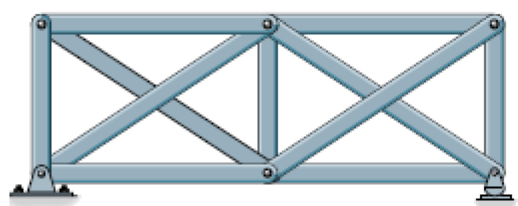
(d)

HW 3-2

Classify each of the following trusses as statically determinate, indeterminate, or unstable. If indeterminate, state its degree.



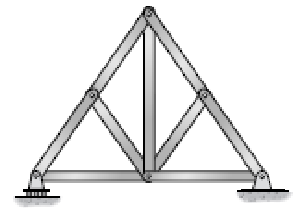
(a)



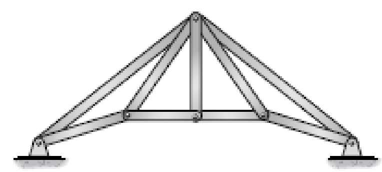
(b)

HW 3-3

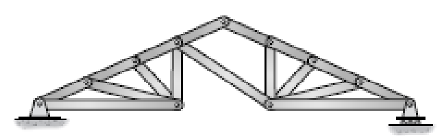
Classify each of the following trusses as stable, unstable, statically determinate, or statically indeterminate. If indeterminate state its degree.



(a)



(b)



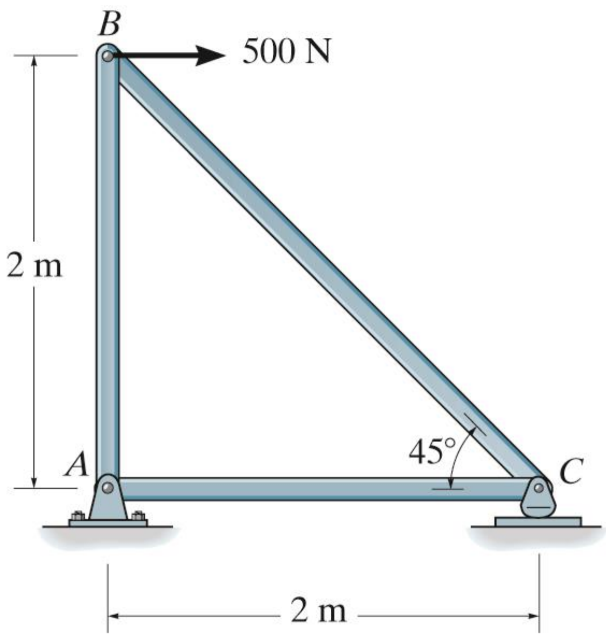
(c)

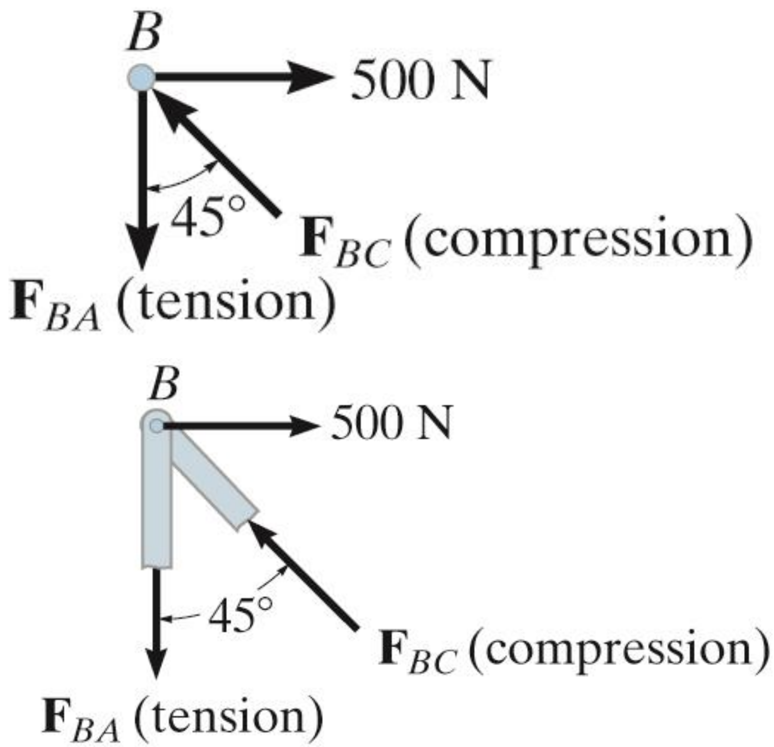
3.3
THE METHOD OF JOINTS

3.3

The Method of Joints

- Satisfying the equilibrium eqns for the forces exerted on the pin at each joint of the truss
- Applications of eqns yields 2 algebraic eqns that can be solved for the 2 unknowns





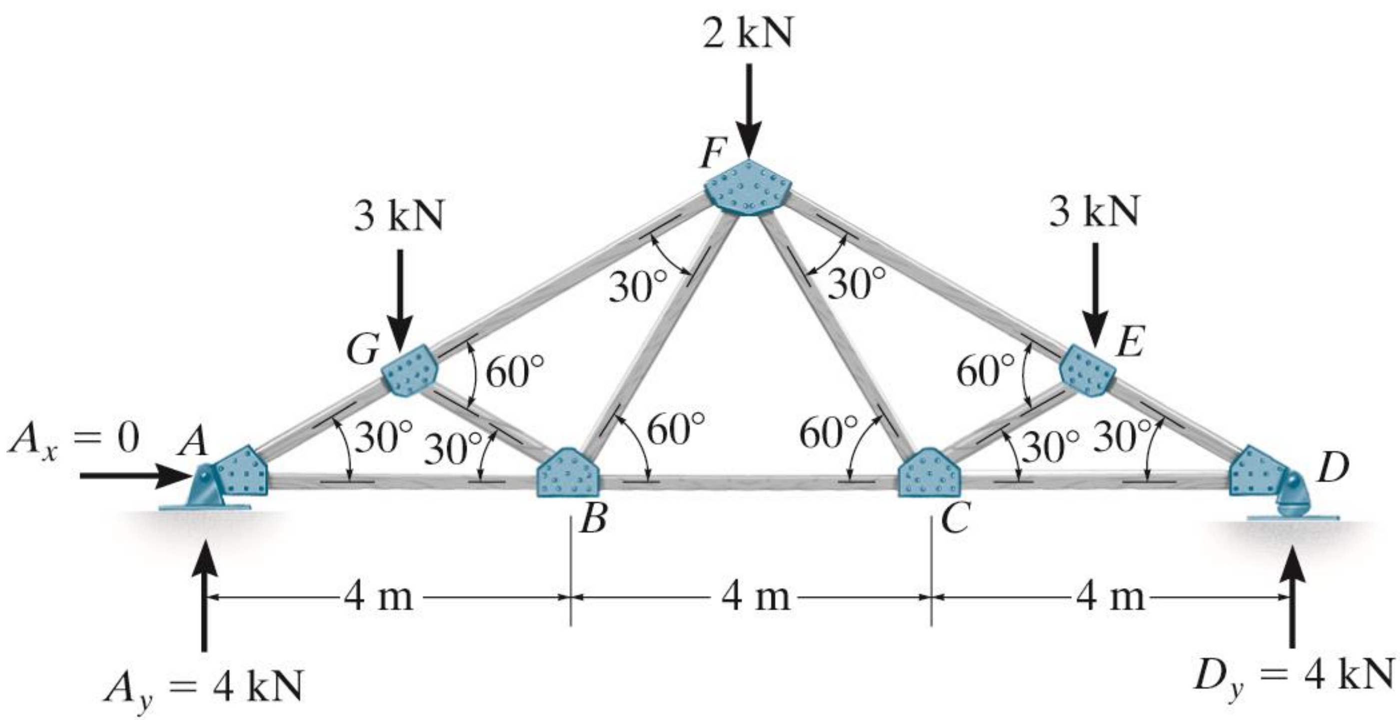
The Method of Joints

- Always assume the unknown member forces acting on the joint's free body diagram to be in tension
- Numerical solution of the equilibrium eqns will yield positive scalars for members in tension & negative for those in compression
- The correct sense of direction of an unknown member force can in many cases be determined by inspection
- A +ve answer indicates that the sense is correct, whereas a –ve answer indicates that the sense shown on the free-body diagram must be reversed

The Method of Joints

Example 3.2

Determine the force in each member of the roof truss shown in the photo. The dimensions and loadings are shown. *State* whether the members are in tension or compression.



The Method of Joints

Example 3.2 (Solution)

Only the forces in half the members have to be determined as the truss is symmetric wrt both loading & geometry,

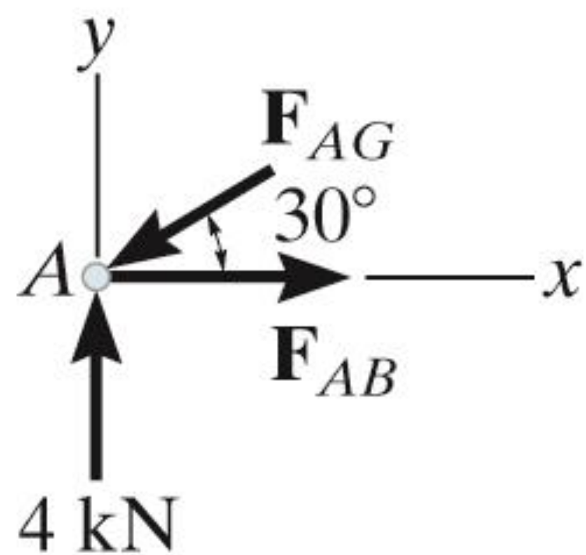
Joint A,

$$+\uparrow \sum F_y = 0; \quad 4 - F_{AG} \sin 30^0 = 0$$

$$F_{AG} = 8\,\text{kN(C)}$$

$$\underset{\rightarrow}{+} \sum F_x = 0; \quad F_{AB} - 8 \cos 30^0 = 0$$

$$F_{AB} = 6.928\,\text{kN(T)}$$



The Method of Joints

Example 3.2 (Solution)

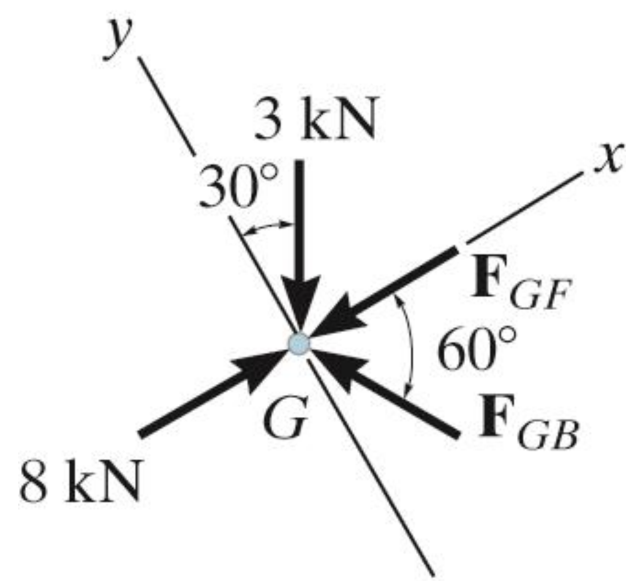
Joint G,

$$+\uparrow \sum F_y = 0;\ \ F_{GB} \sin 60^0 - 3\cos 30^0 = 0$$

$$F_{GB} = 3.00\,\text{kN(C)}$$

$$\underset{\rightarrow}{+} \sum F_x = 0;\ \ 8 - 3\sin 30^0 - 3\cos 60^0 - F_{GF} = 0$$

$$F_{GF} = 5.00\,\text{kN(C)}$$



The Method of Joints

Example 3.2 (Solution)

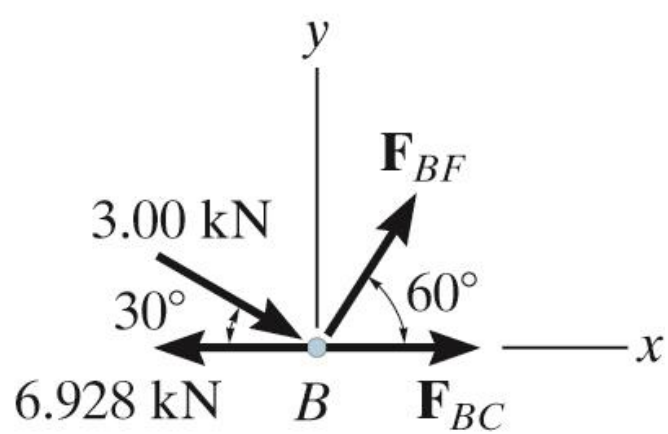
Joint B,

$$+\uparrow \sum F_y = 0; \quad F_{BF}\sin 60^0 - 3.00\sin 30^0 = 0$$

$$F_{BF} = 1.73\,\text{kN(T)}$$

$$\underset{\rightarrow}{+}\sum F_x = 0; \quad F_{BC} + 1.73\cos 60^0 + 3.00\cos 60^0 - 6.928 = 0$$

$$F_{BC} = 3.46\,\text{kN(T)}$$



3.4

ZERO-FORCE MEMBERS

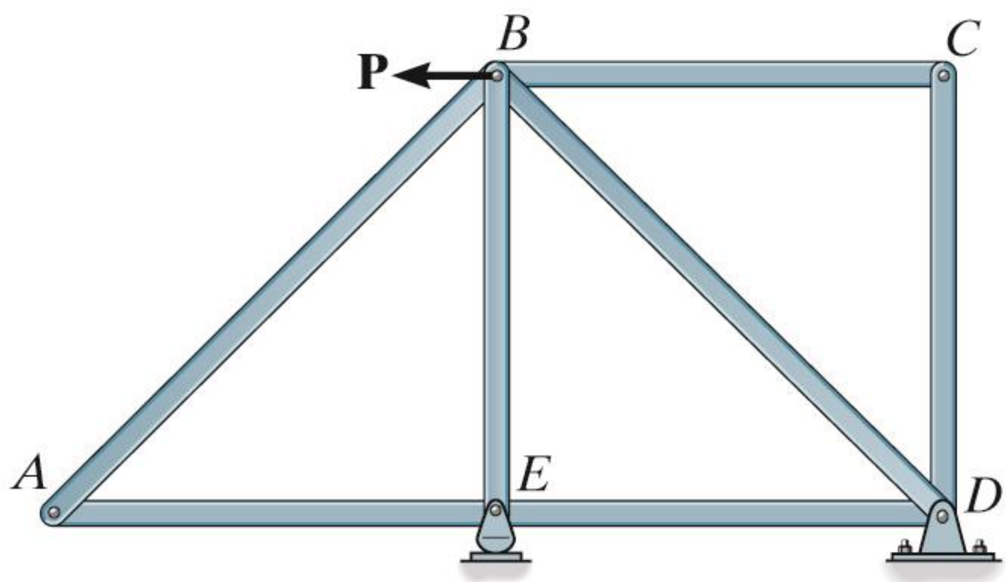
3.4

Zero-Force Members

- Truss analysis using method of joints is greatly simplified if one is able to first determine those members that support no loading
- These zero-force members may be necessary for the stability of the truss during construction & to provide support if the applied loading is changed
- The zero-force members of a truss can generally be determined by inspection of the joints & they occur in 2 cases.

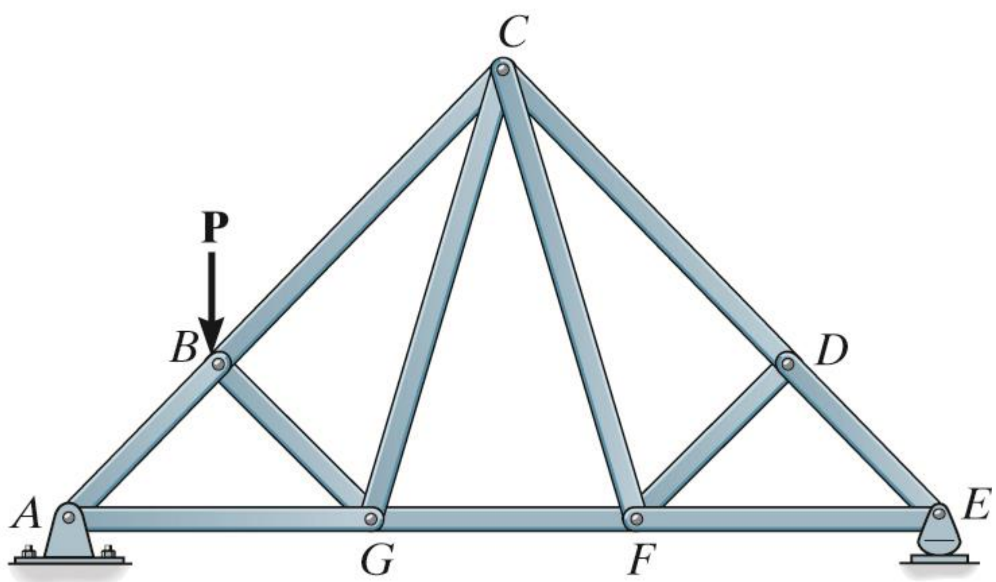
Zero-Force Members

- Case 1
  - The 2 members at joint *C* are connected together at a right angle & there is no external load on the joint
  - The free-body diagram of joint *C* indicates that the force in each member must be zero in order to maintain equilibrium



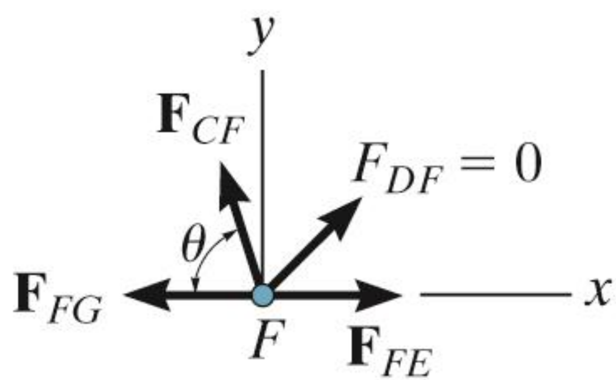
Zero-Force Members

- Case 2
  - Zero-force members also occur at joints having a geometry as joint *D*



Zero-Force Members

- Case 2
  - No external load acts on the joint, so a force summation in the *y*-direction which is perpendicular to the 2 collinear members requires that $F_{DF} = 0$
  - Using this result, *FC* is also a zero-force member, as indicated by the force analysis of joint *F*

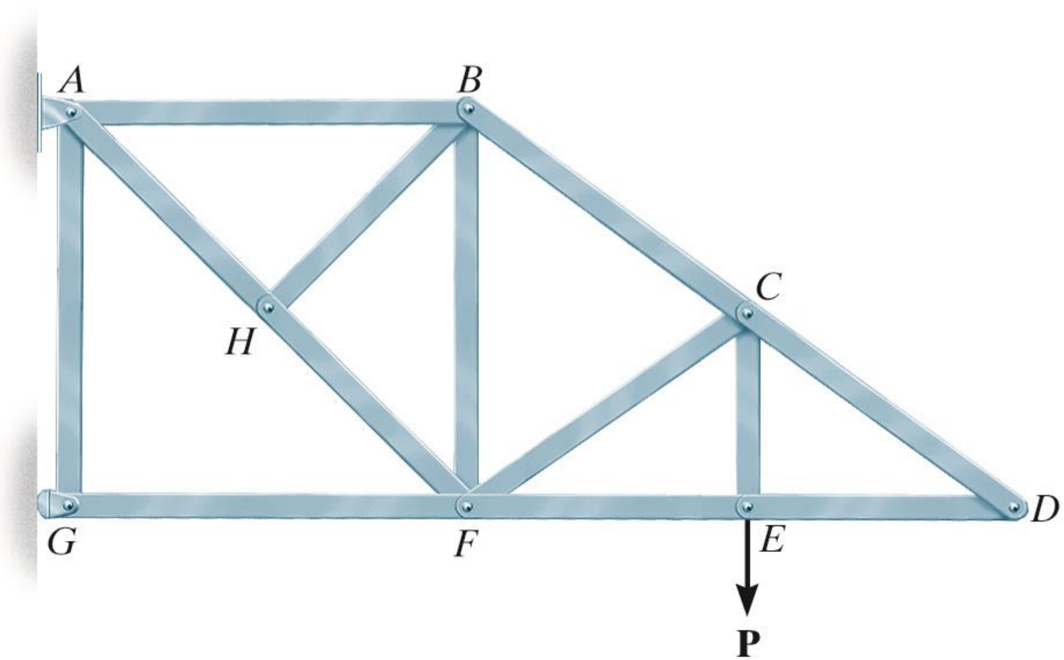


$$+\uparrow \Sigma F_y = 0;\ F_{CF}\sin\theta + 0 = 0$$
$$F_{CF} = 0 \text{ (since } \sin\theta \neq 0)$$

Zero-Force Members

Example 3.4

Using the method of joints, indicate all the members of the truss shown *that have zero force.*



Zero-Force Members

Example 3.4 (Solution)

We have

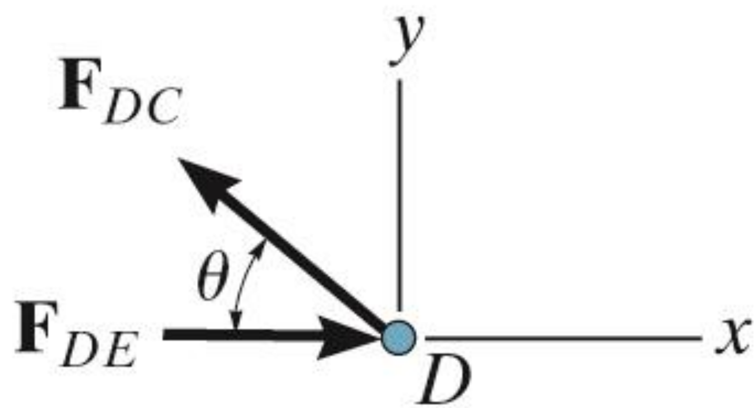
Joint D,

$$+\uparrow \sum F_y = 0; \quad F_{DC} \sin\theta = 0$$

$$F_{DC} = 0$$

$$\underset{\rightarrow}{+} \sum F_x = 0; \quad F_{DE} + 0 = 0$$

$$F_{DE} = 0$$

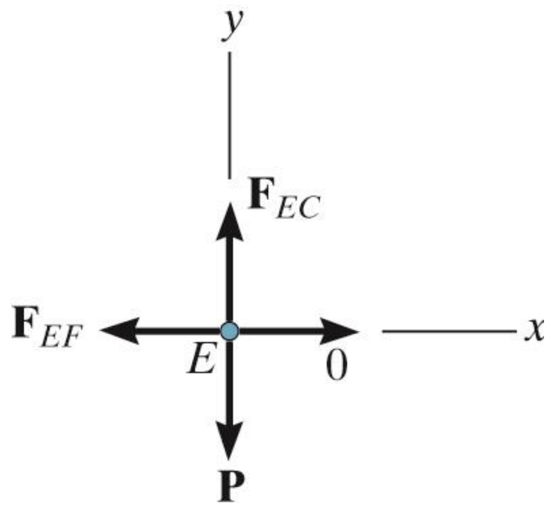


Zero-Force Members

Example 3.4 (Solution)

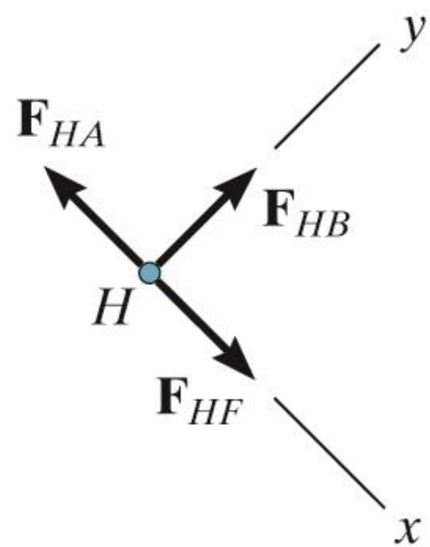
Joint E,

$$\underset{\leftarrow}{+}\sum F_x = 0; \quad F_{EF} = 0$$



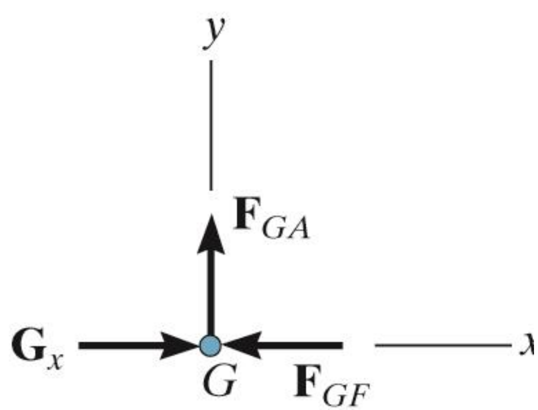
Joint H,

$$+\uparrow\sum F_y = 0; \quad F_{HB} = 0$$



Joint G,

$$+\uparrow\sum F_y = 0; \quad F_{GA} = 0$$



3.5
THE METHOD OF SECTIONS

3.5

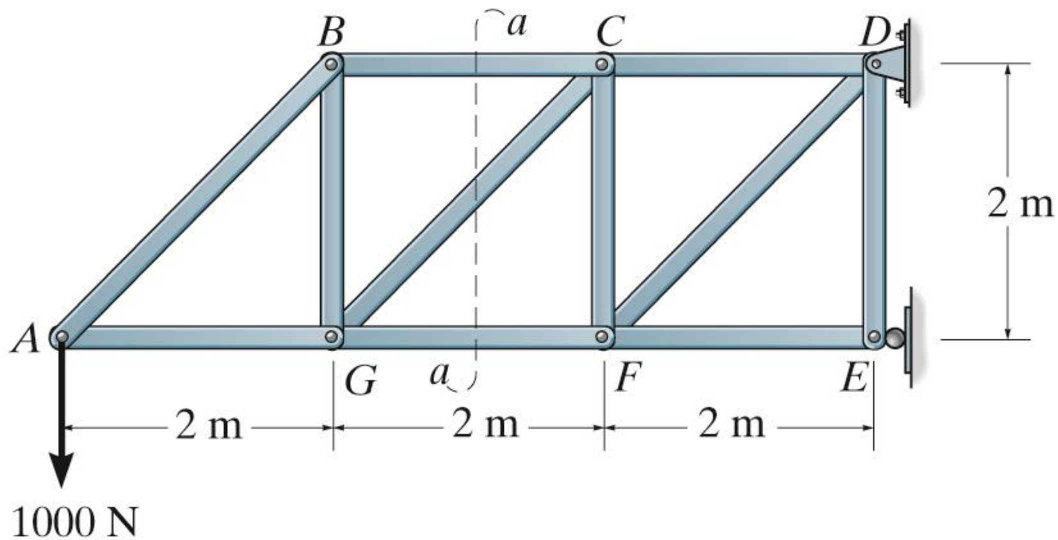
The Method of Sections

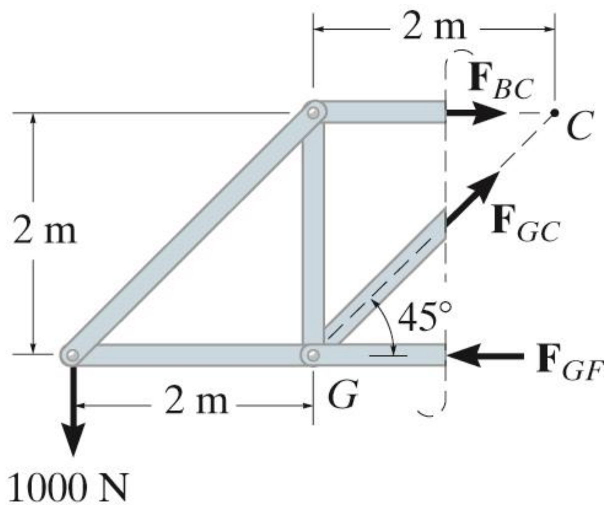
- If the forces in only a few members of a truss are to be found, the method of sections generally provide the most direct means of obtaining these forces
- This method consists of passing an imaginary section through the truss, thus cutting it into 2 parts
- Provided the entire truss is in equilibrium, each of the 2 parts must also be in equilibrium

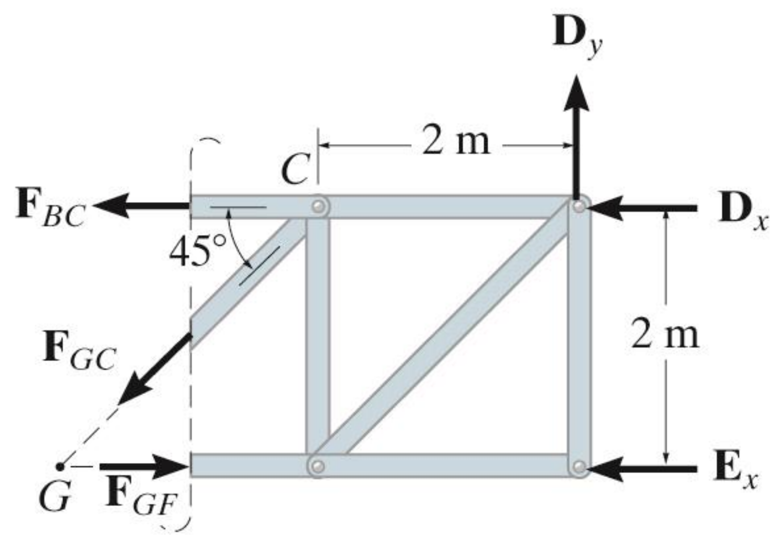
The Method of Sections

- The 3 eqns of equilibrium may be applied to either one of these 2 parts to determine the member forces at the "cut section"
- A decision must be made as to how to "cut" the truss
- In general, the section should pass through not more than 3 members in which the forces are unknown
- If the force in *GC* is to be determined, section *aa* will be appropriate
- Also, the member forces acting on one part of the truss are equal but opposite
- The 3 unknown member forces, F_{BC}, F_{GC} & F_{GF} can be obtained by applying the 3 equilibrium eqns

The Method of Sections







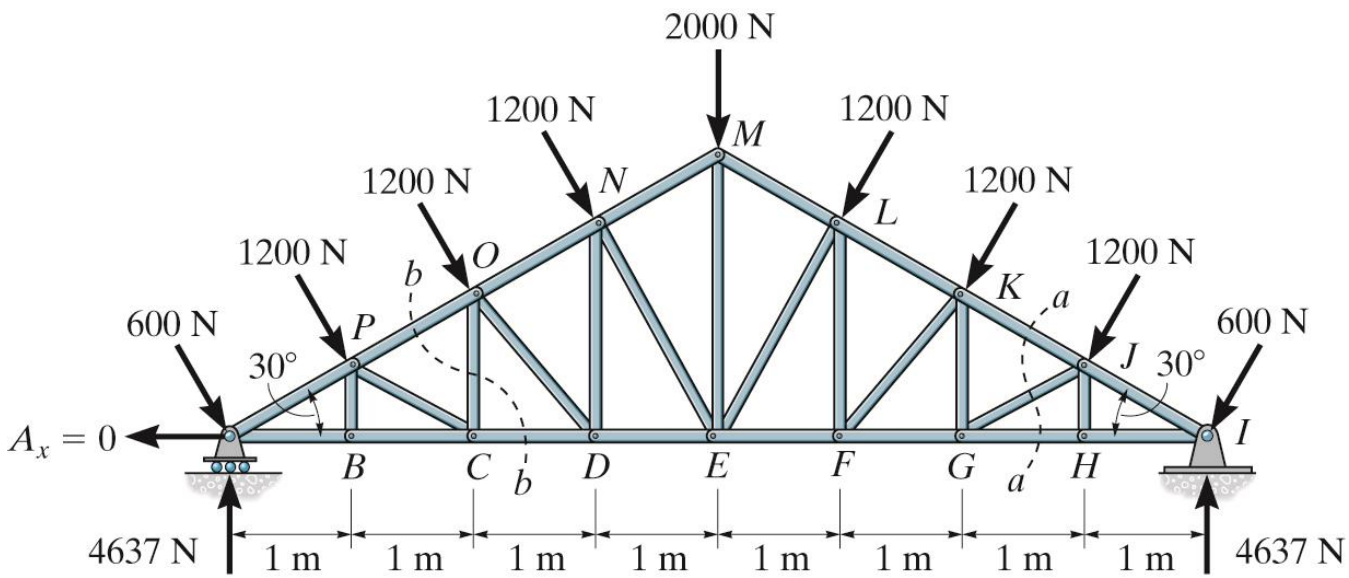
The Method of Sections

- When applying the equilibrium eqns, consider ways of writing the eqns to yield a direct solution for each of the unknowns, rather than to solve simultaneous eqns

The Method of Sections

Example 3.5

Determine the force in members *GJ* and *CO* of the roof truss shown in the photo. The dimensions and loadings are shown. State whether the members are in tension or compression. The reactions at the supports have been calculated.



The Method of Sections

Example 3.5 (Solution)

The free-body diagram of member *GJ* can be obtained by considering the section *aa*,

A direct solution for F_{GJ} can be obtained by applying $\sum M_I = 0$

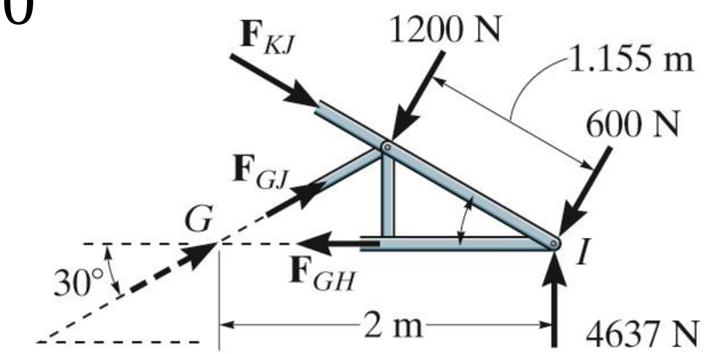
Applying principal of transmissibility,

F_{GJ} is slided to point G for simplicity.

With anti-clockwise moments as +ve, $\sum M_I = 0$

$$-F_{GJ}\sin 30^o(2) + 1200(1.155) = 0$$

$$F_{GJ} = 1386\,\text{N(C)}$$



The Method of Sections

Example 3.5 (Solution)

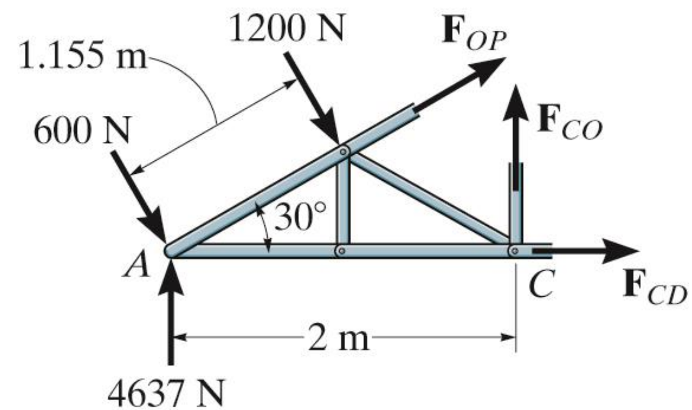
The free-body diagram of member *CO* can be obtained by considering the section *bb*,

Moments will be summed about point A in order to eliminate the unknowns F_{OP} and F_{CD}.

With anti-clockwise moments as +ve, $\sum M_A = 0$

$$-1200(1.155) + F_{CO}(2) = 0$$

$$F_{GC} = 693\,\text{N(T)}$$

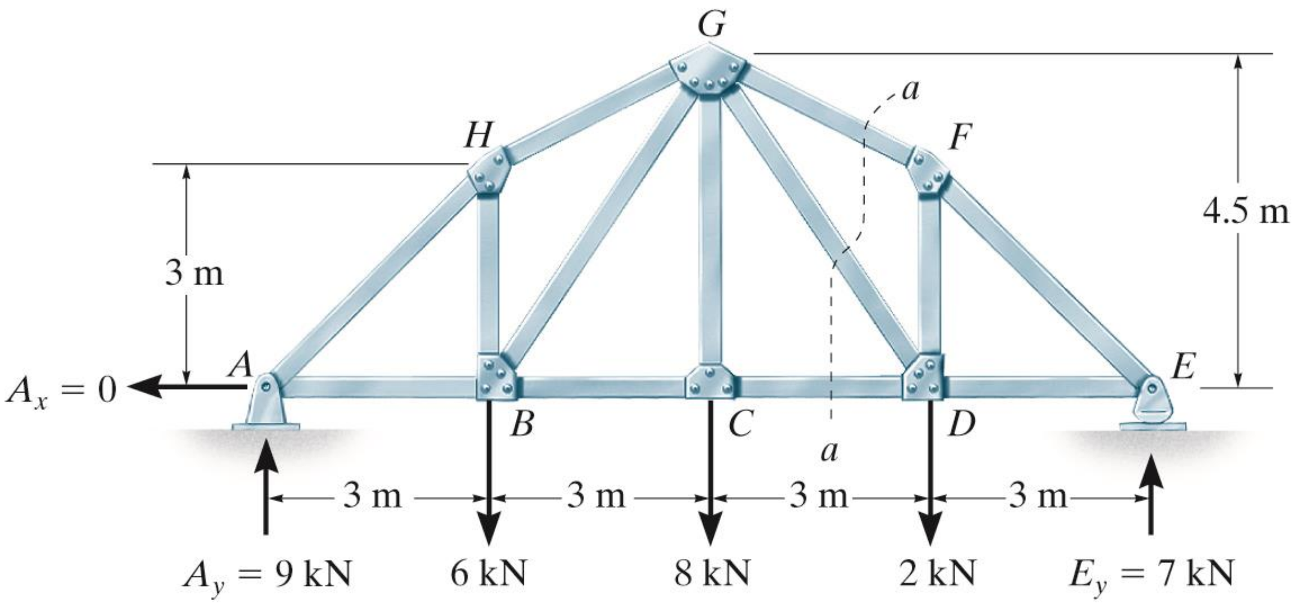


The Method of Sections

Example 3.6

Determine the force in members *GF* and *GD* of the truss shown. State whether the members are in tension or compression.

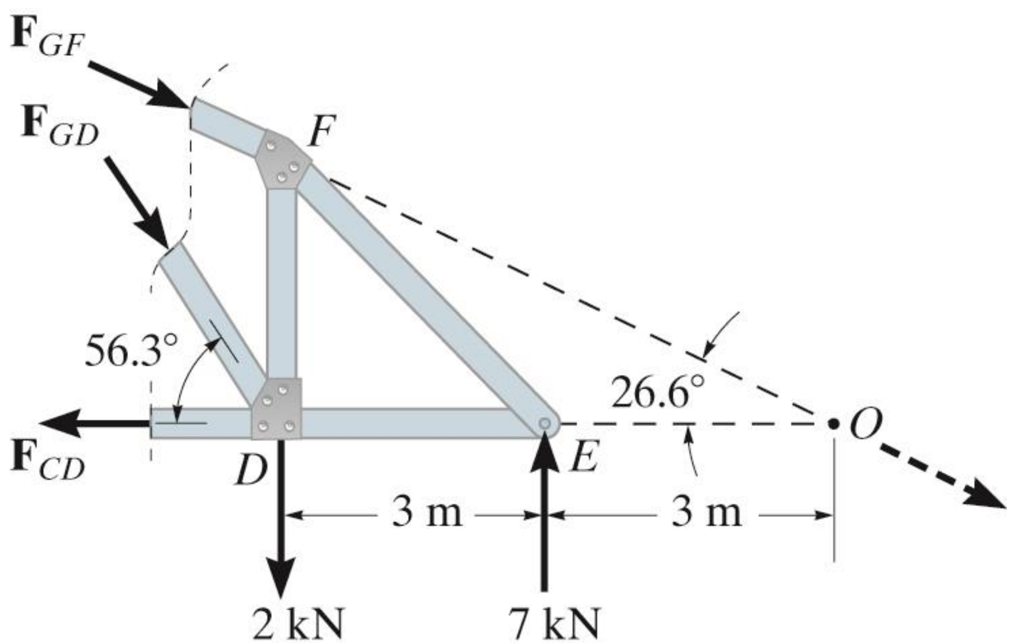
The reactions at the supports have been calculated.



The Method of Sections

Example 3.6 (Solution)

The distance *EO* can be determined by proportional triangles or realizing that member *GF* drops vertically 4.5 – 3 = 1.5 m in 3 m. Hence, to drop 4.5 m from *G* the distance from *C* to *O* must be 9 m



The Method of Sections

Example 3.6 (Solution)

The angles $\mathbf{F}_{GD}$ and $\mathbf{F}_{GF}$ make with the horizontal are

$\tan^{-1}(4.5/3) = 56.3^{o}$

$\tan^{-1}(4.5/9) = 26.6^{o}$

The force in GF can be determined directly by applying

$$\sum M_D = 0$$

F_{GF} is slided to point O.

$$\text{With anti-clockwise moments as +ve}, \sum M_D = 0$$

$$-F_{GF} \sin 26.6^{o}(6) + 7(3) = 0$$

$$F_{GF} = 7.83\,\text{kN(C)}$$

The Method of Sections

Example 3.6 (Solution)

The force in GD can be determined directly by applying

$$\sum M_O = 0$$

F_{GD} is slided to point D.

With anti-clockwise moments as +ve, $\sum M_O = 0$

$$-7(3) + 2(6) + F_{GD} \sin 56.3^o (6) = 0$$

$$F_{GD} = 1.80 \text{ kN(C)}$$

3.6
COMPOUND TRUSSES

3.6

Compound Trusses

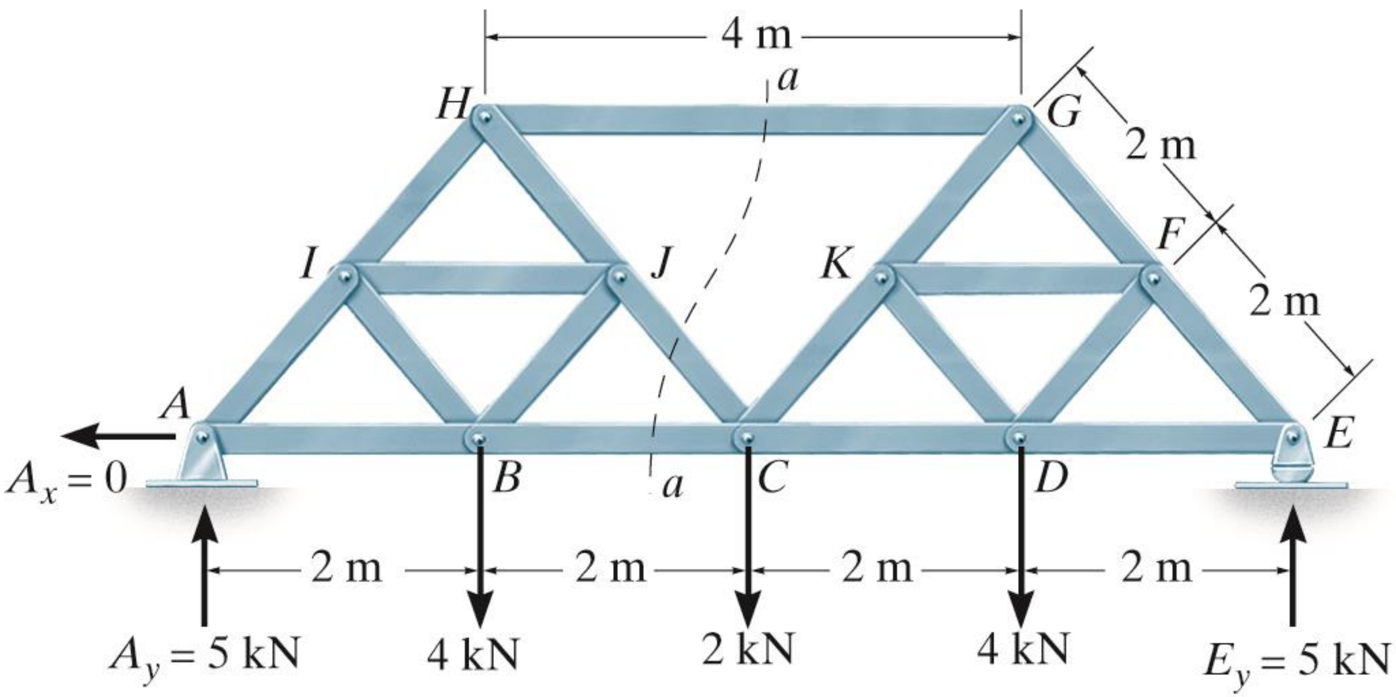
- Compound trusses are formed by connecting two or more simple trusses together either by bars or by joints.

- It is best analyzed by applying *both the* method of joints and the method of sections.

The Method of Sections

Example 3.8

Indicate how to analyze the compound truss as shown. *The* reactions at the supports have been calculated.



The Method of Sections

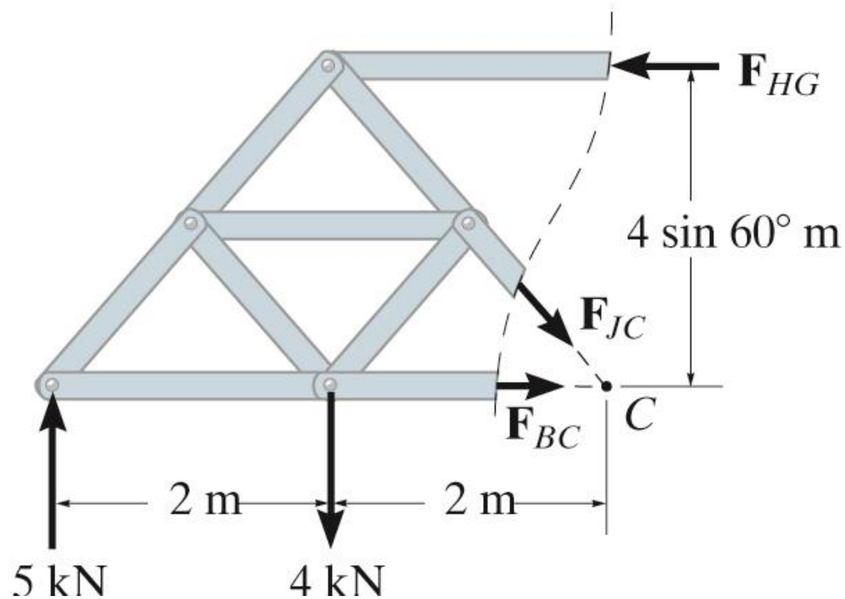
Example 3.8 (Solution)

The force in HG is determined as

$$\text{With anti-clockwise moments as +ve,} \sum M_O = 0;$$

$$-5(4) + 4(2) + F_{HG}(4\sin 60^o) = 0$$

$$F_{HG} = 3.46\,\text{kN(C)}$$



The Method of Sections

Example 3.8 (Solution)

The joints of this truss can be analyzed in the following sequence:

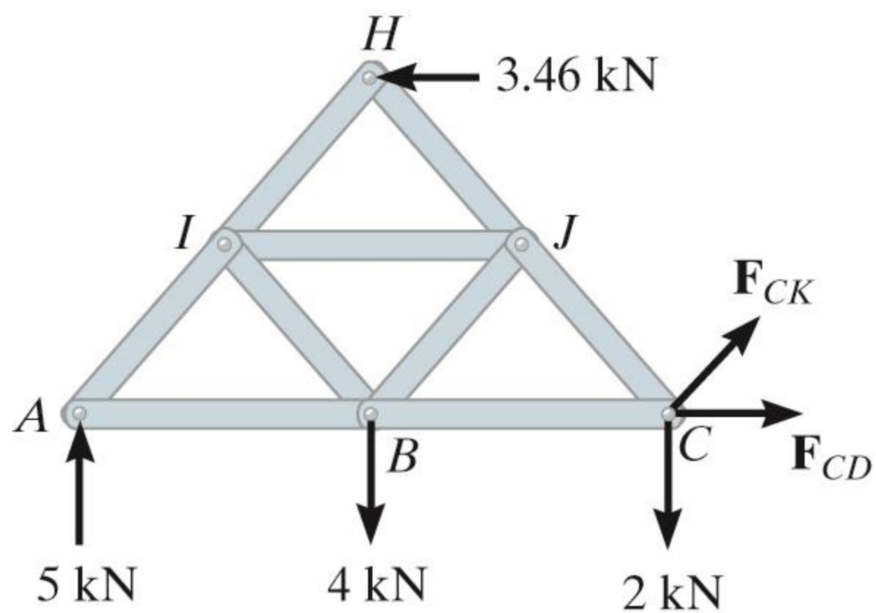
Joint A: Determine the force in AB and AI.

Joint H: Determine the force in HI and HJ.

Joint I: Determine the force in IJ and IB.

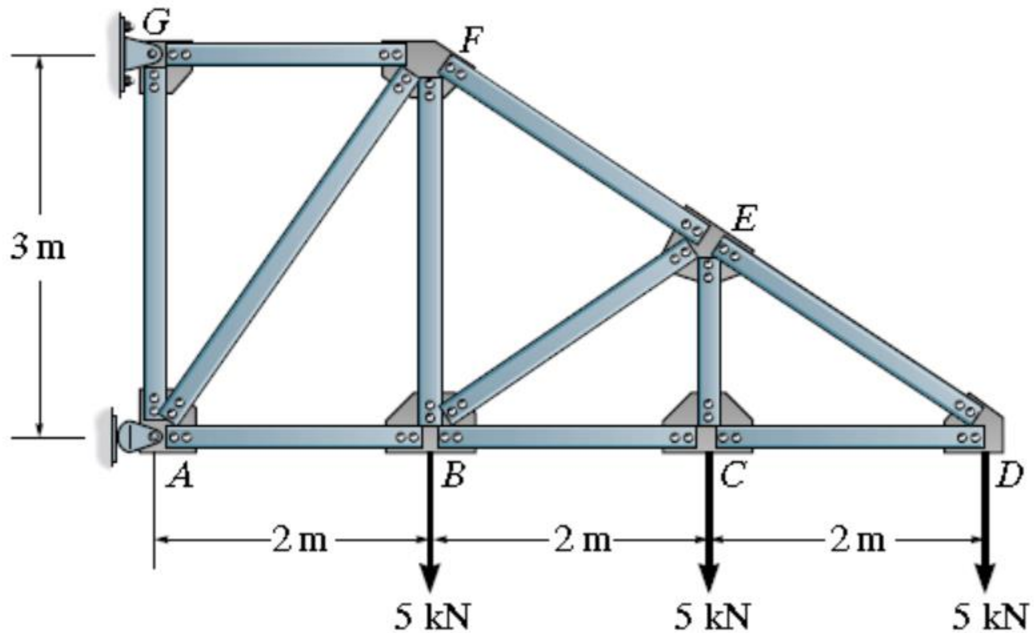
Joint B: Determine the force in BC and BJ.

Joint J: Determine the force in JC.



HW 3-4

Determine the force in each member of the truss. State if the members are in tension or compression. Assume all members are pin connected.

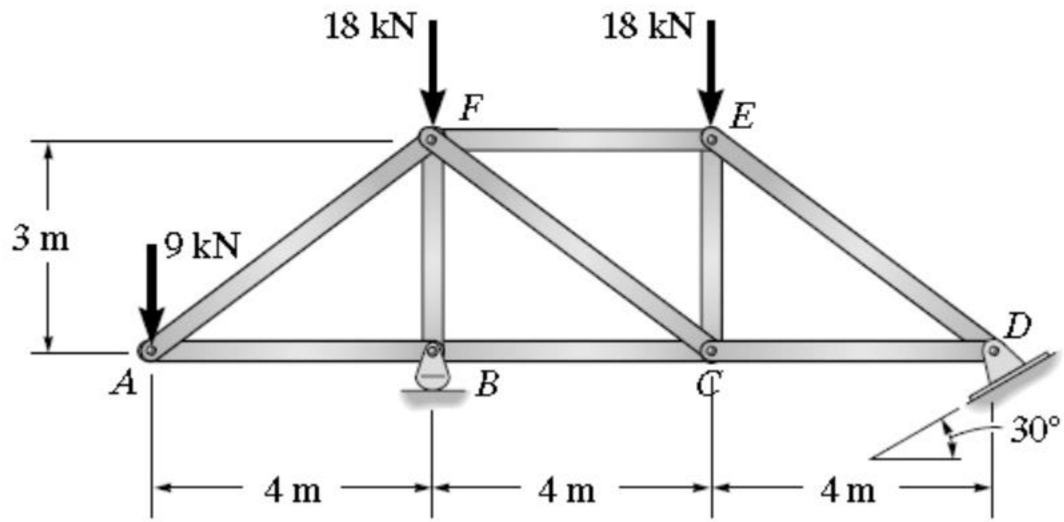


Ans.

$F_{ED} = 8.33$ kN (T); $F_{CD} = 6.67$ kN (C);
$F_{BC} = 6.67$ kN (C); $F_{CE} = 5$ kN (T);
$F_{GF} = 20$ kN (T); $F_{GA} = 15$ kN (T);
$F_{AF} = 18.0$ kN (C); $F_{AB} = 10.0$ kN (C);
$F_{BE} = 4.17$ kN (C); $F_{FB} = 7.50$ kN (T);
$F_{FE} = 12.5$ kN (T)

HW 3-5

Determine the force in each member of the truss.
State if the members are in tension or compression.



Ans.

$F_{AF} = 15$ kN (T) $\quad F_{AB} = 12$ kN (C)

$F_{BF} = 40.5$ kN (C) $\quad F_{BC} = 12$ kN (C)

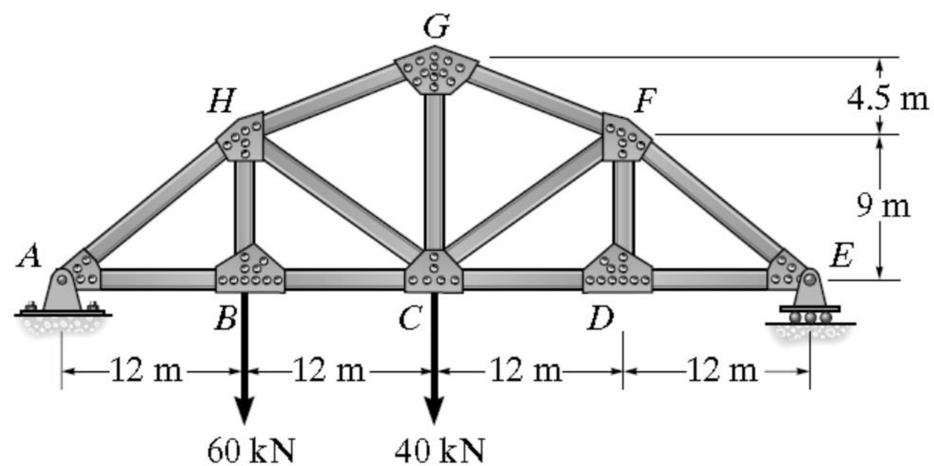
$F_{FC} = 22.5$ kN (T) $\quad F_{FE} = 6$ kN (C)

$F_{CE} = 13.5$ kN (C) $\quad F_{CD} = 6$ kN (T)

$F_{DE} = 7.5$ kN (C)

HW 3-6

Determine the force in members *GF*, *FC*, and *CD* of the bridge truss. State if the members are in tension of compression. Assume all members are pin connected..



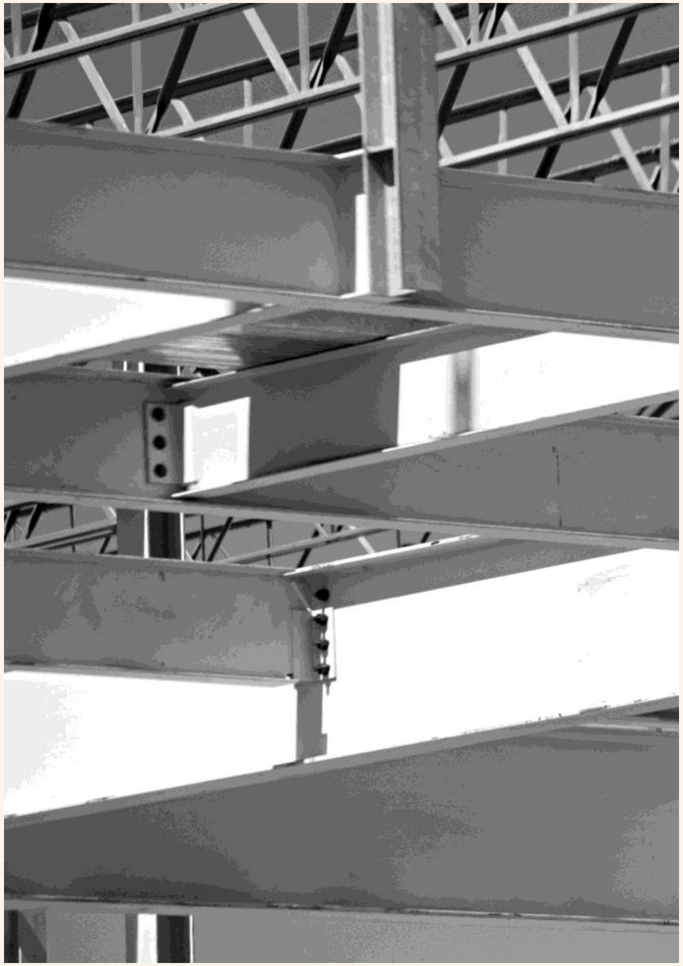
Ans.

$F_{DC} = 46.7 \text{ kN (T)}$

$F_{FG} = 66.5 \text{ kN (C)}$

$F_{FC} = 19.4 \text{ kN (T)}$

CHAPTER 4:
INTERNALLY LOADINGS DEVELOPED IN STRUCTURAL MEMBERS



4

Chapter Outline

4.1

INTERNAL LOADINGS AT A SPECIFIED POINT

4.1

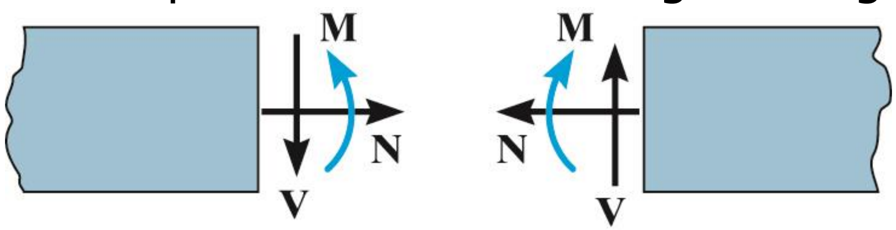
Internal Loadings at a Specified Point

- The internal load at a specified point in a member can be determined by using the method of sections

- This consists of:
  - **N**, normal force
  - **V**, shear force
  - **M**, bending moment

Internal Loadings at a Specified Point

- Sign convention
  - Although the choice is arbitrary, the convention shown has been widely accepted in structural engineering



Internal Loadings at a Specified Point

- Procedure for analysis
  - Determine the support reactions before the member is "cut"
  - If the member is part of a pin-connected structure, the pin reactions can be determine using the methods of section 2-5
  - Keep all distributed loadings, couple moments & forces acting on the member in their exact location
  - Pass an imaginary section through the member, perpendicular to its axis at the point where the internal loading is to be determined
  - Then draw a free-body diagram of the segment that has the least no. of loads on it
  - Indicate the unknown resultants **N**, **V** & **M** acting in their positive directions

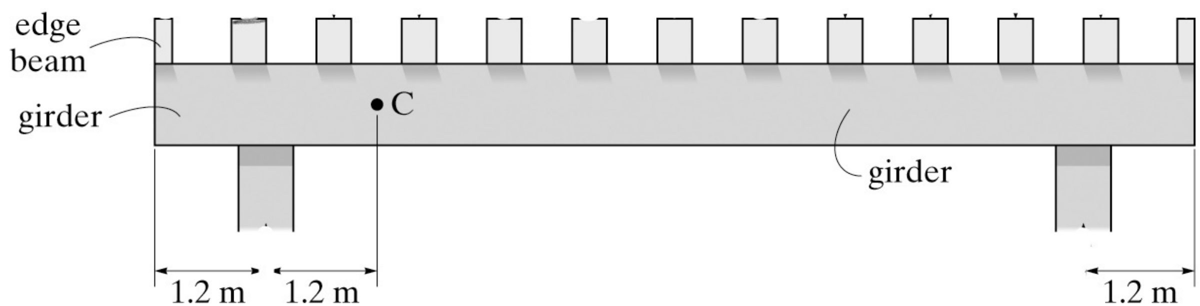
Internal Loadings at a Specified Point

- Procedure for analysis
  - Moments should be summed at the section about axes that pass through the centroid of the member's cross-sectional area in order to eliminate **N** & **V**, thereby solving **M**
  - If the solution of the equilibrium eqn yields a quantity having a –ve magnitude, then the assumed directional sense of the quantity is opposite to that shown on the free-body diagram

Internal Loadings at a Specified Point

Example 4.1

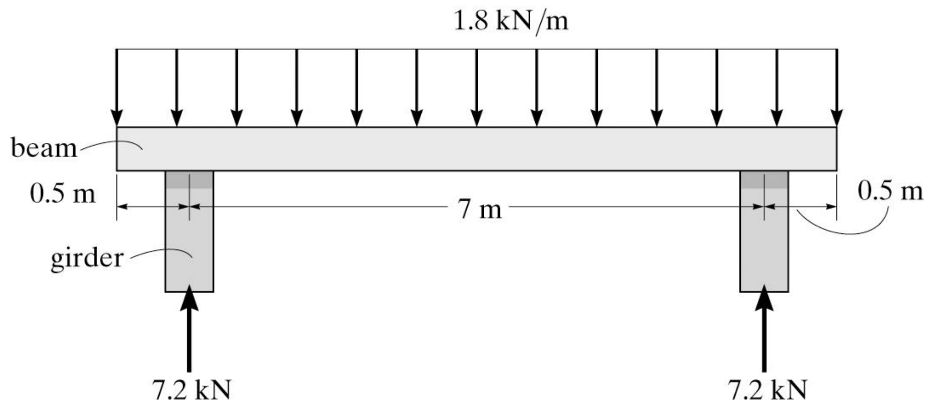
The building roof shown in the photo has a weight of 1.8 kN/m^2 and is supported on 8-m long simply supported beams that are spaced 1 m apart. Each beam as shown transmits its loading to two girders, located at the front and back of the building. Determine the internal shear and moment in the front girder at point *C.* Neglect the weight of the members.



Internal Loadings at a Specified Point

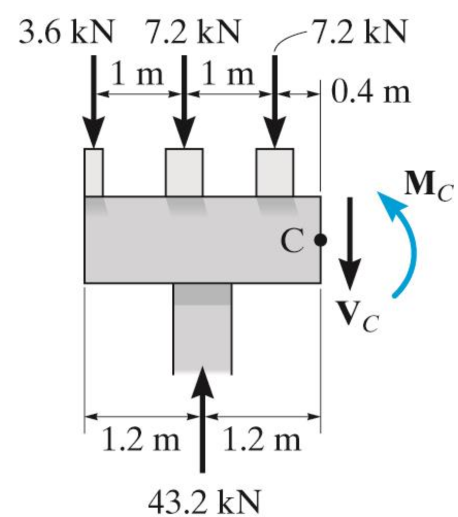
Example 4.1 (Solution)

Roof loading is transmitted to each beam as a one-way slab (L2/L1 =8>2)



Tributary load on each interior beam = (1.8 kN/m2)(1 m) = 1.8 kN/m

Reaction on girder = (1.8 kN/m)(8 m)/2 = 7.2 kN



The two edge beams support 0.9 kN/m

Each column reaction is [2(3.6 kN) + 11(7.2 kN)]/2 = 43.2 kN

$$+\uparrow \sum F_y = 0;$$

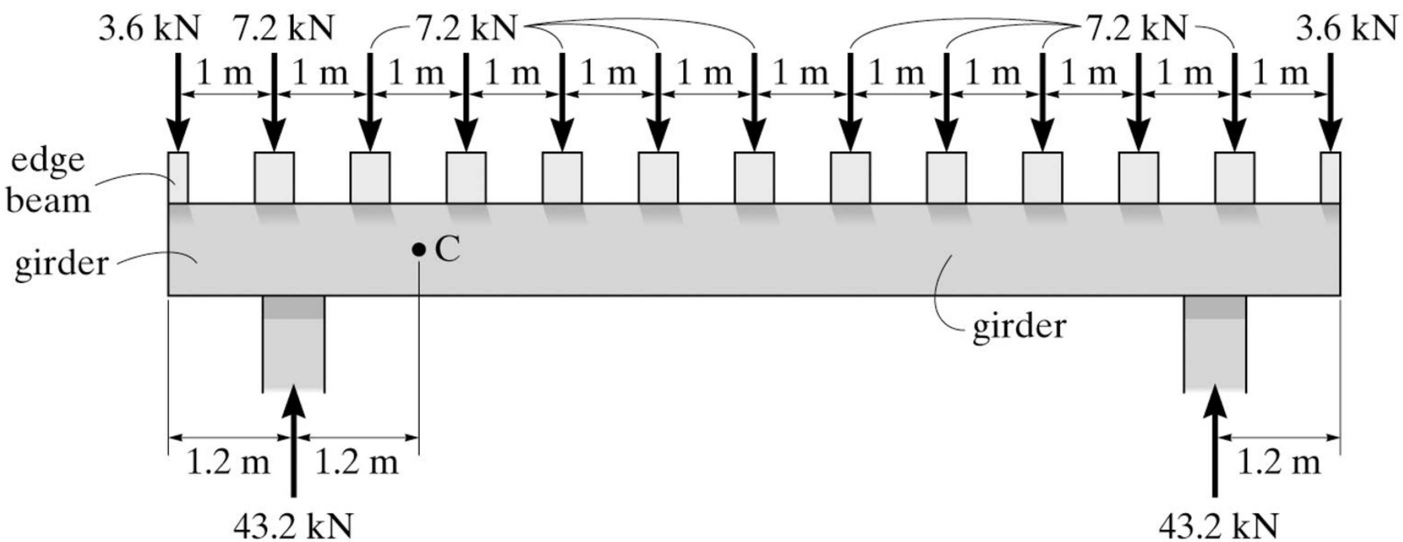
$$43.2 - 3.6 - 2(7.2) - V_C = 0 \Rightarrow V_C = 25.2\,\text{kN}$$

With moments in the anti-clockwise as positive,

$$\sum M_C = 0$$

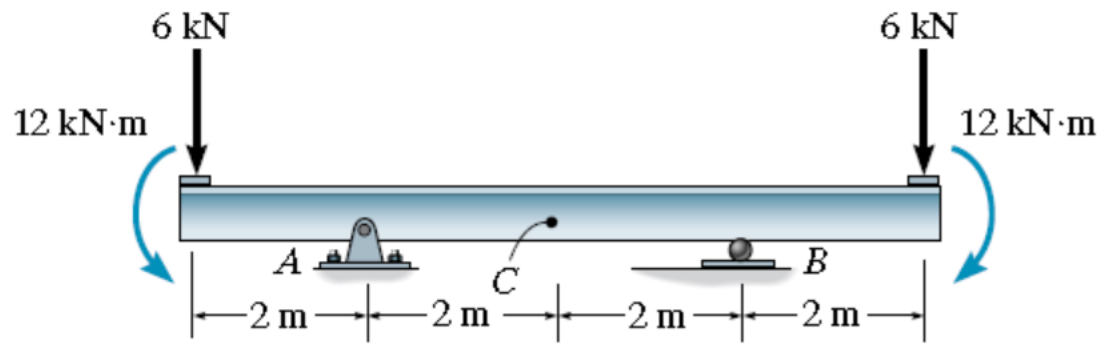
$$M_C + 7.2(0.4) + 7.2\ \ (1.4) + 3.6(2.4) - 43.2(1.2) = 0$$

$$\Rightarrow M_c = 30.2\,\text{kN} \bullet \text{m}$$



HW 4-1

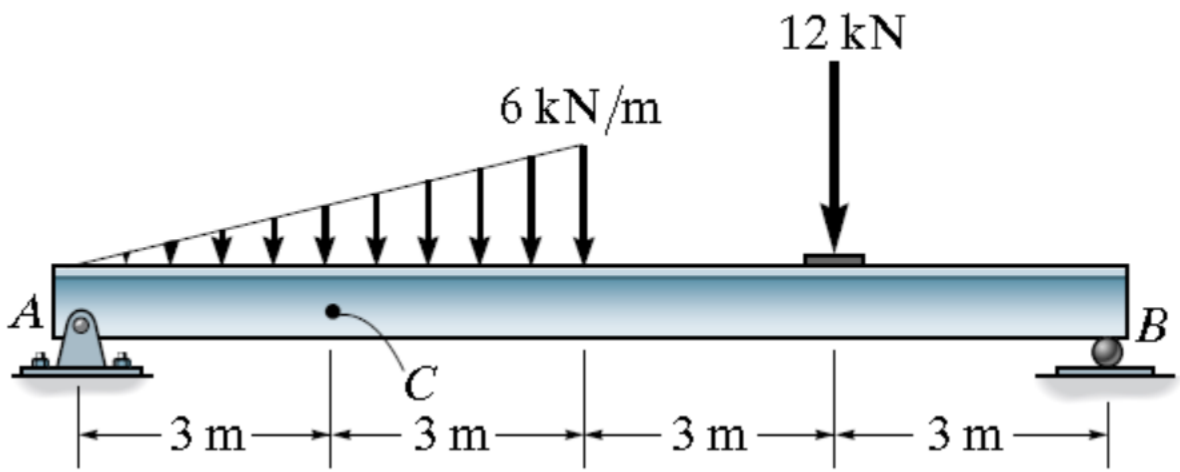
Determine the internal normal force, shear force, and bending moment at point *C*.



Ans.
NC = 0;
VC = 0;
MC = -24.0 kN.m

HW 4-2

Determine the internal normal force, shear force, and bending moment at point *C*.



Ans.
NC = 0;
VC = 10.5 kN;
MC = 40.5 kN .m

4.2
SHEAR AND MOMENT FUNCTIONS

4.2

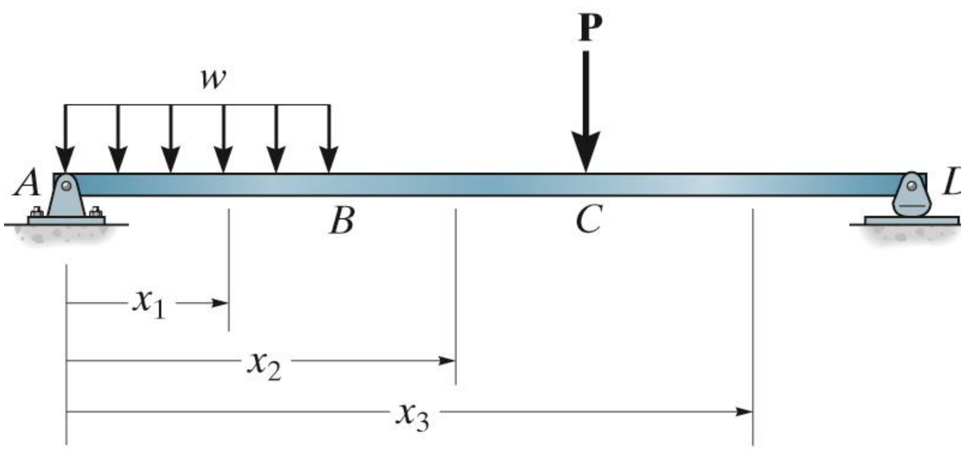
Shear and Moment Functions

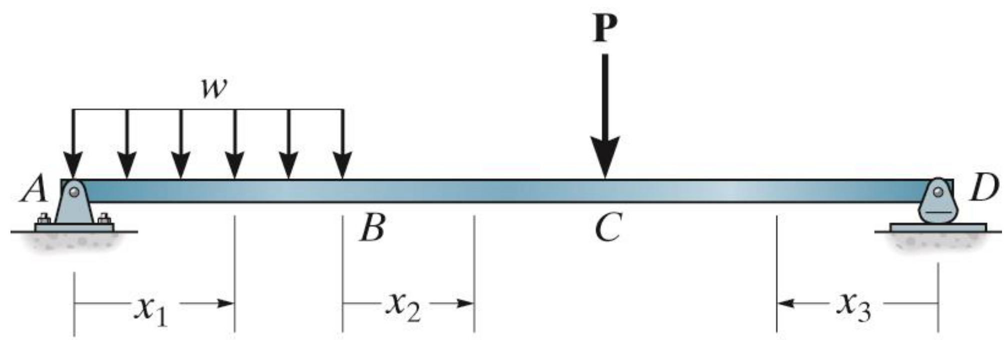
- Design of beam requires detailed knowledge of the variations of V & M

- Internal N is generally not considered as:
  - The loads applied to a beam act perpendicular to the beam's axis
  - For design purposes, a beam's resistance to shear & bending is more important than its ability to resist normal force
  - An exception is when it is subjected to compressive axial force where buckling may occur

Shear and Moment Functions

- In general, the internal shear & moment functions will be discontinuous or their slope will be discontinuous at points where:
  - The type or magnitude of the distributed load changes
  - Concentrated forces or couple moments are applied





Shear and Moment Functions

- Procedure for Analysis
  - Determine the support reactions on the beam
  - Resolve all the external forces into components acting perpendicular & parallel to beam's axis
  - Specify separate coordinates *x* and associated origins, extending into:
    - Regions of the beam between concentrated forces and/or couple moments; or
    - Discontinuity of distributed loading

Shear and Moment Functions

- Procedure for Analysis
  - Section the beam perpendicular to its axis at each distance *x*
  - From the free-body diagram of one of the segments, determine the unknowns *V* & *M*
  - On the free-body diagram, *V* & *M* should be shown acting in their +ve directions
  - *V* is obtained from $\sum F_y = 0$
  - *M* is obtained by $\sum M_s = 0$

Shear and Moment Functions

- Procedure for Analysis
  - The results can be checked by noting that:

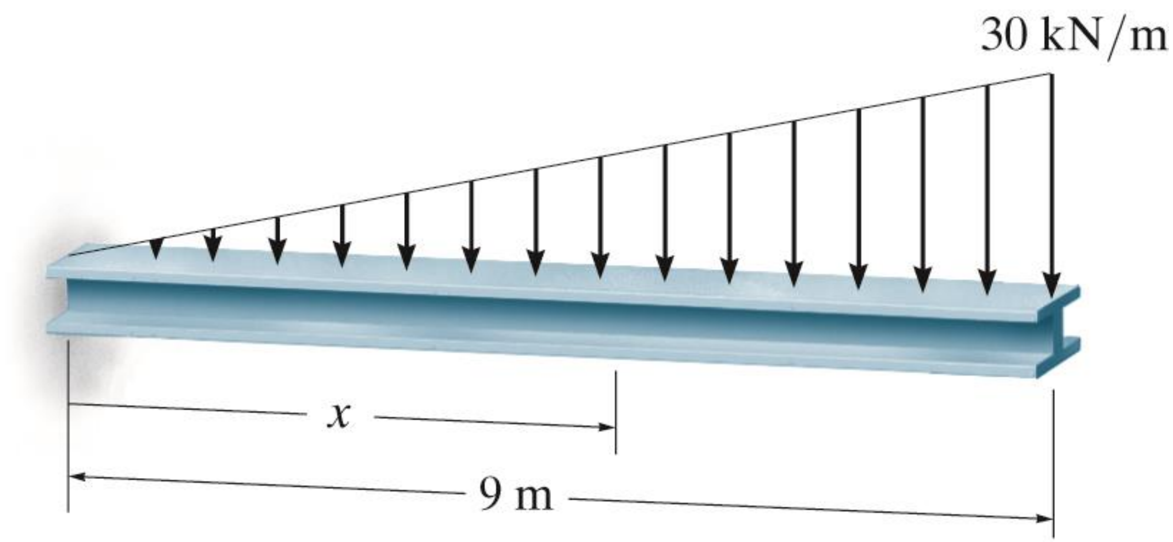
$$\frac{dM}{dx} = V$$

$$\frac{dV}{dx} = w$$

Shear and Moment Functions

Example 4.4

Determine the shear and moment in the beam shown *as a f*unction of *x.*

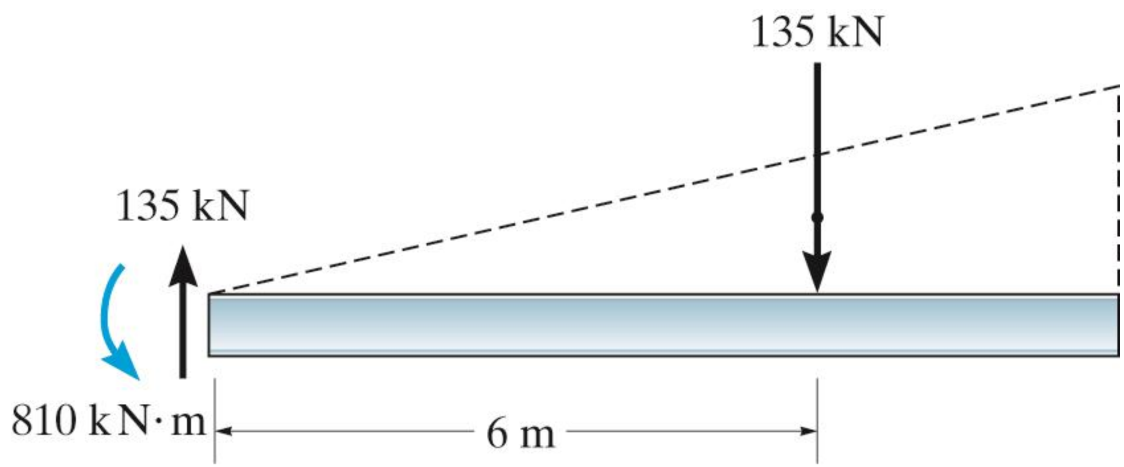


Shear and Moment Functions

Example 4.4 (Solution)

Support reactions:

For the purpose of computing the support reactions, the distributed load is replaced by its resultant force of 135 kN. However, this resultant is not the actual load on the beam



Shear and Moment Functions

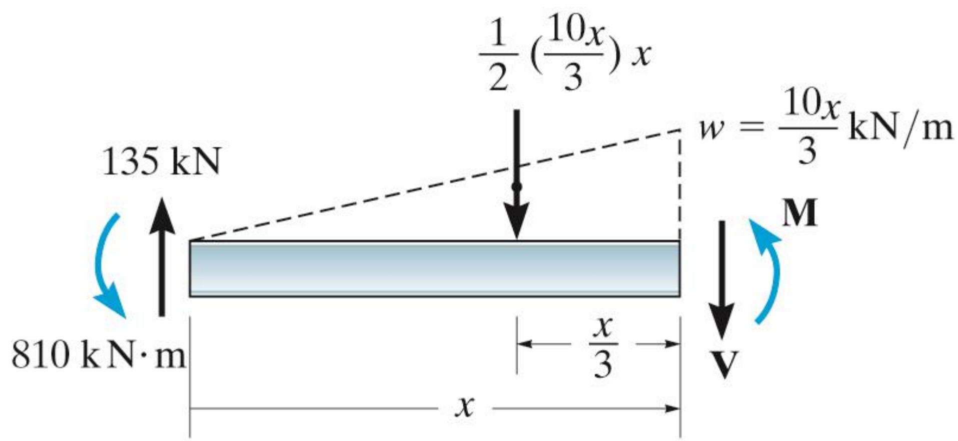
Example 4.4 (Solution)

Shear & moment functions:

A free-body diagram of the beam segment of length *x* is shown.

Note that the intensity of the triangular load at the section is found by proportion.

With the load intensity known, the resultant of the distributed load is found in the usual manner.



Shear and Moment Functions

Example 4.4 (Solution)

Shear & moment functions:

$$+\uparrow \sum F_y = 0;\ 135 - \frac{1}{2}\left(\frac{10x}{3}\right)x - V = 0$$

$$V = 135 - 1.667x^2$$

With anti-clockwise moments as +ve:

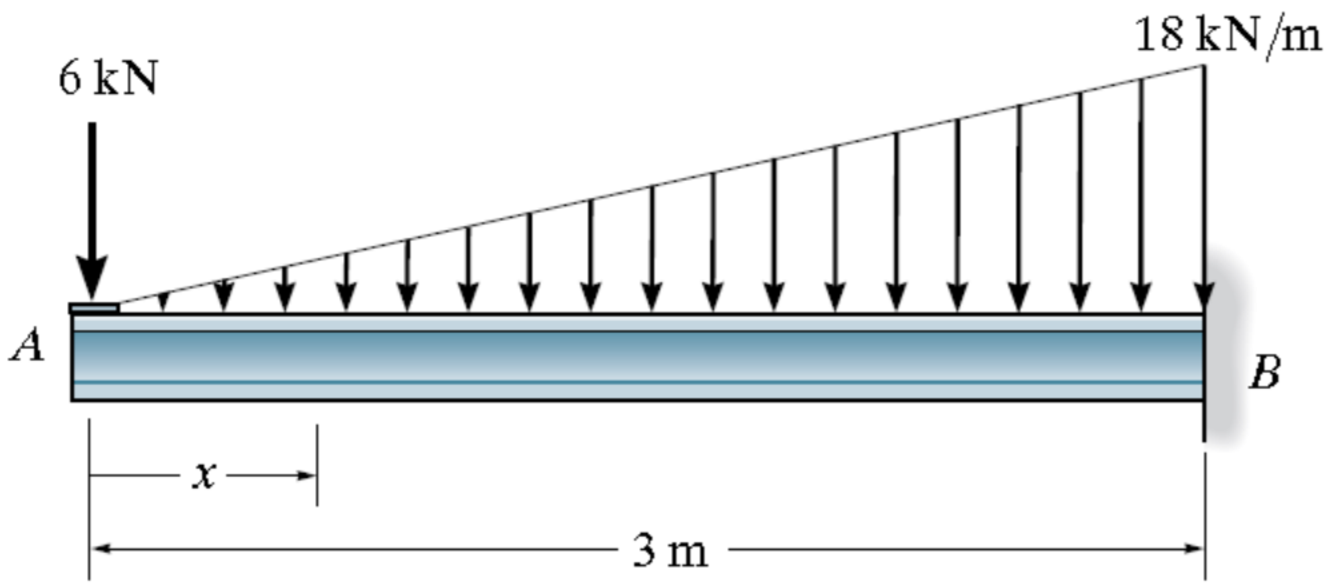
$$\sum M_s = 0;\ -810 - 135x + \left[\frac{1}{2}\left(\frac{10x}{3}\right)x\right]\frac{x}{3} + M = 0$$

$$M = -810 + 135x - 0.556x^3$$

Note that $dM/dx = V$ and $dV/dx = \dfrac{-10x}{3} = w$ serves as a check of the results

HW 4-3

Determine the internal shear and moment in the beam as a function of *x*.



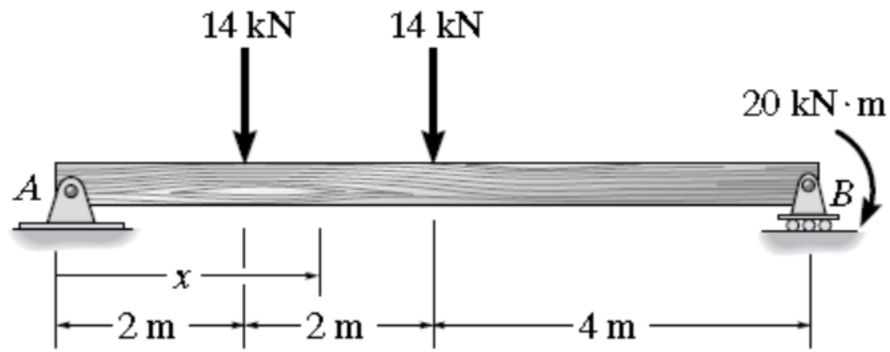
Ans.

$V = \{-3x^2 - 6\}$ kN

$M = \{-x^3 - 6x\}$ kN.m

HW 4-4

Determine the shear and moment in the beam as a function of x, where 2 m < x < 4 m.



Ans.
V = 1.00 kN
M = (x + 28) kN . m

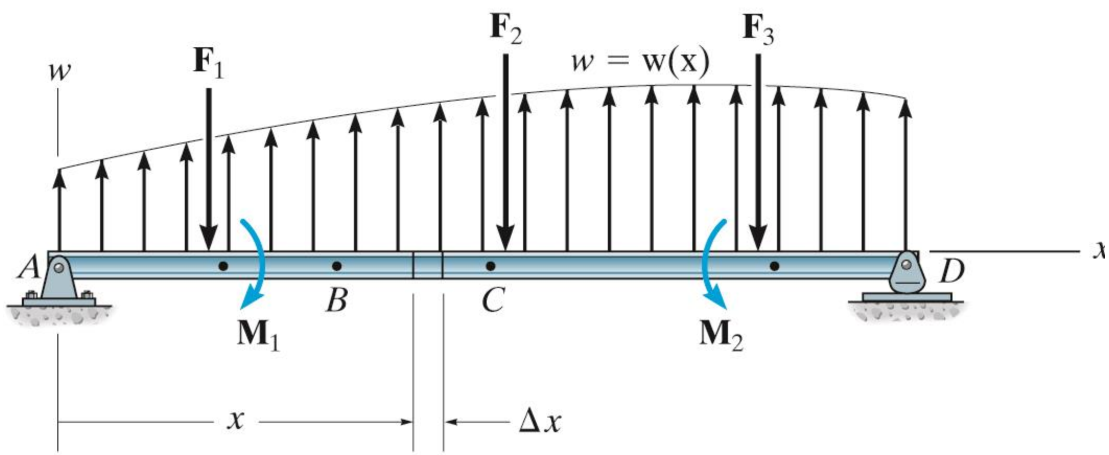
4.3

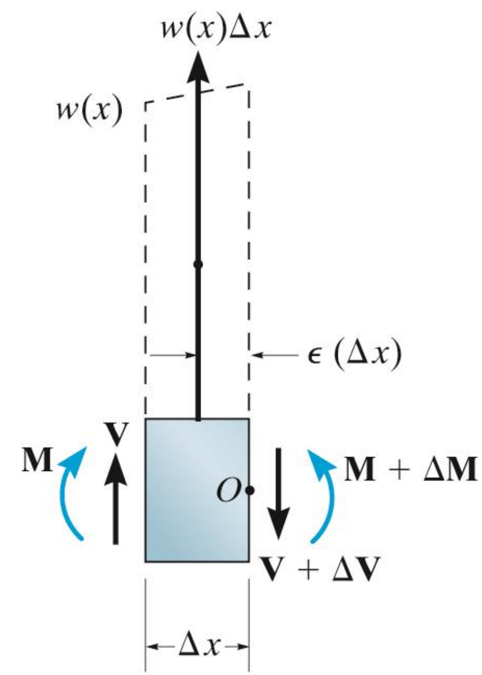
SHEAR AND MOMENT DIAGRAMS FOR A BEAM

4.3

Shear and Moment Diagrams for a Beam

- If the variations of *V* & *M* are plotted, the graphs are termed the shear diagram and moment diagram





Shear and Moment Diagrams for a Beam

- Applying the eqn of equilibrium, we have:

$$+\uparrow \sum F_y = 0;$$

$$V + w(x)\Delta x - (V + \Delta V) = 0$$
$$\Delta V = w(x)\Delta x$$

With anti-clockwise moments taken as +ve:

$$\sum M_o = 0;$$

$$-V\Delta x - M - w(x)\Delta x \varepsilon(\Delta x) + (M + \Delta M) = 0$$
$$\Delta M = V\Delta x + w(x)\varepsilon(\Delta x)^2$$

Shear and Moment Diagrams for a Beam

- Dividing by Δx & taking the limit as $\Delta x \rightarrow 0$, the previous eqns become:

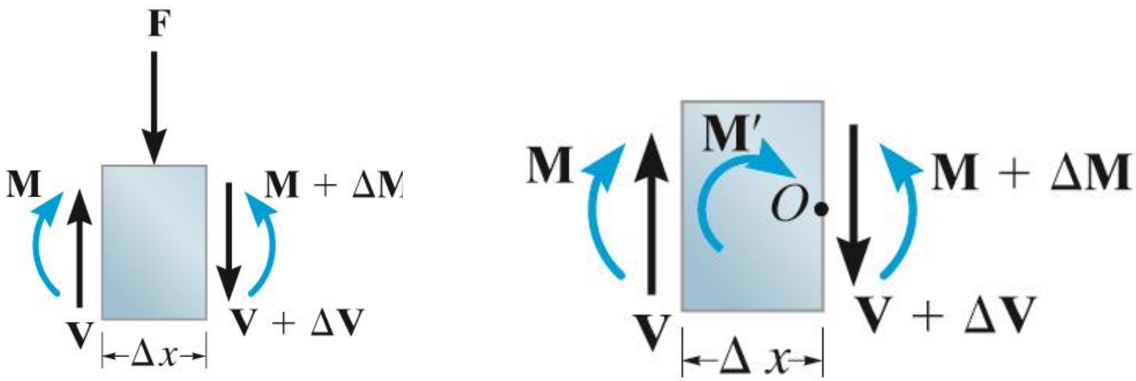
$$\frac{dV}{dx} = w(x) \quad , \quad \frac{dM}{dx} = V$$

- Integrating from one point to another between concentrated forces or couples in which case

$$\Delta V = \int \mathrm{w(x)dx} \quad , \quad \Delta M = \int V\mathrm{(x)dx}$$

Shear and Moment Diagrams for a Beam

- In order to account for concentrated force and moment, consider the free-body diagrams of differential elements of the beam



- It is seen that force equilibrium requires the change in shear to be

$$\Delta V = -F$$

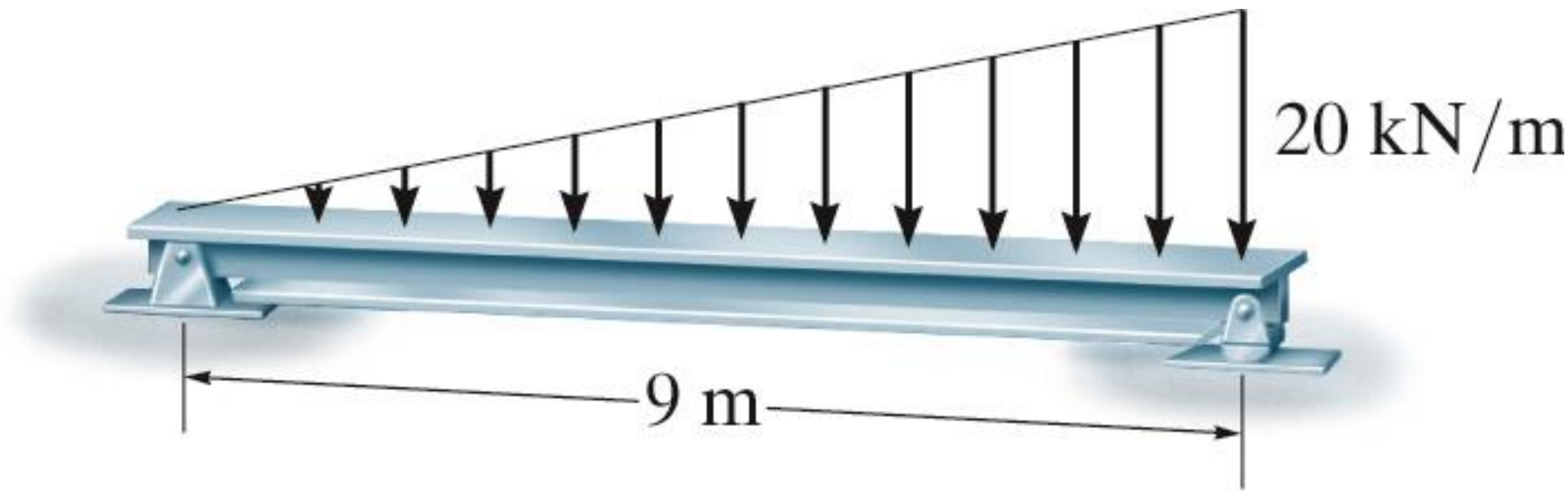
- Moment equilibrium requires the change in moment to be:

$$\Delta M = M'$$

Shear and Moment Diagrams for a Beam

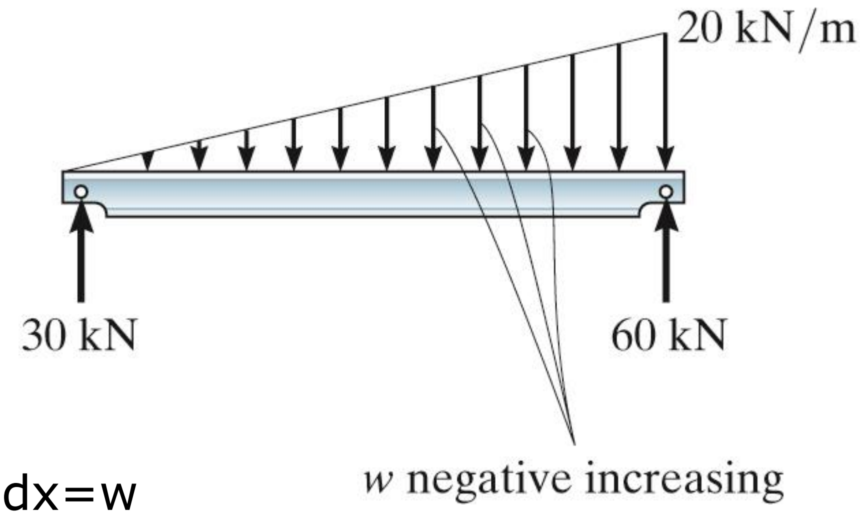
Example 4.8

Draw the shear and moment diagrams for the beam.



Shear and Moment Diagrams for a Beam

Example 4.8 (Solution)



At end points:

x = 0, V = 30 kN

x = 9 m, V = -60 kN

The load w is –ve & linearly increasing, dV/dx=w

The point of zero shear can be found by using method of sections from a beam segment of length x,

$$V = 0$$

$$+\uparrow \sum F_y = 0 \Rightarrow 30 - \frac{1}{2}\left[20\left(\frac{x}{9}\right)\right]x = 0$$

$$x = 5.20\,\text{m}$$

Shear and Moment Diagrams for a Beam

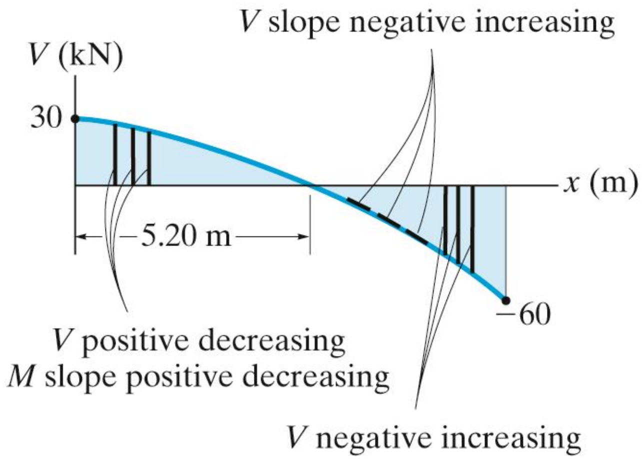
Example 4.8 (Solution)

From the shear diagram, for 0<x<5.20 m, the value of shear is +ve but ↓ so *dM*/*dx*=*V* is also +ve and↓

At *x* = 5.20m, *dM*/*dx*=0

Likewise for 5.20 m<x<9 m, the shear & so the slope of the moment diagram are –ve ↑

Max *M* is at *x* = 5.20 m since *dM*/*dx* =*V*=0



Shear and Moment Diagrams for a Beam

Example 4.8 (Solution)

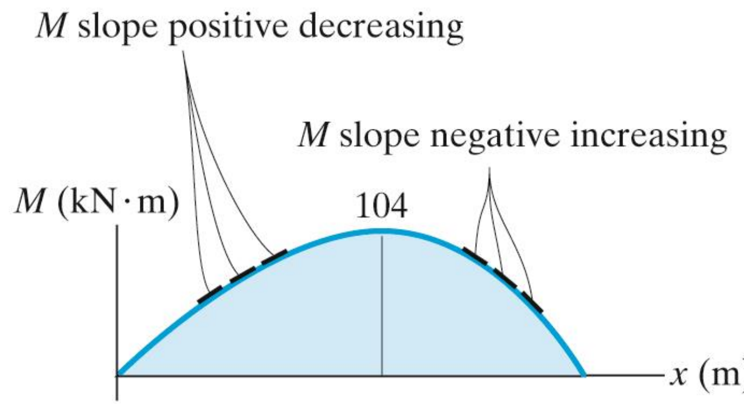
We have

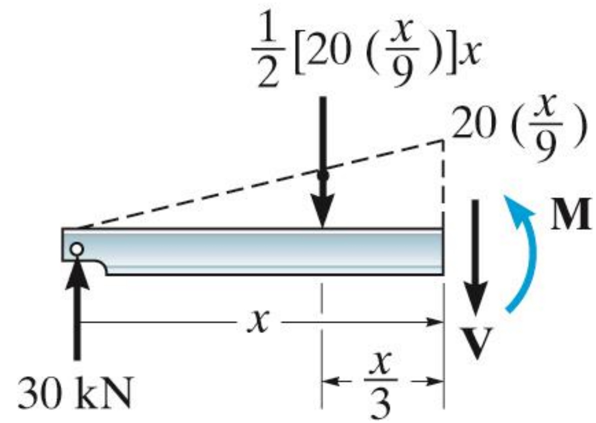
Anticlockwise moment as -ve:

$$\sum M_s = 0$$

$$-30(5.20) + \frac{1}{2}\left[20\left(\frac{5.20}{9}\right)\right]5.20\left(\frac{5.20}{3}\right) + M = 0$$

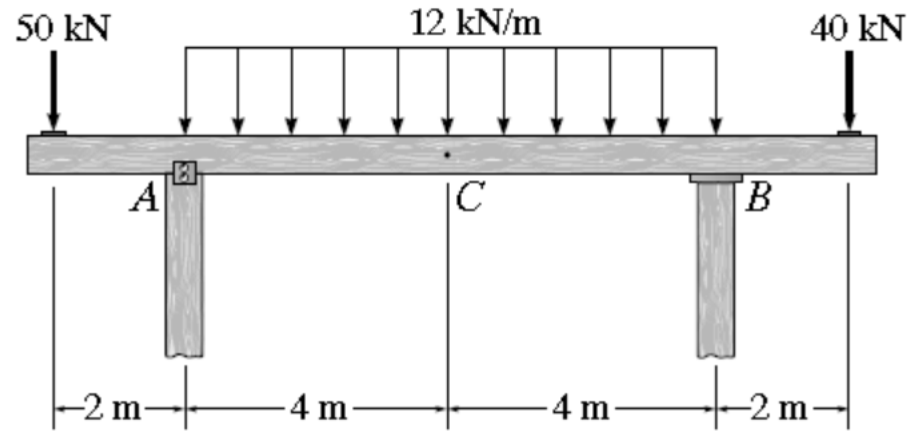
$$M = 104\,\text{kN}\bullet\text{m}$$





HW 4-5

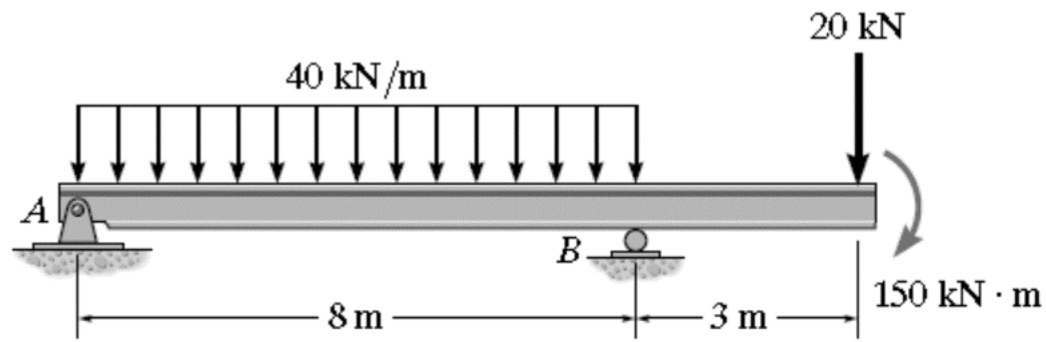
Draw the shear and moment diagrams for the beam.



Ans.
Vmax=-50.5 KN
Mmax=-100 KN.m

HW 4-6

Draw the shear and moment diagrams for the beam.



Ans.
Vmax=-186.25 KN
Mmax=223.6 KN.m

4.4

SHEAR AND MOMENT DIAGRAMS FOR A FRAME

4.4

Shear and Moment Diagrams for a Frame

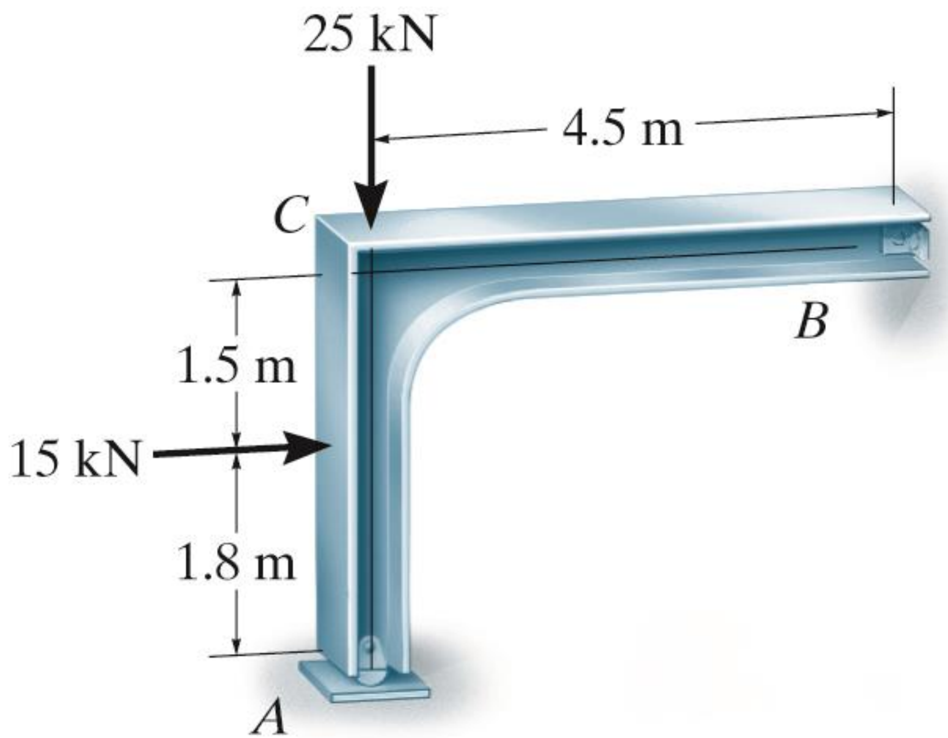
- A frame is composed of several connected members that are either fixed or pin connected at their ends.

- We will use the opposite sign convention and always draw the moment diagram positive on the compression side of the member.

Shear and Moment Diagrams for a Frame

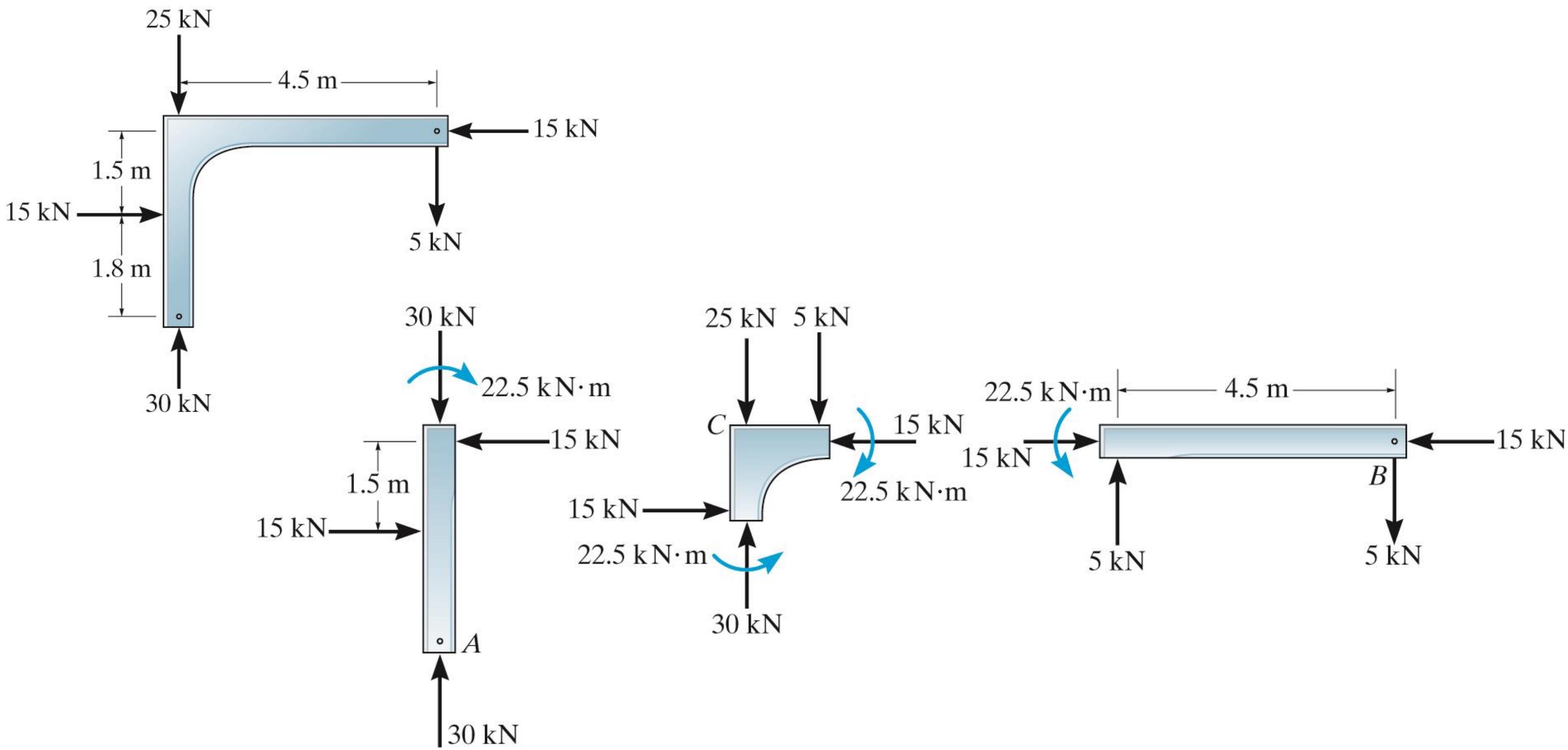
Example 4.13

Draw the moment diagram for the tapered frame shown. Assume the support at *A* is a roller and *B* is a pin.



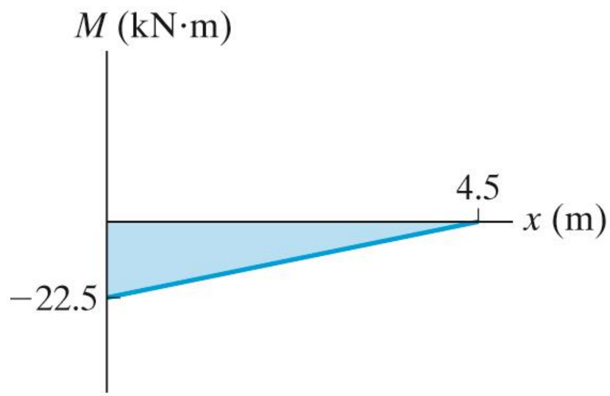
Shear and Moment Diagrams for a Frame

Example 4.13 (Solution)

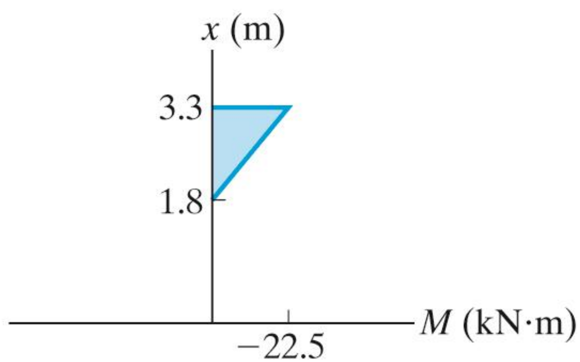


Shear and Moment Diagrams for a Frame

Example 4.13 (Solution)



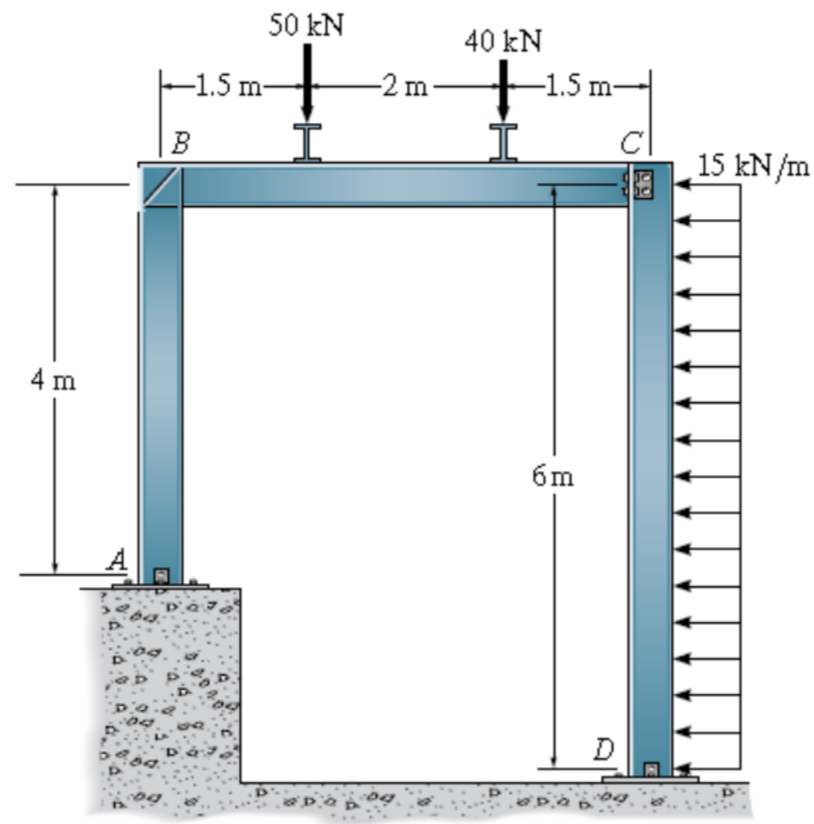
member *CB*



member *AC*

HW 4-7

Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at *A*, *C*, and *D* and there is a fixed joint at *B*.



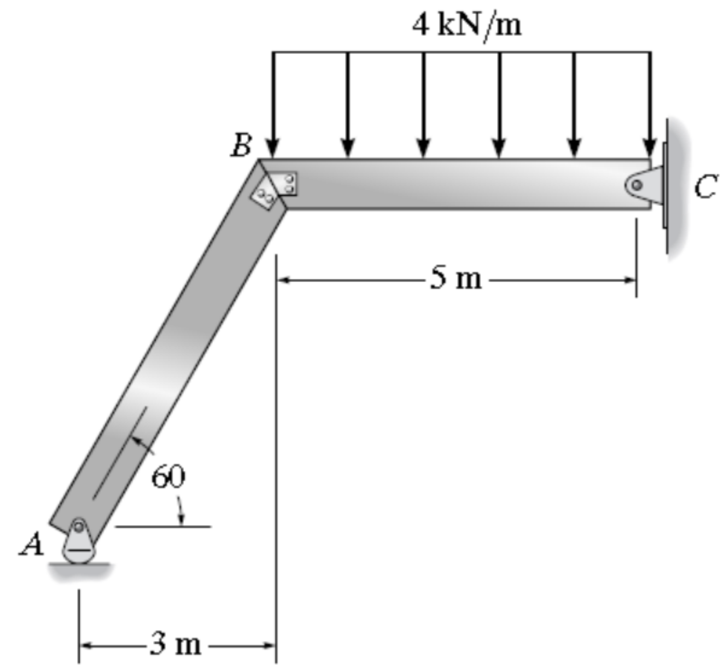
Ans.

Vmax=83 KN

Mmax=-180 KN.m

HW 4-8

Draw the shear and moment diagrams for each member of the frame. The joint at *B* is fixed connected.



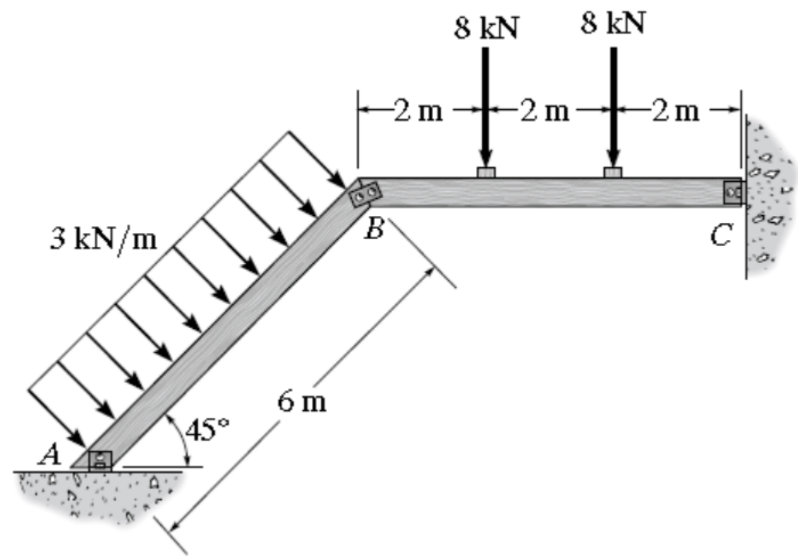
Ans.

V_{max} = -13.75 kN;

M_{max} = 23.6 kN.m

HW 4-9

Draw the shear and moment diagrams for each member of the frame. The members are pin connected at *A*, *B*, and *C*.



Ans.

Vmax = 9.00 kN;

Mmax = 16.0 kN.m

4.5

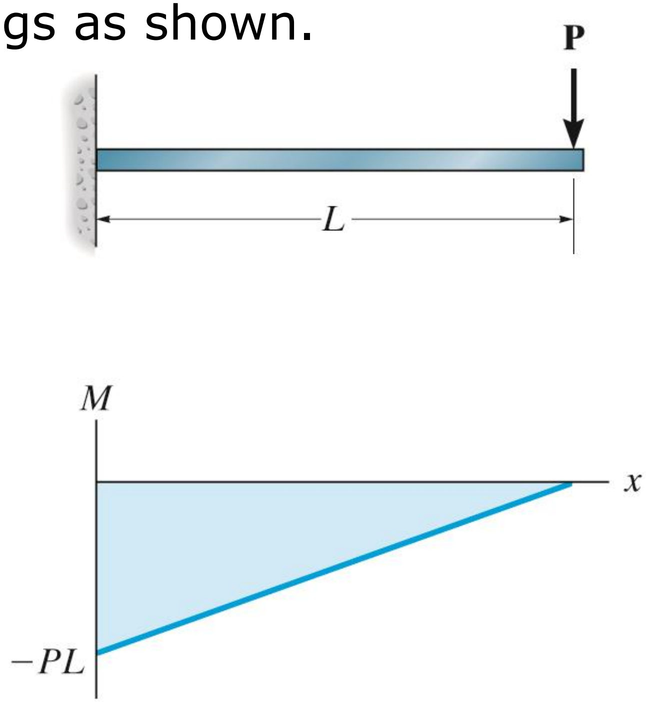
MOMENT DIAGRAMS CONSTRUCTED BY THE METHOD OF SUPERPOSITION

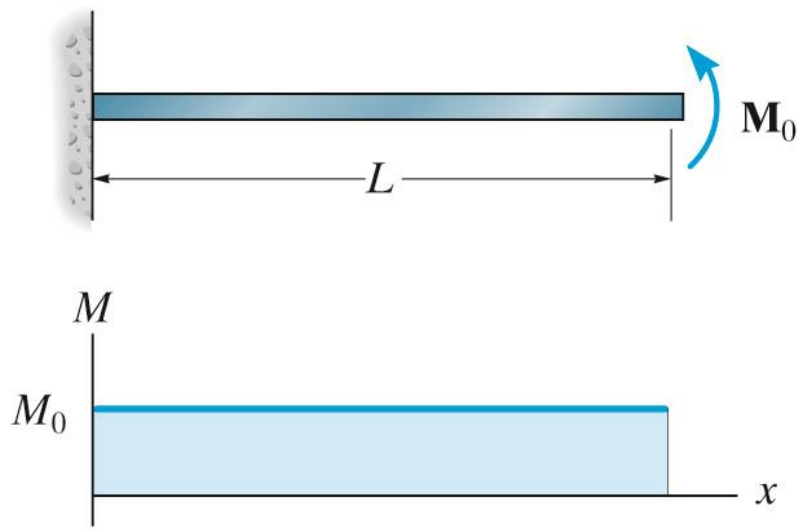
4.5

Moment Diagrams Constructed by the Method of Superposition

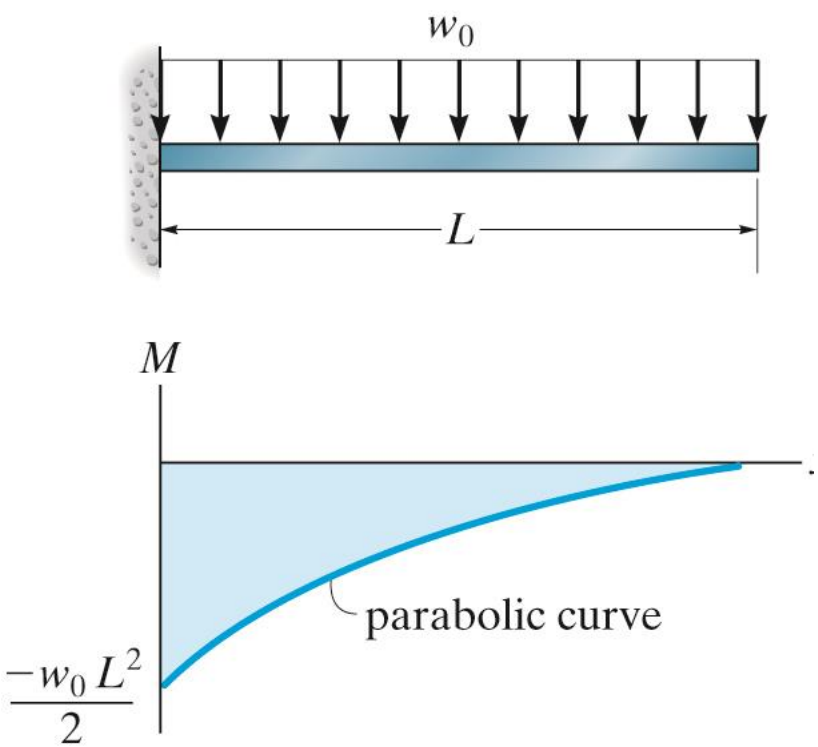
- Beams are used primarily to resist bending stress, it is important that the moment diagram accompany the solution for their design.

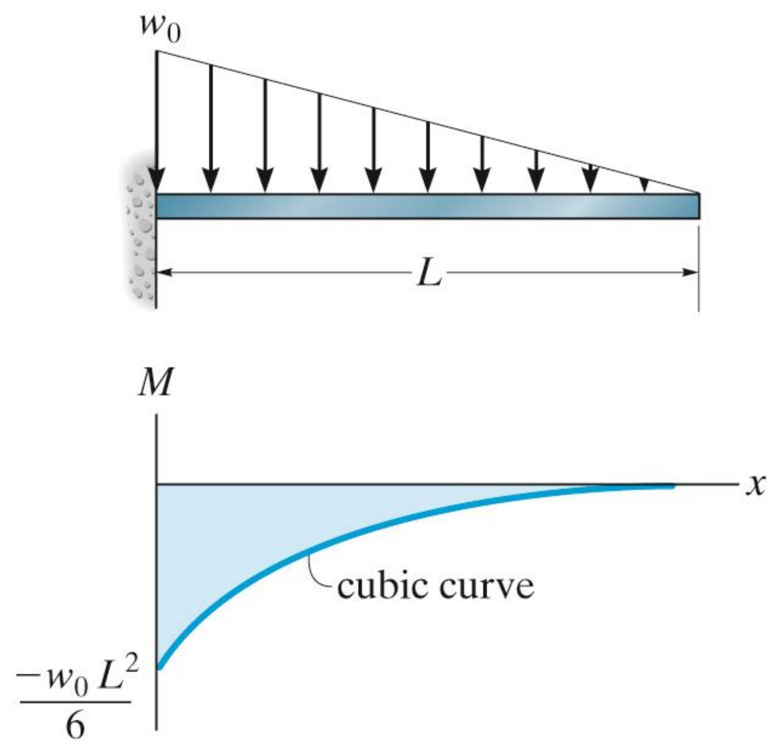
- Most loadings on beams in structural analysis will be a combination of the loadings as shown.





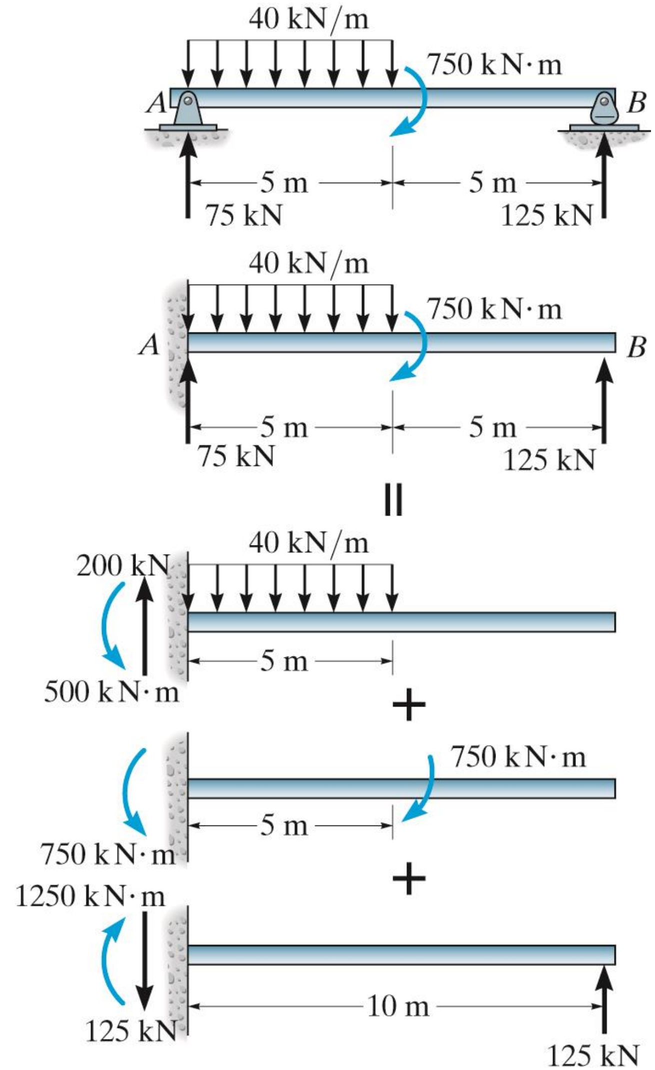
Moment Diagrams Constructed by the Method of Superposition





Moment Diagrams Constructed by the Method of Superposition

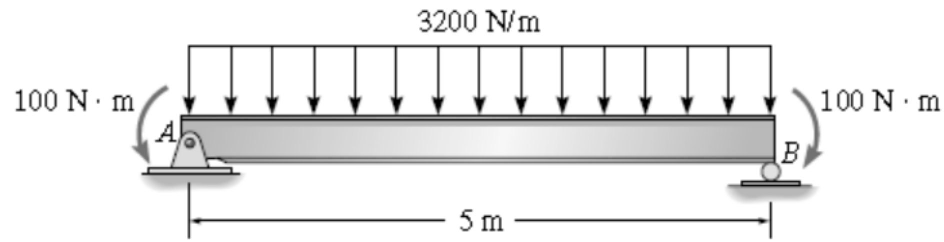
- Following show the method of superposition for simply supported beam.



superposition of cantilevered beams

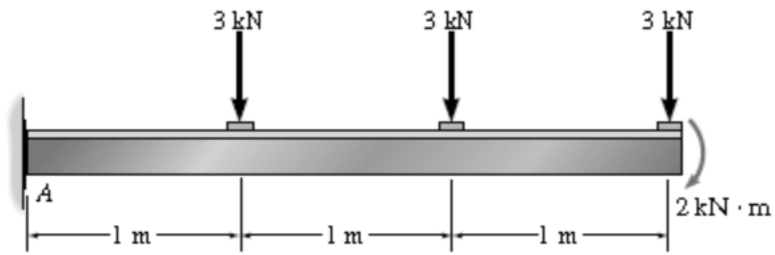
HW 4-10

Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be simply supported at *A* and *B* as shown.



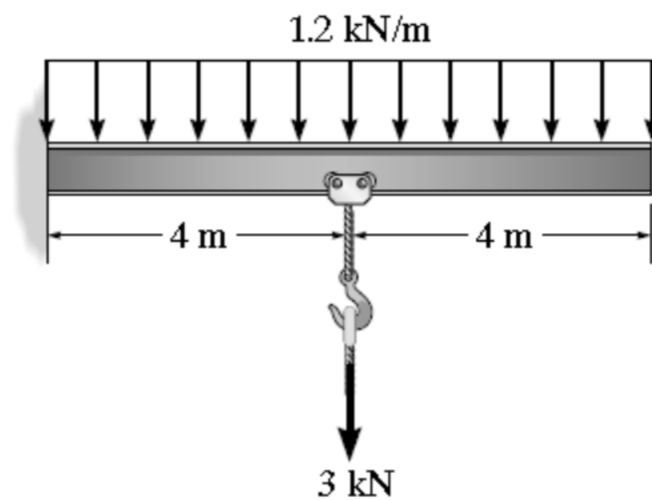
HW 4-11

Draw the moment diagrams for the beam using the method of superposition. The beam is cantilevered from *A*.

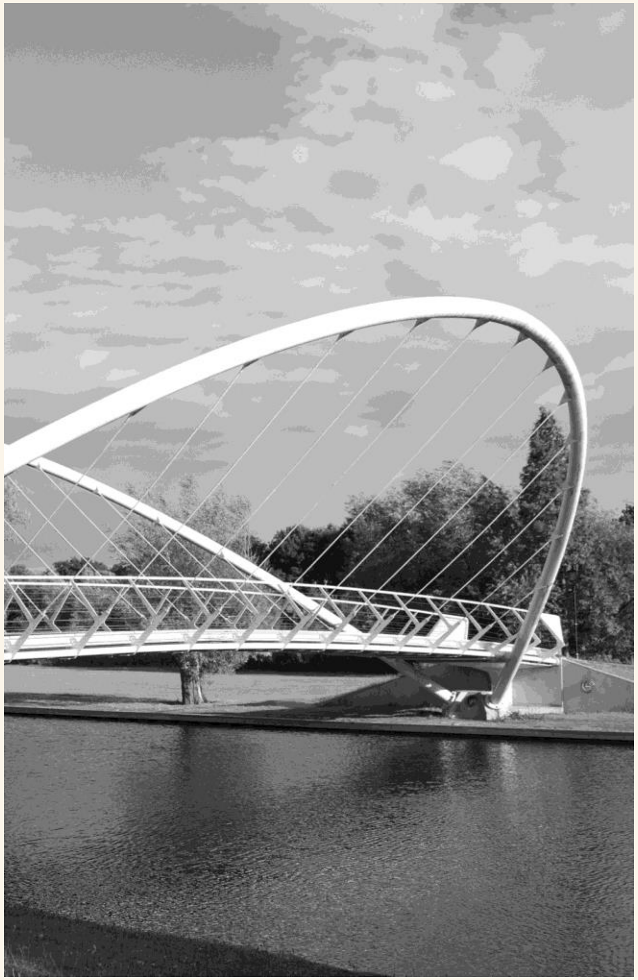


HW 4-12

Draw the moment diagrams for the beam using the method of superposition.



CHAPTER 5:
CABLES AND ARCHES

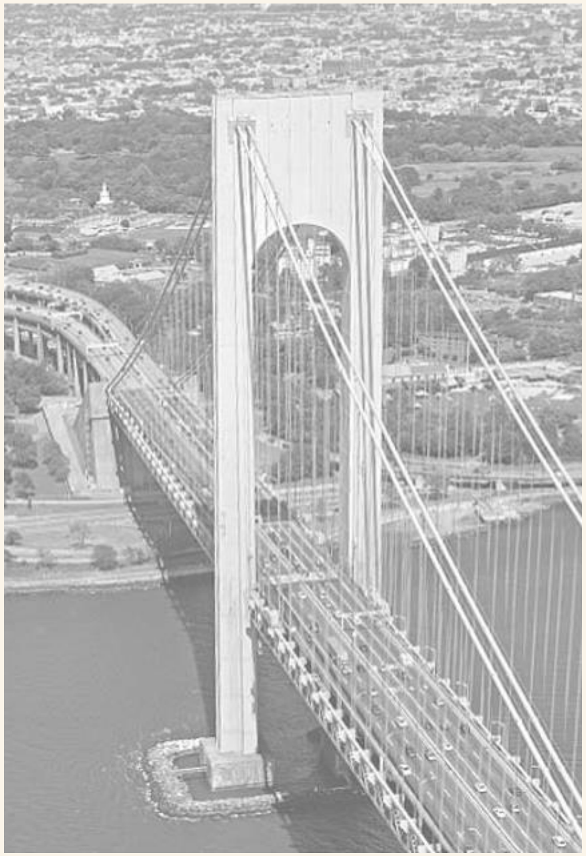


5

Chapter Outline

5.1

CABLES



5.1

Cables

- Assumptions when deriving the relations between force in cable & its slope
- Cable is perfectly flexible & inextensible
- Due to its flexibility, cable offers no resistance to shear or bending
- The force acting the cable is always tangent to the cable at points along its length

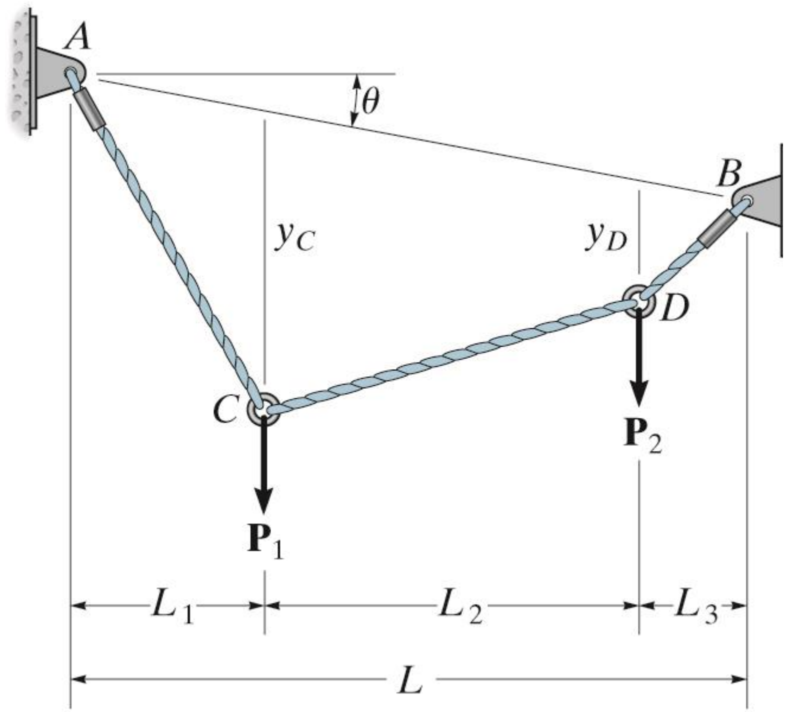
5.2

CABLES SUBJECTED TO CONCENTRATED LOADS

5.2

Cable Subjected to Concentrated Loads

- When a cable of negligible weight supports several concentrated loads, the cable takes the form of several straight line segments
- Each of the segment is subjected to a constant tensile force
- θ specifies the angle of the cord *AB*
- *L* = cable length



Cable Subjected to Concentrated Loads

- If L_1, L_2 & L_3 and loads P_1 & P_2 are known, determine the 9 unknowns consisting of the tension in each of the 3 segments, the 4 components of reactions at *A* & *B* and the sags y_C & y_D

- For solutions, we write 2 eqns of equilibrium at each of 4 points *A*, *B*, *C* & *D*

- Total 8 eqns

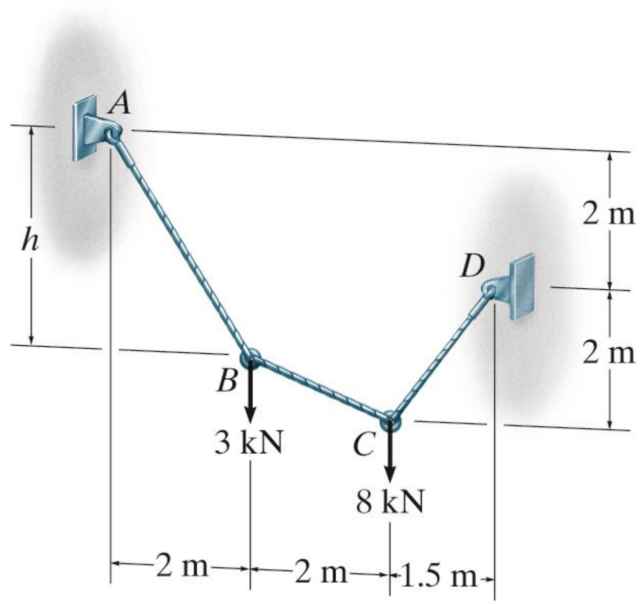
- The last eqn comes from the geometry of the cable

Cable Subjected to Concentrated Loads

Example 5.1

Determine the tension in each segment of the cable shown in Figure.

Also, what is the dimension *h*?



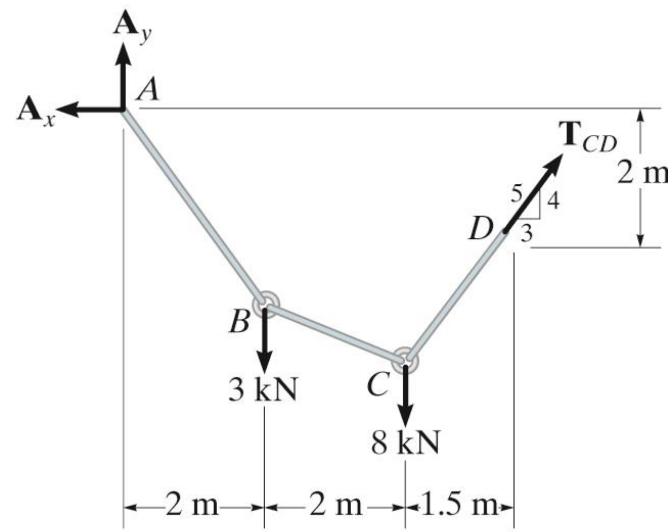
Cable Subjected to Concentrated Loads

Example 5.1 (Solution)

By inspection, there are
➔ 4 unknown external reactions (A_x, A_y, D_x and D_y)
➔ 3 unknown cable tensions

These unknowns and sag, *h* can be determined from available equilibrium eqns applied to points *A* through *D*.

A more direct approach to the solution is to recognize that the slope of cable *CD* is specified.



Cable Subjected to Concentrated Loads

Example 5.1 (Solution)

With anti-clockwise moment as +ve

$$\Sigma M_A = 0$$

$$T_{CD}(3/5)(2\,\text{m}) + T_{CD}(4/5)(5.5\,\text{m}) - 3\,\text{kN}(2\,\text{m}) - 8\,\text{kN}(4\,\text{m}) = 0$$

$$T_{CD} = 6.79\,\text{kN}$$

Now we can analyze the equilibrium of points C and B in sequence.

Point C (Fig 5.2c)

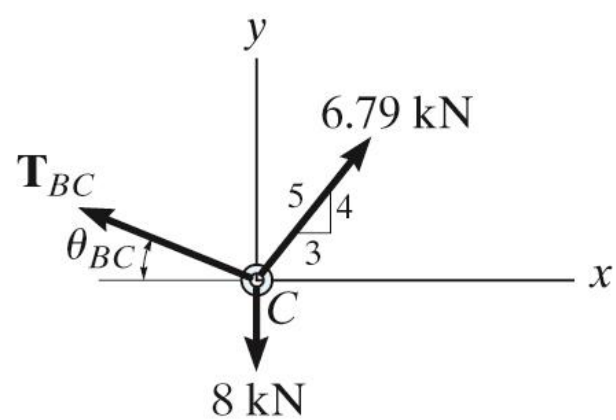
$$\underset{\rightarrow}{+}\Sigma F_x = 0$$

$$6.79\,\text{kN}(3/5) - T_{BC}\cos\theta_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0$$

$$6.79\,\text{kN}(4/5) - 8\,\text{kN} + T_{BC}\sin\theta_{BC} = 0$$

$$\theta_{BC} = 32.3^o \quad \text{and} \quad T_{BC} = 4.82\,\text{kN}$$



Cable Subjected to Concentrated Loads

Example 5.1 (Solution)

Point B (Fig 5.2d)

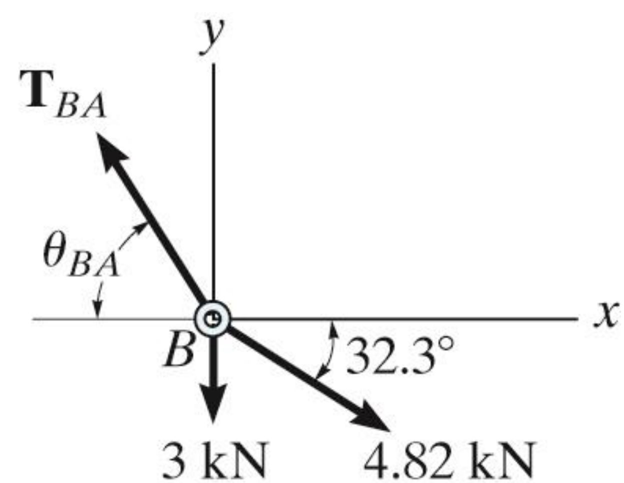
$$\underset{\rightarrow}{+}\Sigma F_x = 0$$

$$-T_{BA}\cos\theta_{BA} + 4.82\,\text{kN}\cos 32.3^o = 0$$

$$+\uparrow \Sigma F_y = 0$$

$$T_{BA}\sin\theta_{BA} - 4.82\,\text{kN}\sin 32.3^o - 3\,\text{kN} = 0$$

$$\theta_{BA} = 53.8^o \text{ and } T_{BA} = 6.90\,\text{kN}$$

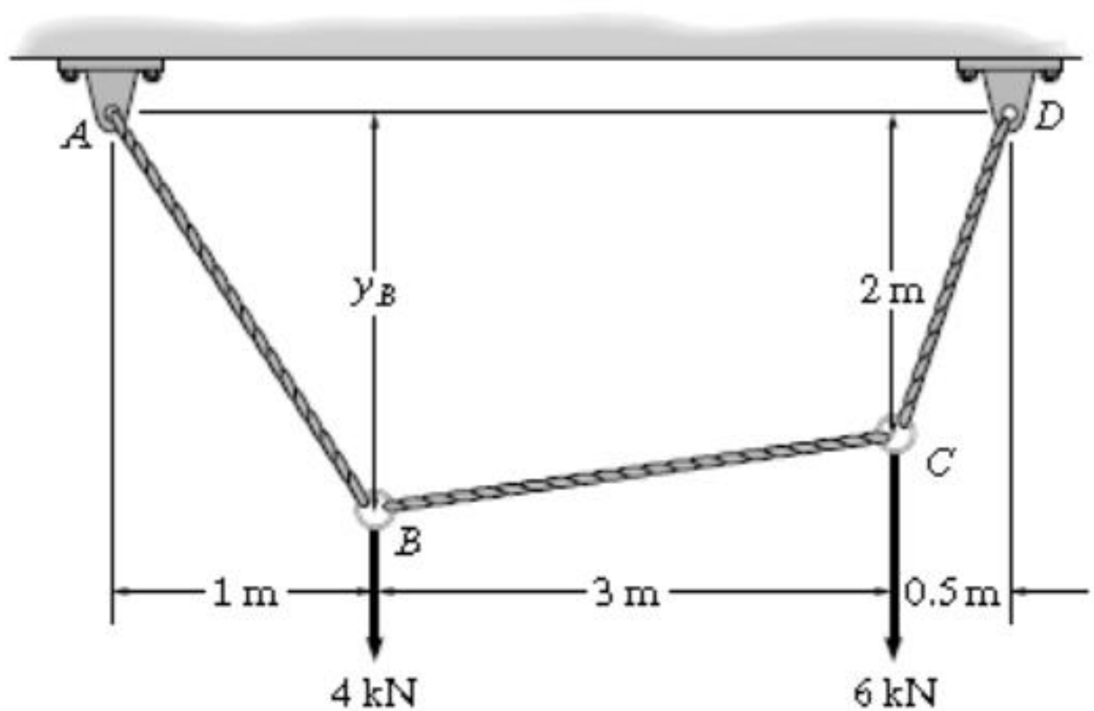


Hence from Fig 5.2(a)

$$h = (2\,\text{m})\tan 53.8^o = 2.74\,\text{m}$$

HW 5-1

Cable *ABCD* supports the loading shown. Determine the maximum tension in the cable and the sag of point *B*.



Ans.

T_{CD} = 6.414 kN = 6.41 kN (Max)

T_{BC} = 1.571 kN = 1.57 kN

θ_{BC} = 8.130°

T_{AB} = 4.086 kN = 4.09 kN

θ_{AB} = 67.62°

y_B = 2.43 m

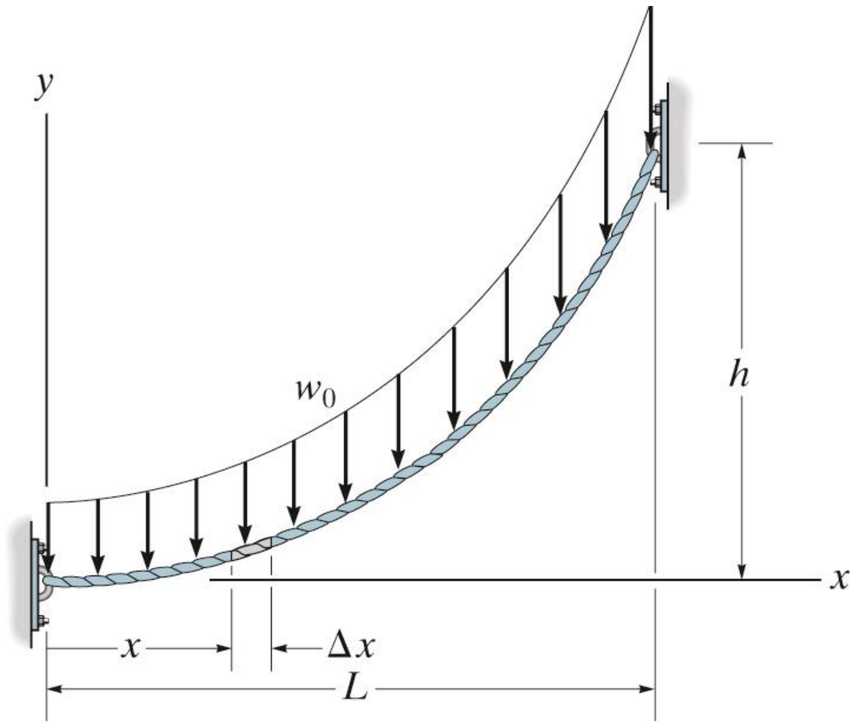
5.3

CABLES SUBJECTED TO A UNIFORM DISTRIBUTED LOAD

5.3

Cable Subjected to a Uniform Distributed Load

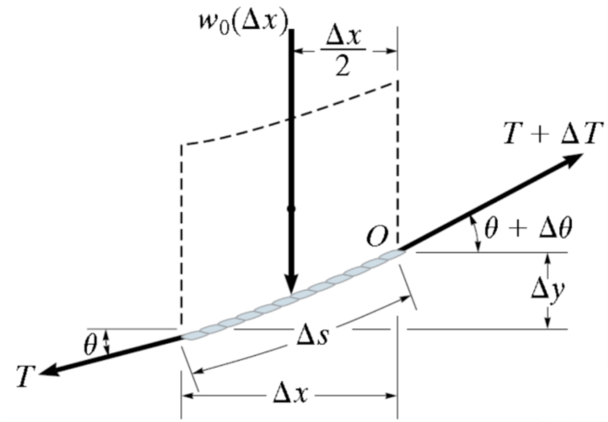
- The *x*,*y* axes have their origin located at the lowest point on the cable such that the slope is zero at this point
- Since the tensile force in the cable changes continuously in both magnitude & direction along the cable's length, this change is denoted by ΔT



Cable Subjected to a Uniform Distributed Load

- The distributed load is represented by its resultant force $w_o \Delta x$ which acts at $\Delta x/2$ from point O

- Applying eqns of equilibrium yields:



$$\xrightarrow{+} \Sigma F_x = 0$$

$$-T\cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$$

$$+\uparrow \Sigma F_y = 0$$

$$-T\sin\theta - w_o(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$$

With anti-clockwise moment as +ve

$$\Sigma M_O = 0$$

$$w_o(\Delta x)(\Delta x/2) - T\cos\theta\Delta y + T\sin\theta\Delta x = 0$$

Cable Subjected to a Uniform Distributed Load

- Dividing each of these eqn by Δx and taking the limit as $\Delta x \to 0$, hence, $\Delta y \to 0$, $\Delta\theta \to 0$ and $\Delta T \to 0$, we obtain:

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad \text{eqn 1}$$

$$\frac{d(T\sin\theta)}{dx} = w_o \qquad \text{eqn 2}$$

$$\boxed{\frac{dy}{dx} = \tan\theta} \qquad \text{eqn 3}$$

Cable Subjected to a Uniform Distributed Load

- Integrating Eqn 1 where $T = F_H$ at $x = 0$, we have:

$$\boxed{T\cos\theta = F_H} \quad \text{eqn 4}$$

- Which indicates the horizontal component of force at any point along the cable remains constant

- Integrating Eqn 2 realizing that $T \sin\theta = 0$ at $x = 0$, we have:

$$T\sin\theta = w_o x \quad \text{eqn 5}$$

Cable Subjected to a Uniform Distributed Load

- Dividing Eqn 5 by Eqn 5.4 eliminates *T*

- Then using Eqn 3, we can obtain the slope at any point

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \quad \text{eqn 6}$$

- Performing a second integration with *y* = 0 at *x* = 0 yields

$$y = \frac{w_o}{2F_H} x^2 \quad \text{eqn 7}$$

Cable Subjected to a Uniform Distributed Load

- This is the eqn of a parabola
- The constant F_H may be obtained by using the boundary condition $y = h$ at $x = L$
- Thus

$$F_H = \frac{w_o L^2}{2h} \quad \text{eqn 8}$$

- Substituting into Eqn 7

$$y = \frac{h}{L^2} x^2 \quad \text{eqn 9}$$

Cable Subjected to a Uniform Distributed Load

- From Eqn 4, the max tension in the cable occurs when θ is max, i.e. at $x=L$
- From Eqn 4 and 5

$$T_{max} = \sqrt{F^2{}_H + (w_o L)^2} \qquad \text{eqn 10}$$

- Using Eqn 8 we can express T_{max} in terms of w_o

$$T_{max} = w_o L \sqrt{1 + (L/2h)^2} \qquad \text{eqn 11}$$

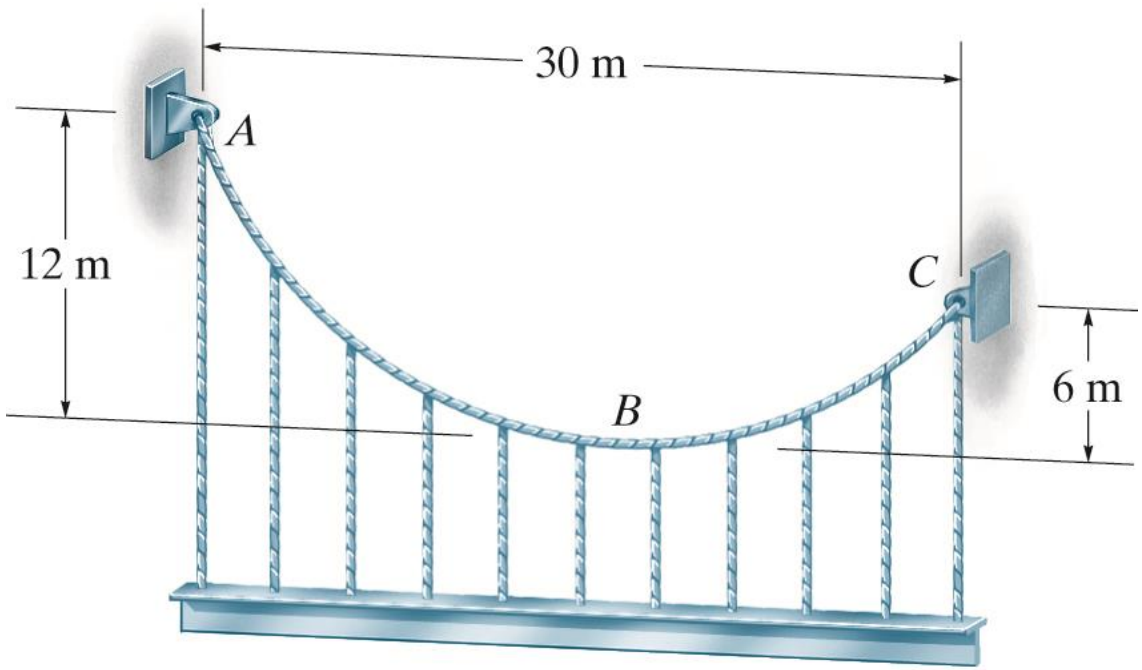
Cable Subjected to a Uniform Distributed Load

- We have neglected the weight of the cable which is uniform along the length
- A cable subjected to its own weight will take the form of a catenary curve
- If the sag-to-span ratio is small, this curve closely approximates a parabolic shape

Cable Subjected to a Uniform Distributed Load

Example 5.2

The cable supports a girder which weighs 12 kN/m. Determine the tension in the cable at points *A*, *B* & *C*.

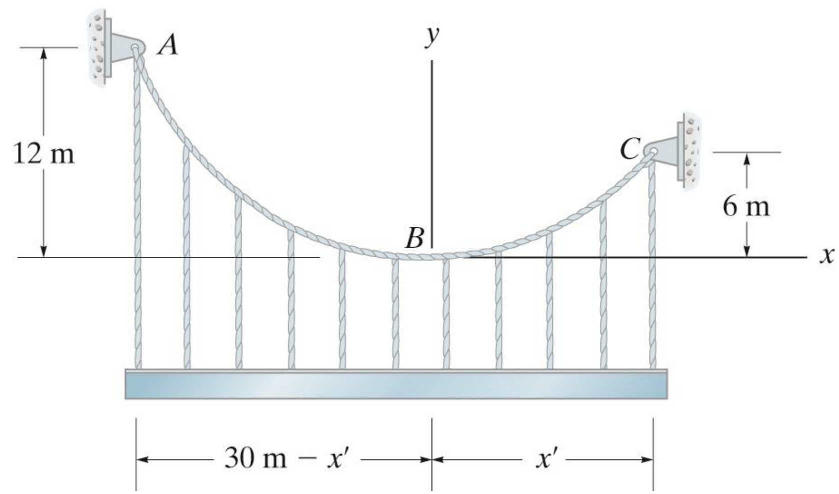


Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

The origin of the coordinate axes is established at point *B*, the lowest point on the cable where slope is zero,

$$y = \frac{w_o}{2F_H}x^2 = \frac{12\,\text{kN/m}}{2F_H}x^2 = \frac{6}{F_H}x^2 \quad (1)$$



Assuming point *C* is located *x*' from *B* we have:

$$6 = \frac{6}{F_H}x'^2 \Rightarrow F_H = 1.0x'^2 \quad (2)$$

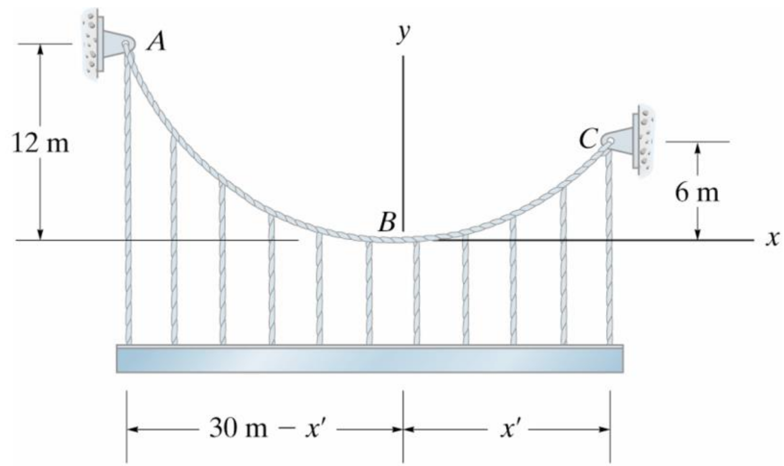
Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

For point A,

$$12 = \frac{6}{F_H}[-(30 - x')]^2$$

$$12 = \frac{6}{1.0x'^2}[-(30 - x')]^2$$



$$x'^2 + 60x' - 900 = 0 \Rightarrow x' = 12.43\,\text{m}$$

Thus from eqn 2 and 1, we have:

$$F_H = 1.0(12.43)^2 = 154.4\,\text{kN}$$

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \longrightarrow \frac{dy}{dx} = \frac{12}{154.4}x = 0.7772x \quad (3)$$

Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

At point *A*,

$$x = -(30 - 12.43) = -17.57\,\text{m}$$

$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{x=-17.57} = 0.7772(-17.57) = -1.366$$

$$\theta_A = -53.79^o$$

We have,

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{154.4}{\cos(-53.79^o)} = 261.4\,\text{kN}$$

Cable Subjected to a Uniform Distributed Load

Example 5.2 (Solution)

At point *B*, *x* = 0

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_{x=0} = 0 \Rightarrow \theta_B = 0^o$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{154.4}{\cos 0^o} = 154.4\,\text{kN}$$

At point *C*, *x* = 12.43 m

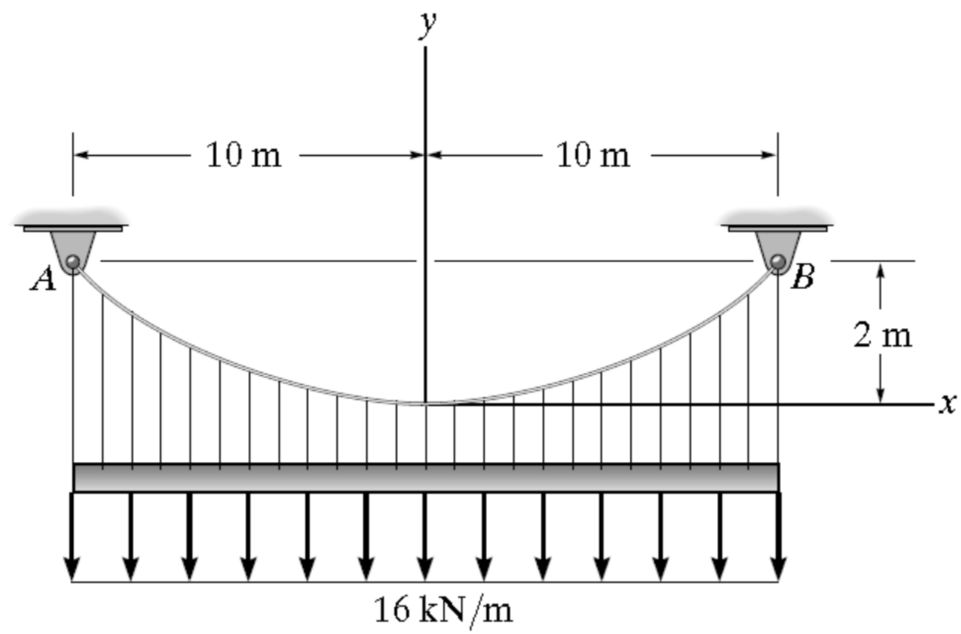
$$\tan\theta_C = \left.\frac{dy}{dx}\right|_{x=12.43} = 0.7772(12.43) = 0.9660$$

$$\theta_C = 44.0^o$$

$$T_C = \frac{F_H}{\cos\theta_C} = \frac{154.4}{\cos 44.0^o} = 214.6\,\text{kN}$$

HW 5-2

Determine the maximum and minimum tension in the cable.



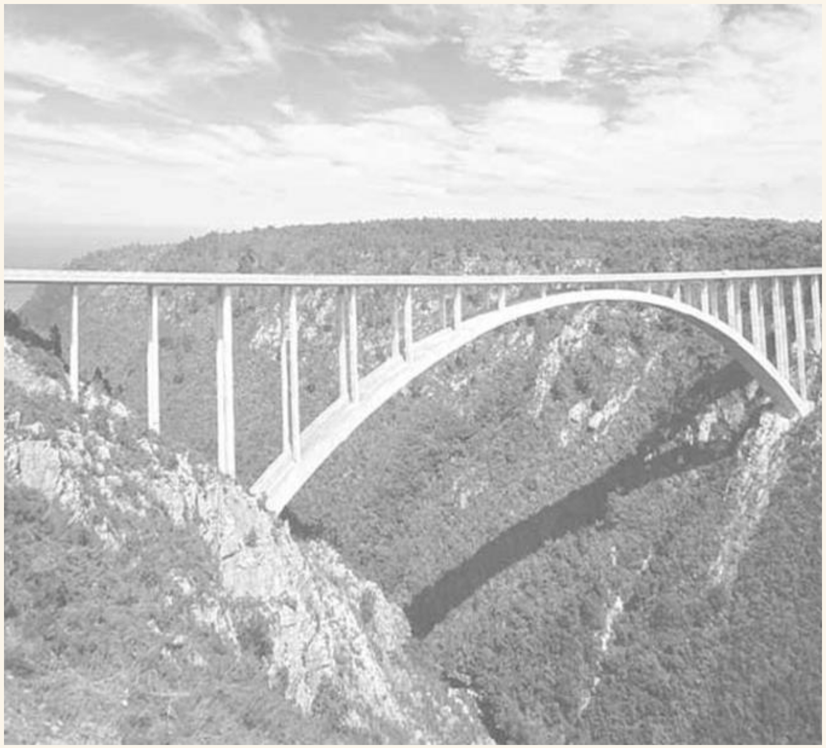
Ans.

T_{min} = 400 KN

T_{max}= 431 KN

5.4

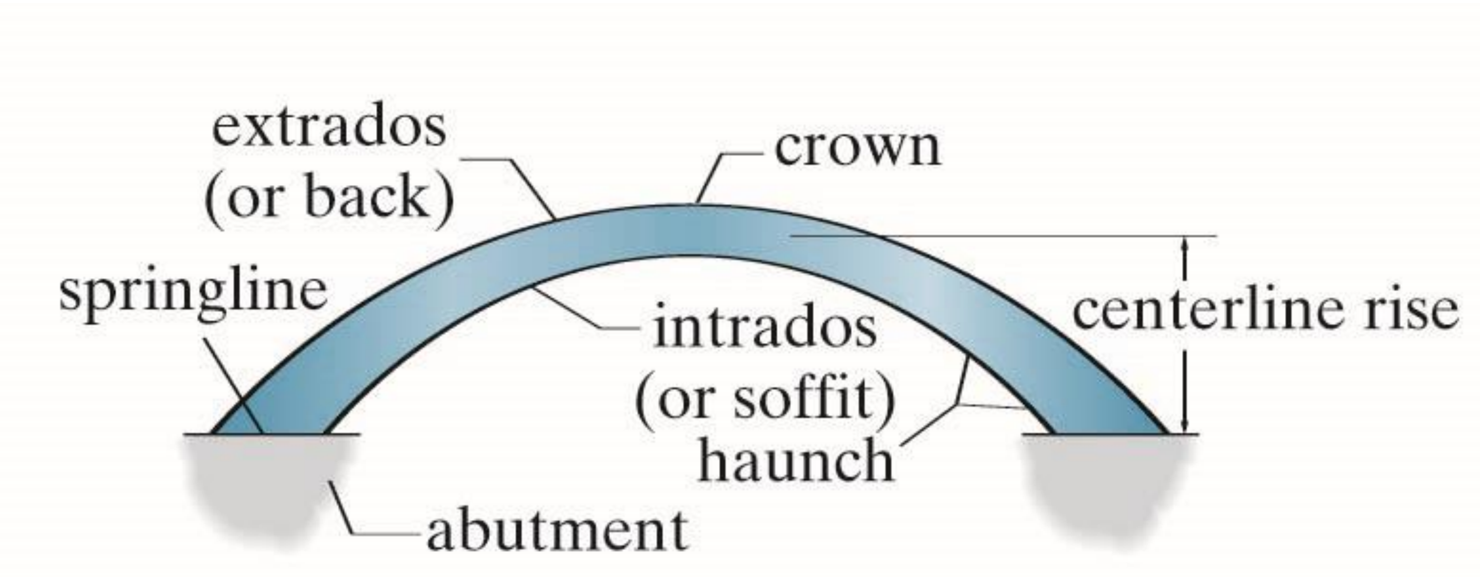
ARCHES



5.4

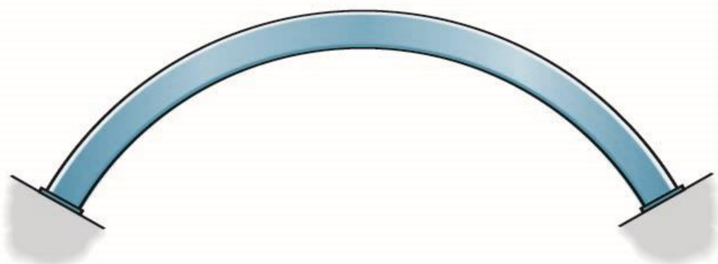
Arches

- An arch acts as inverted cable so it receives loading in compression
- Because of its rigidity, it must also resist some bending and shear depending upon how it is loaded & shaped

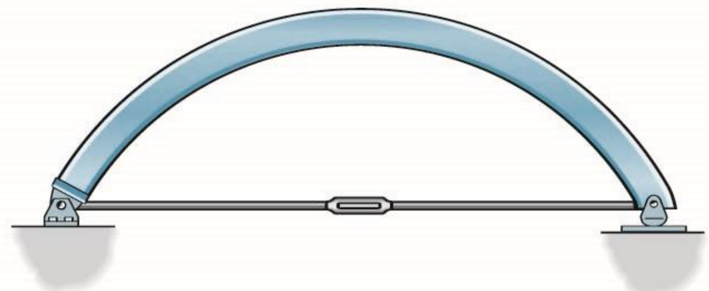


Arches

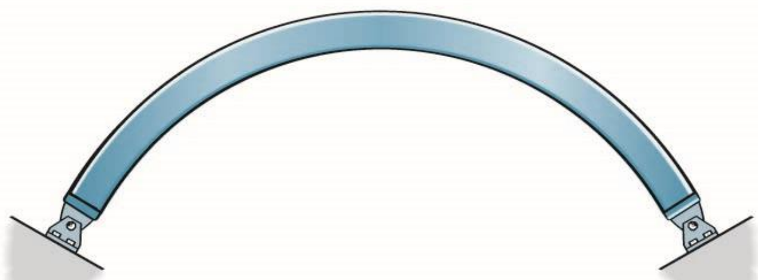
- Depending on its uses, several types of arches can be selected to support a loading



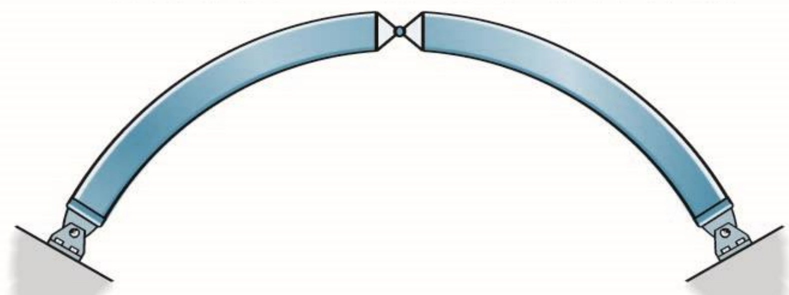
fixed arch



tied arch



two-hinged arch



three-hinged arch

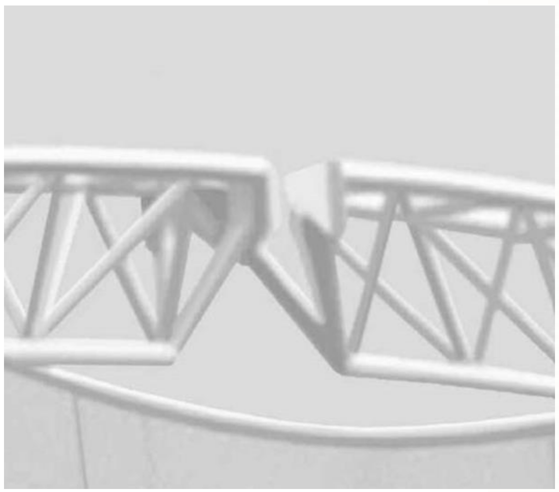
5.5

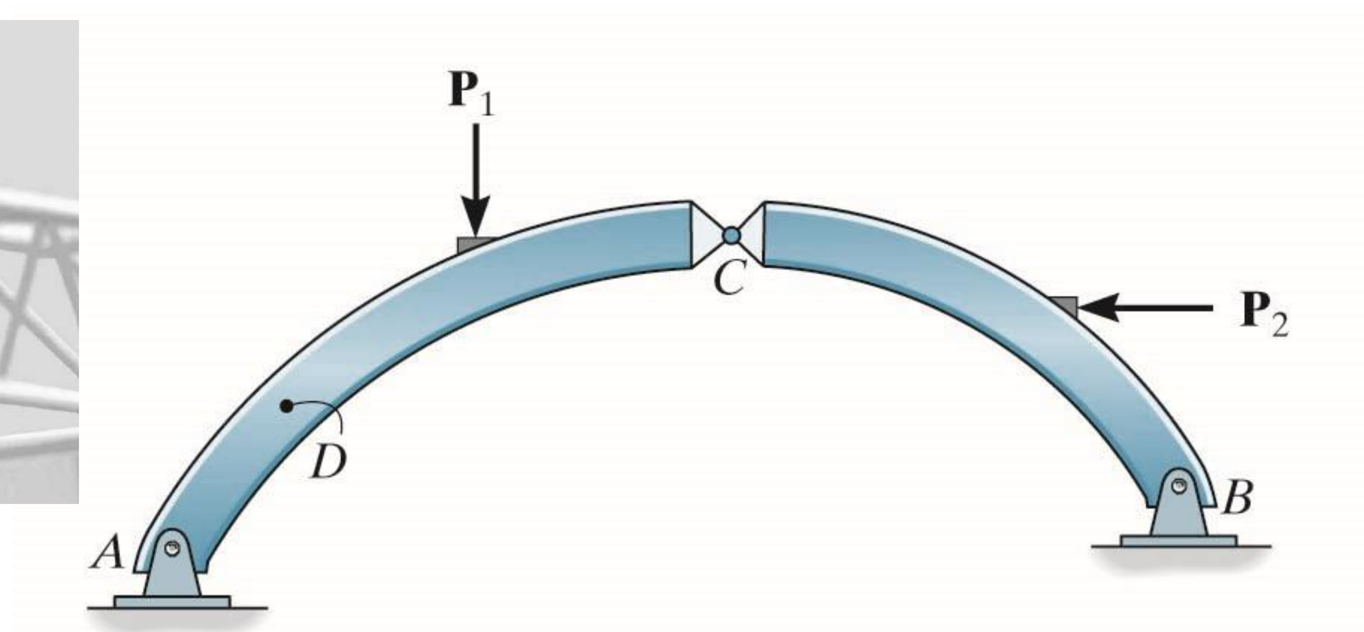
THREE-HINGED ARCH

5.5

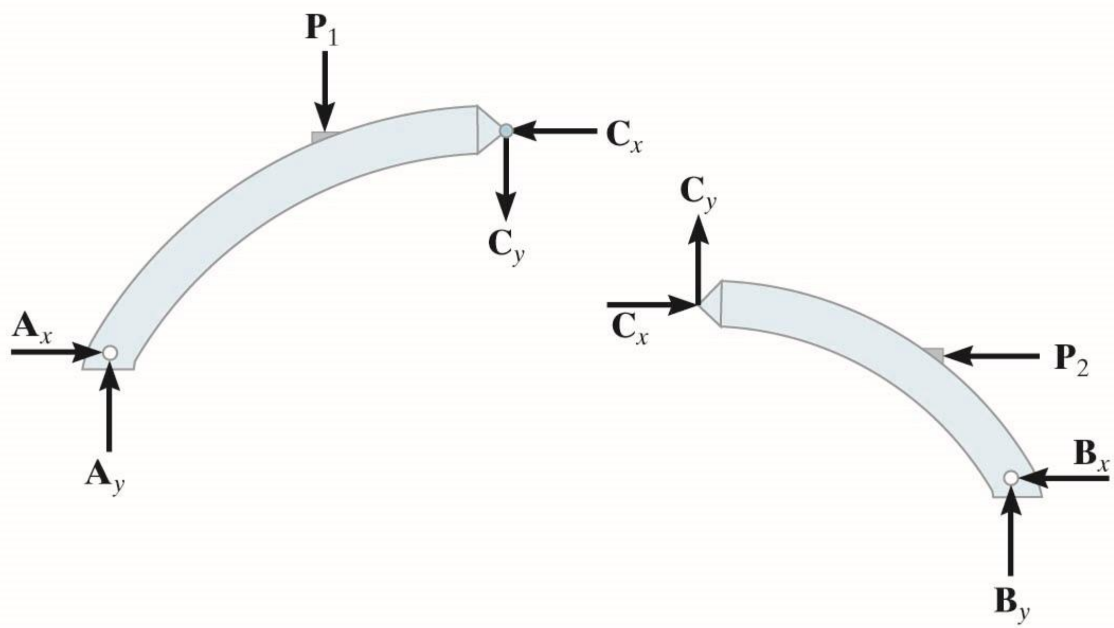
Three-Hinged Arch

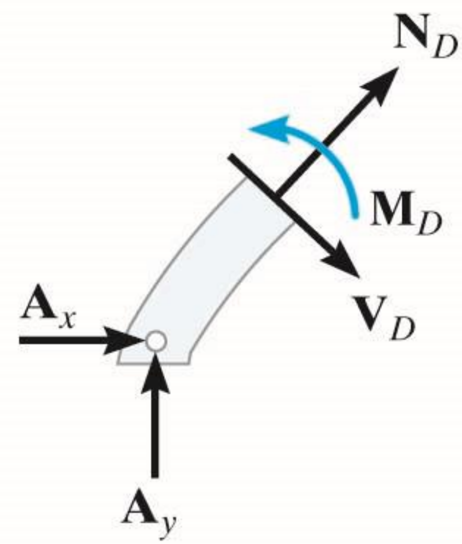
- The third hinge is located at the crown & the supports are located at different elevations
- To determine the reactions at the supports, the arch is disassembled





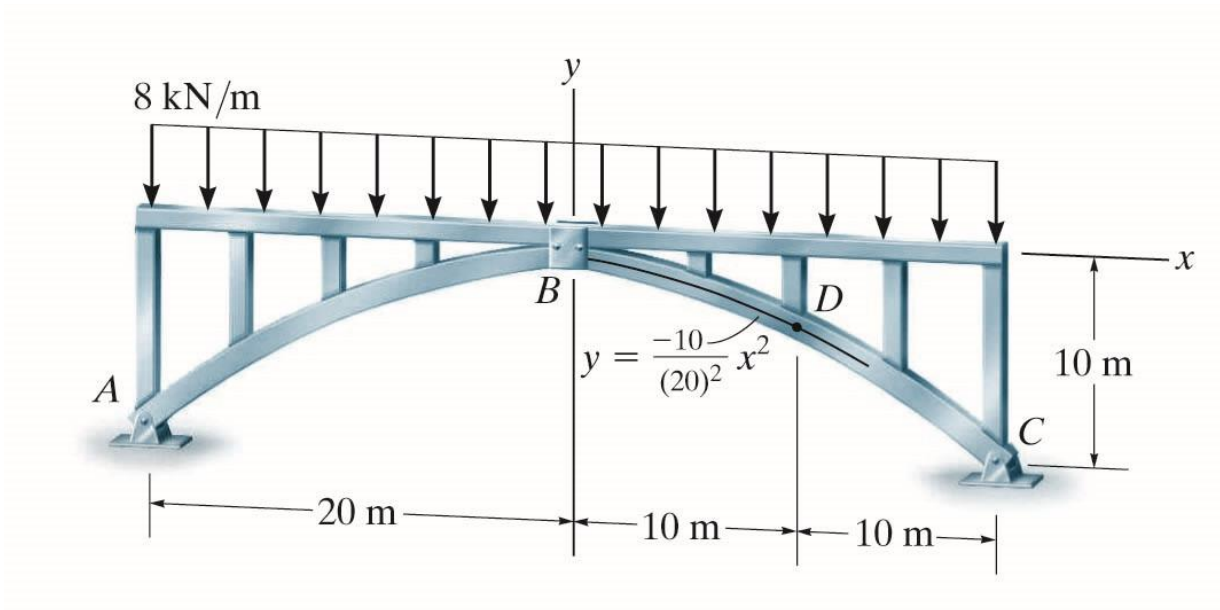
Three-Hinged Arch





Three-Hinged Arch

Example 5.4
The three-hinged open-spandrel arch bridge has a parabolic shape and supports a uniform load. Show that the parabolic arch is subjected *only to axial compression* at an intermediate point such as point *D*. Assume the load is uniformly transmitted to the arch ribs.



Three-Hinged Arch

Example 5.4 (Solution)

Applying the eqns of equilibrium, we have:

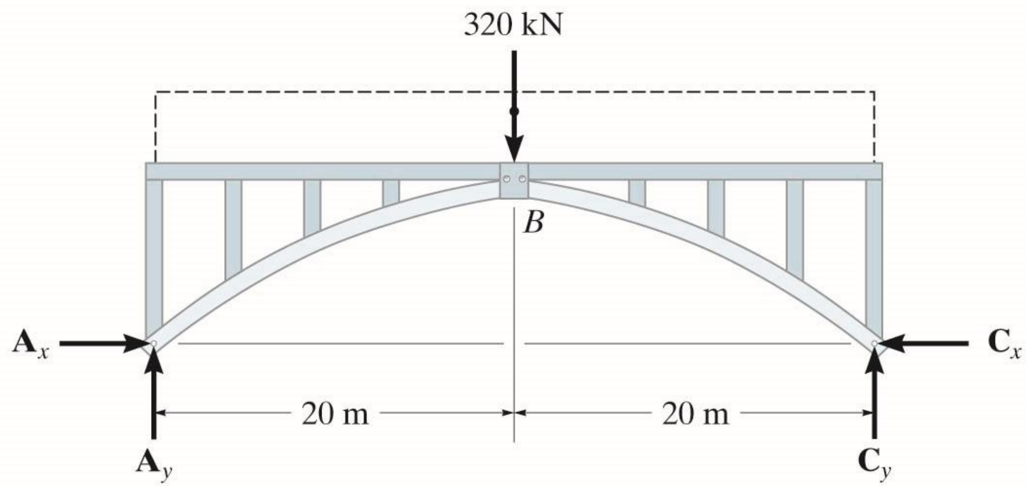
Entire Arch :

With anti - clockwise direction moments as + ve,

$$\sum M_A = 0$$

$$C_y(40\,\text{m}) - 320\,\text{kN}(20\,\text{m}) = 0$$

$$C_y = 160\,\text{kN}$$



Three-Hinged Arch

Example 5.4 (Solution)

Arch segment BC :

With anti - clockwise direction moments as + ve,

$$\sum M_B = 0$$

$$-160\text{ kN}(10\text{ m}) + 160\text{ kN}(20\text{ m}) - C_x(10\text{ m}) = 0$$

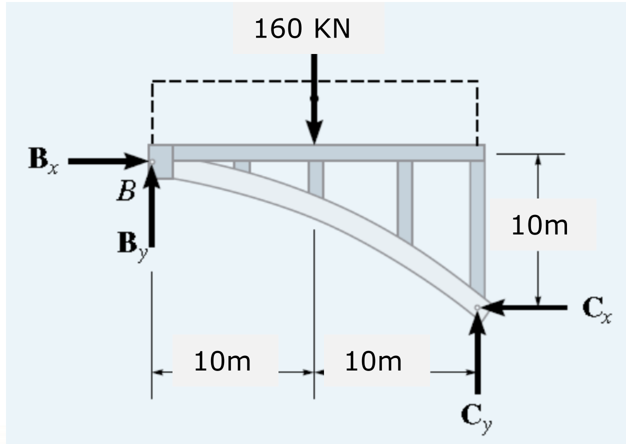
$$C_x = 160\text{ kN}$$

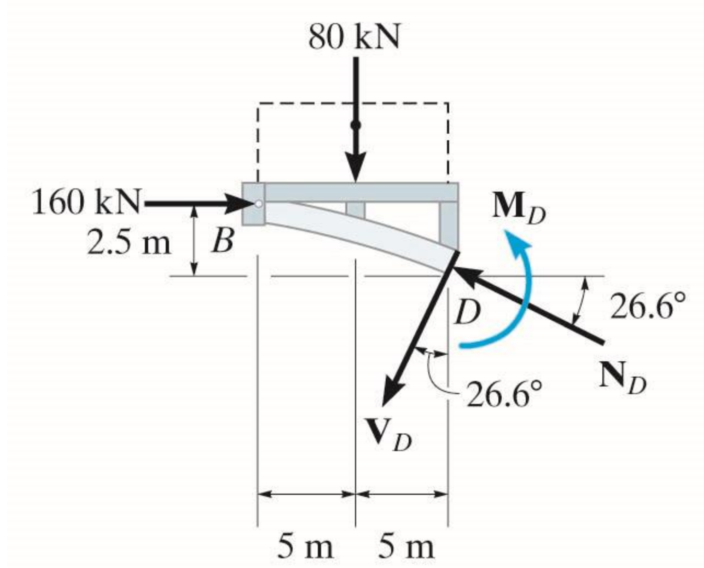
$$\underset{\rightarrow}{+}\sum F_x = 0 \Rightarrow B_x = 160\text{ kN}$$

$$+\uparrow \sum F_y = 0$$

$$B_y - 160\text{ kN} + 160\text{ kN} = 0$$

$$B_y = 0$$





Three-Hinged Arch

Example 5.4 (Solution)

A section of the arch taken through point *D*

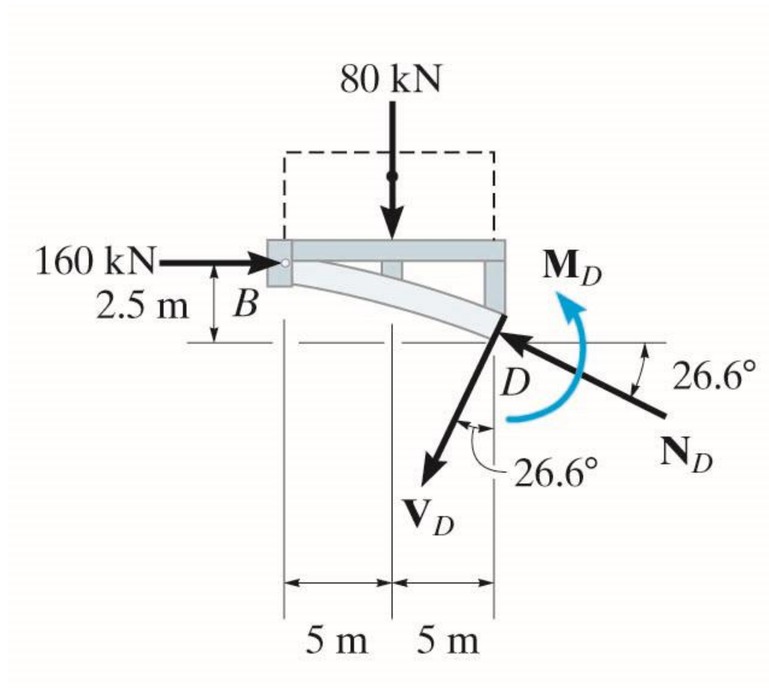
$$x = 10\,\text{m}$$

$$y = -10(10)^2/(20)^2 = -2.5\,\text{m}$$

The slope of the segment at D is:

$$\tan\theta = \frac{dy}{dx} = \frac{-20}{(20)^2}x\bigg|_{x=10\,\text{m}} = -0.5$$

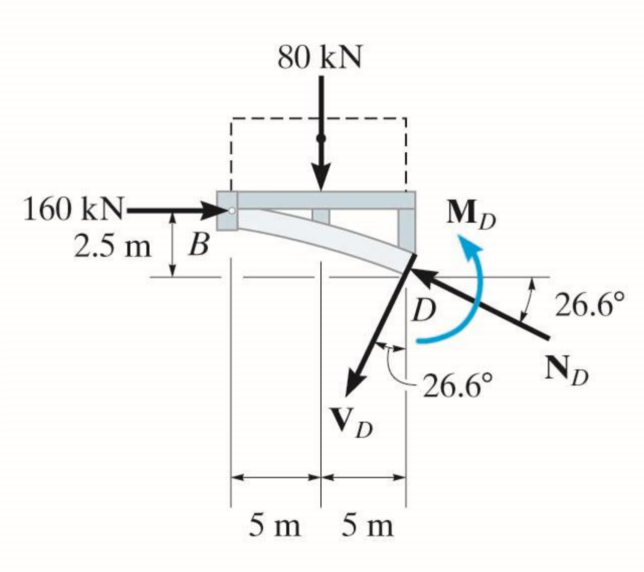
$$\theta = 26.6^\circ$$



Three-Hinged Arch

Example 5.4 (Solution)

Applying the eqn of equlibrium, Fig 5.10(d), we have:



$$\overset{+}{\rightarrow}\sum F_x = 0$$

$$160\text{ kN} - N_D \cos 26.6^o - V_D \sin 26.6^o = 0$$

$$+\uparrow \sum F_y = 0$$

$$-80\text{ kN} + N_D \sin 26.6^o - V_D \cos 26.6^o = 0$$

$$\Rightarrow N_D = 178.9kN$$

With anti-clockwise moments as +ve:

$$\Rightarrow V_D = 0$$

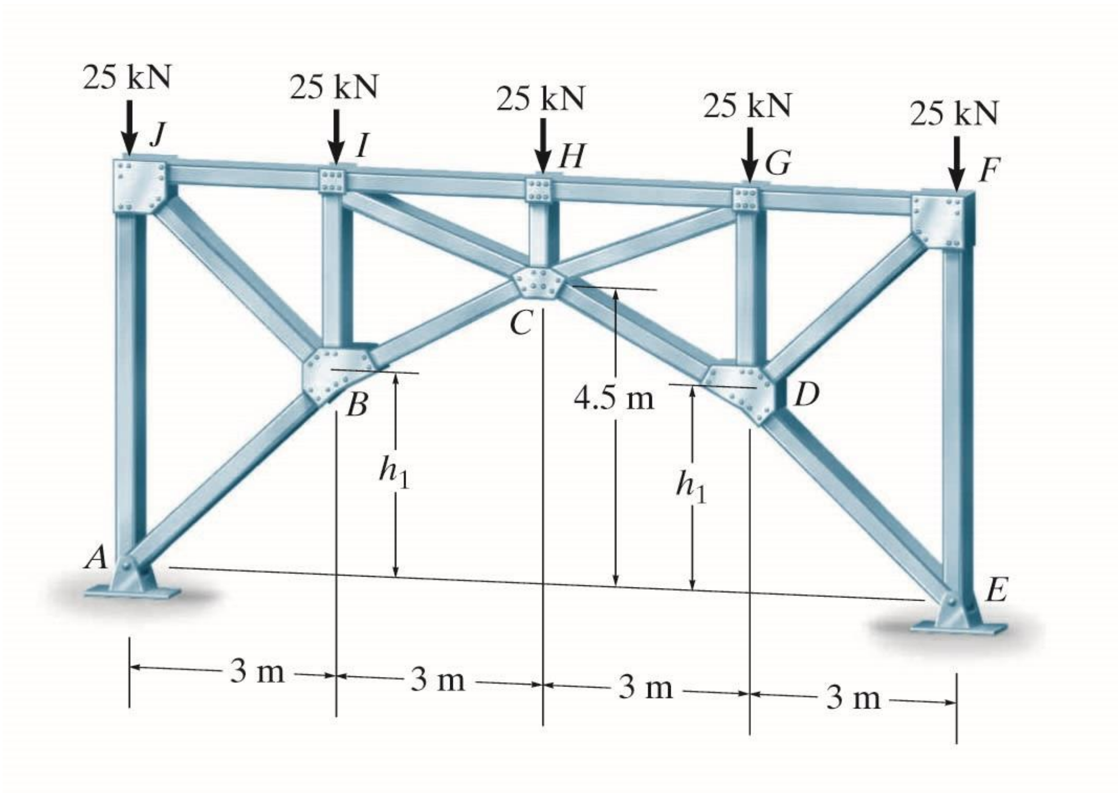
$$\sum M_D = 0$$

$$M_D + 80\text{ kN}(5\text{ m}) - 160\text{ kN}(2.5\text{ m}) = 0$$

$$\Rightarrow M_D = 0$$

Three-Hinged Arch

Example 5.6
The three-hinged trussed arch supports the symmetric loading. Determine the required height of the joints *B* and *D*, so that the arch takes a funicular shape. Member *HG* is intended to carry no force.

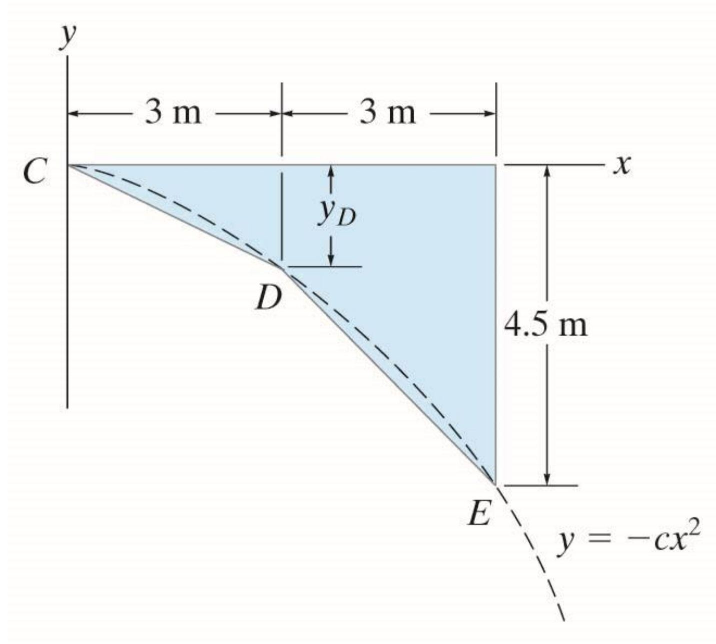


Three-Hinged Arch

Example 5.6 (Solution)

For a symmetric loading, the funicular shape for the arch must be parabolic as indicated by the dashed line. Here we must find the eqn which fits this shape.

With the *x*, *y* axes having an origin at *C*, the eqn is of the form of $y = -cx^2$. To obtain the constant *c*, we require:



$$-(4.5\,\text{m}) = -c(6\,\text{m})^2$$

$$c = 0.125/\text{m}$$

Therefore,

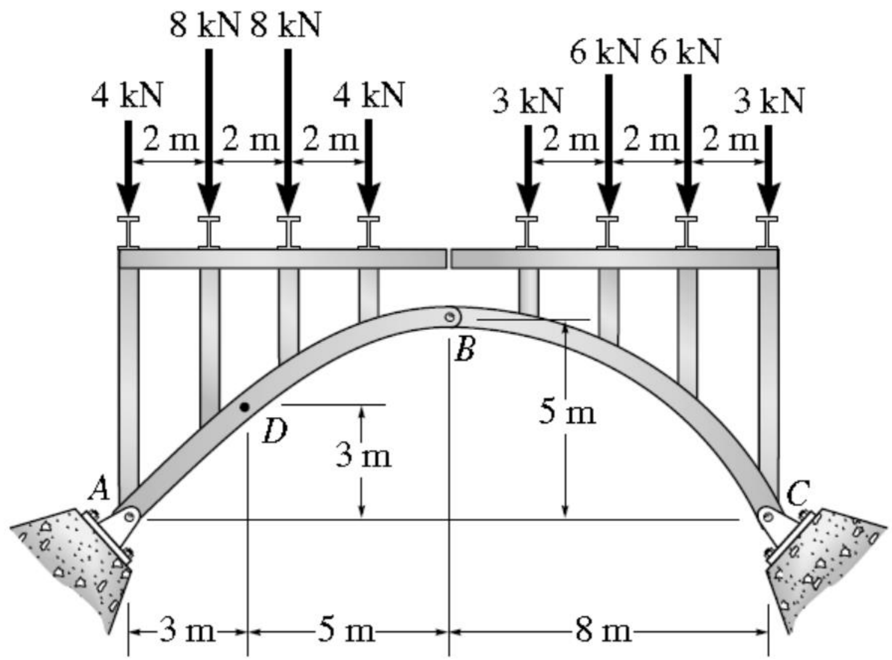
$$y_D = -(0.125/\text{m})(3\,\text{m})^2 = -1.125\,\text{m}$$

From Fig 5.12(a)

$$h_1 = 4.5\,\text{m} - 1.125\,\text{m} = 3.375\,\text{m}$$

HW 5-3

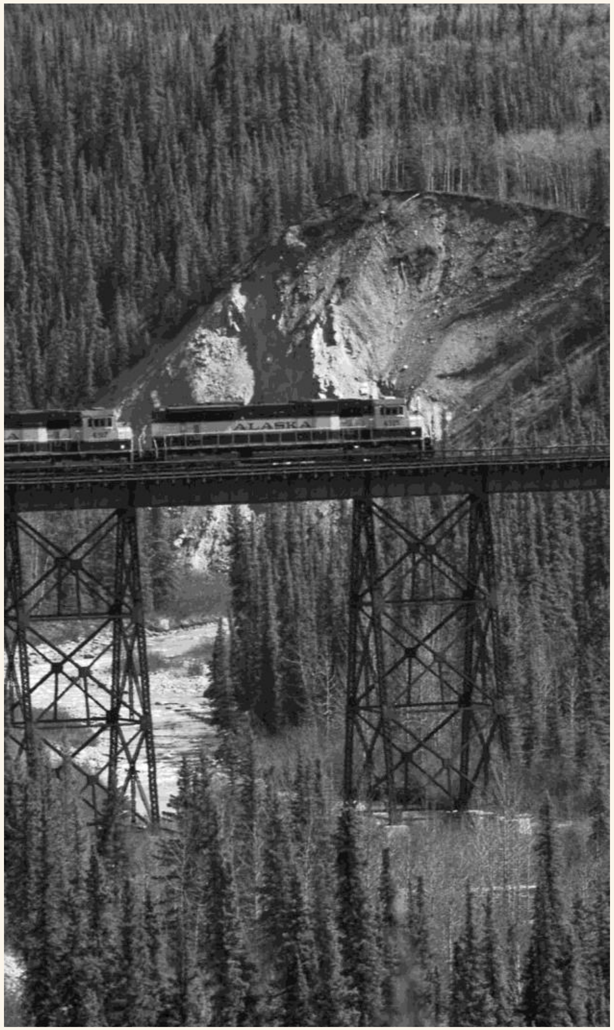
The three-hinged spandrel arch is subjected to the loading shown. Determine the internal moment in the arch at point *D*.



Ans.

M_D = 10.8 KN.m

CHAPTER 6:
INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES



6

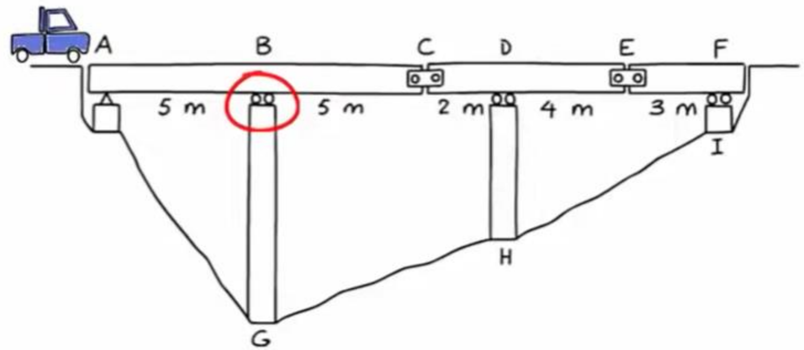
Chapter Outline

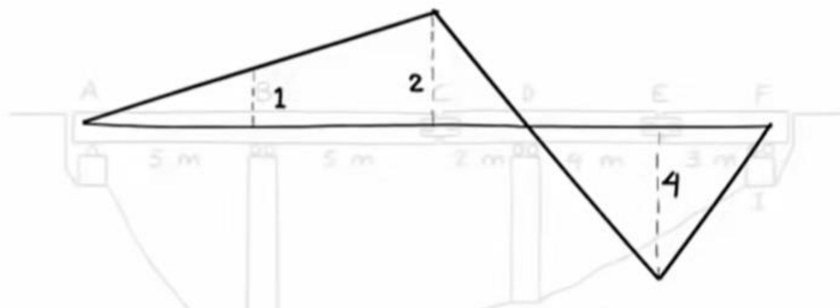
6.1
INFLUENCE LINES

6.1

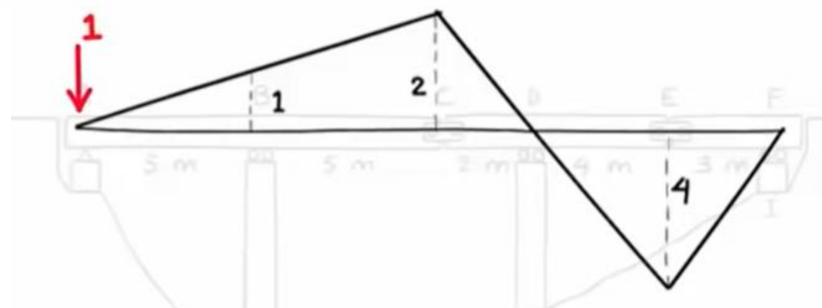
Influence Lines

- If a structure is subjected to a moving load, the variation of shear & bending moment is best described using the influence line
- One can tell at a glance, where the moving load should be placed on the structure so that it creates the greatest influence at a specified point
- The magnitude of the associated shear, moment or deflection at the point can then be calculated using the ordinates of the influence-line diagram

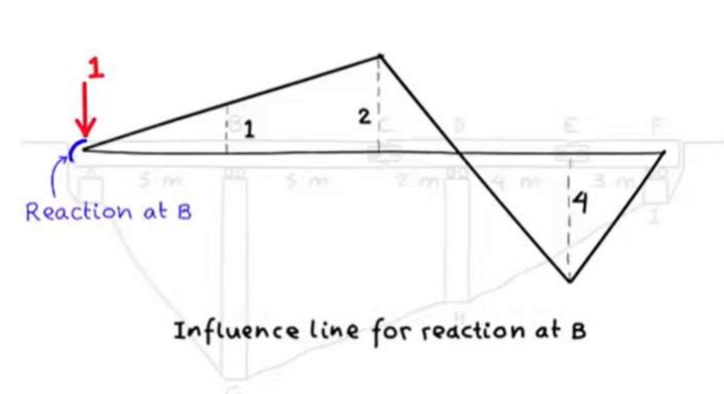


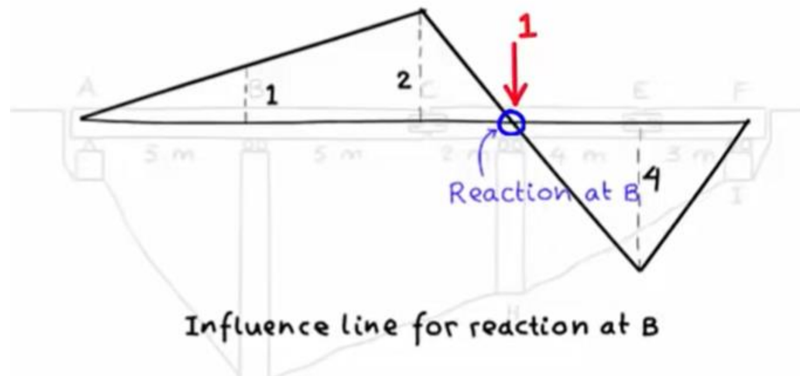


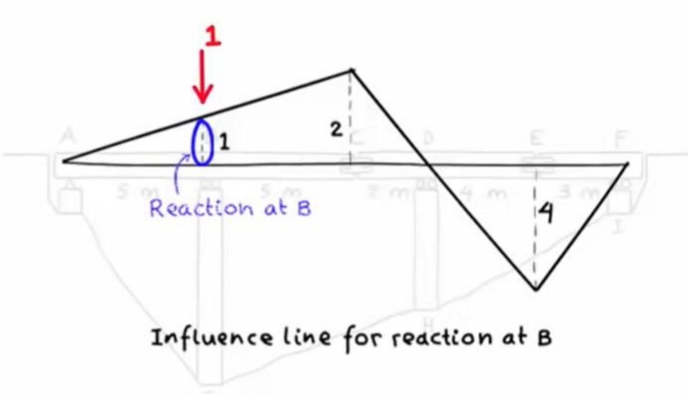
Influence line for reaction at B

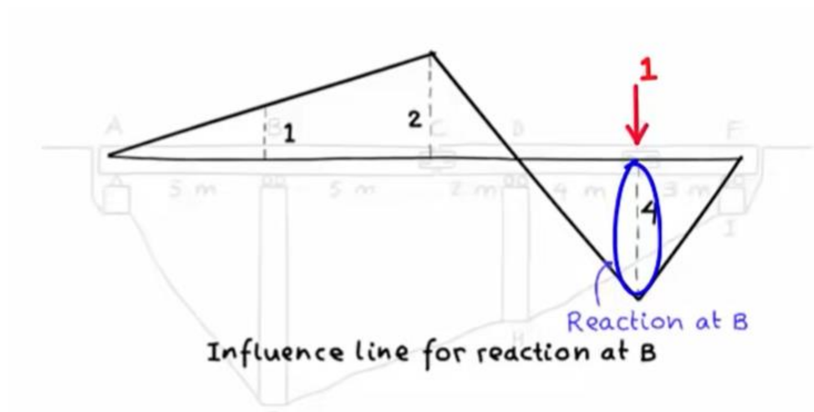


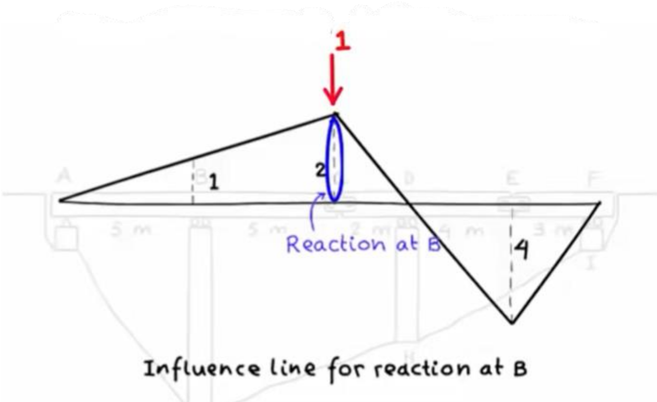
Influence line for reaction at B

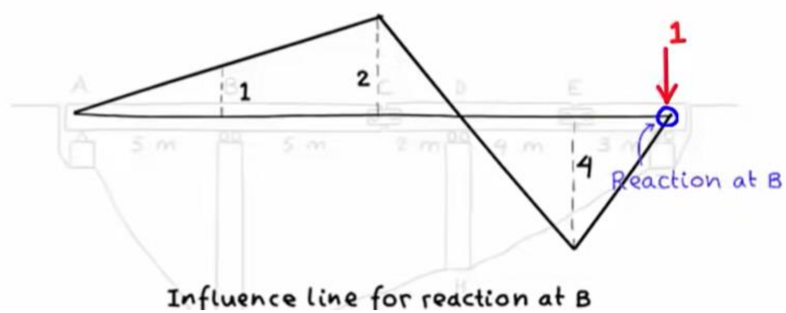

1
1
2
4
Reaction at B
Influence line for reaction at B

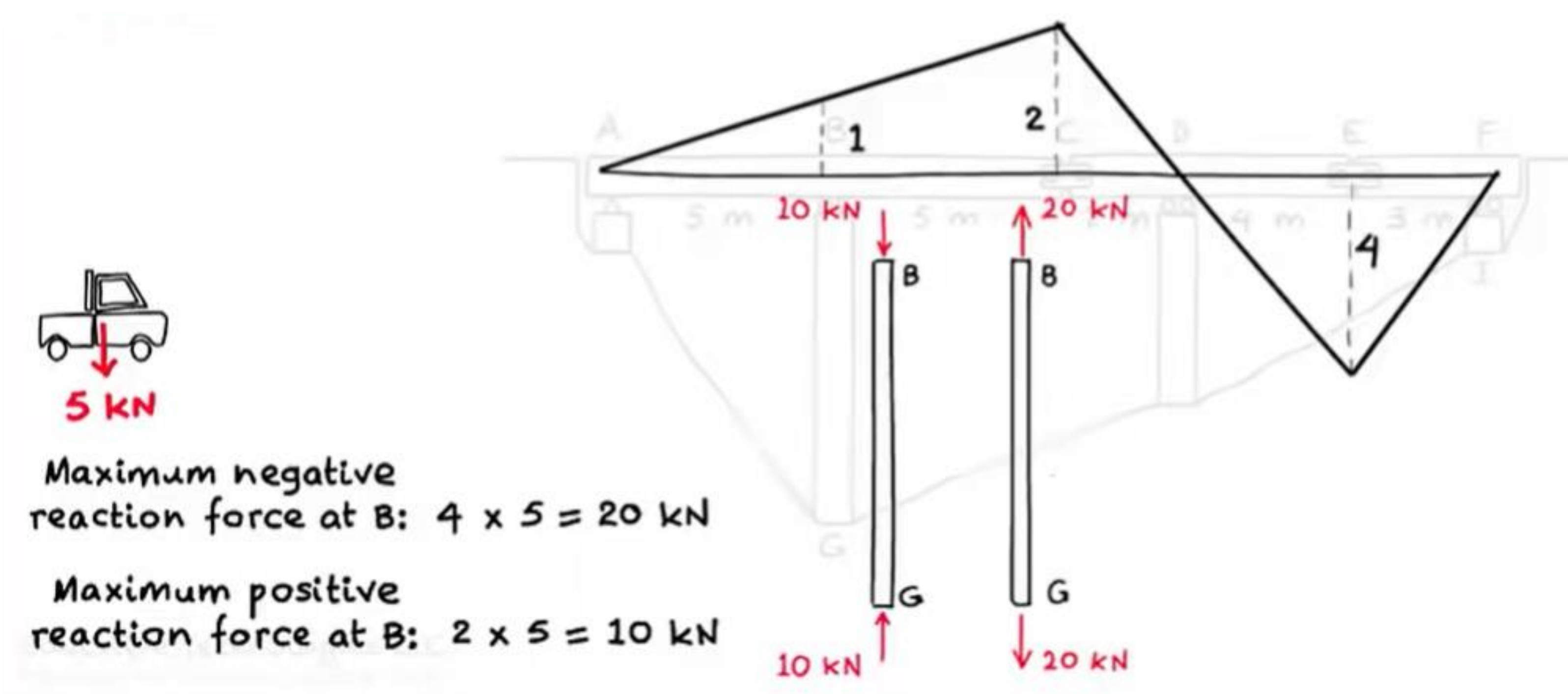

1
1
2
4
Reaction at B
Influence line for reaction at B


1
1
2
4
Reaction at B
Influence line for reaction at B


1
1
2
4
Reaction at B
Influence line for reaction at B


1
1
2
4
Reaction at B
Influence line for reaction at B


1
1
2
4
Reaction at B
Influence line for reaction at B



1
2
4
10 kN
20 kN
B
B
G
G
10 kN
20 kN
5 kN
Maximum negative
reaction force at B: 4 x 5 = 20 kN
Maximum positive
reaction force at B: 2 x 5 = 10 kN

Influence Lines

- One should be clear of the difference between Influence Lines & shear or moment diagram

- Influence line represent the effect of a moving load only at a specific point

- Shear or moment diagrams represent the effect of fixed loads at all points along the axis of the member

- Procedure for Analysis
  - Tabulate Values
  - Influence-Line equations

Influence Lines

- Tabulate Values
  - Place a unit load at various locations, x, along the member
  - At each location use statics to determine the value of function at the specified point
  - If the influence line for a vertical force reaction at a point on a beam is to be constructed, consider the reaction to be +ve at the point when it acts upward on the beam
  - If a shear or moment influence line is to be drawn for a point, take the shear or moment at the point as +ve according to the same sign convention used for drawing shear & moment diagram
  - All statically determinate beams will have influence lines that consist of straight line segments

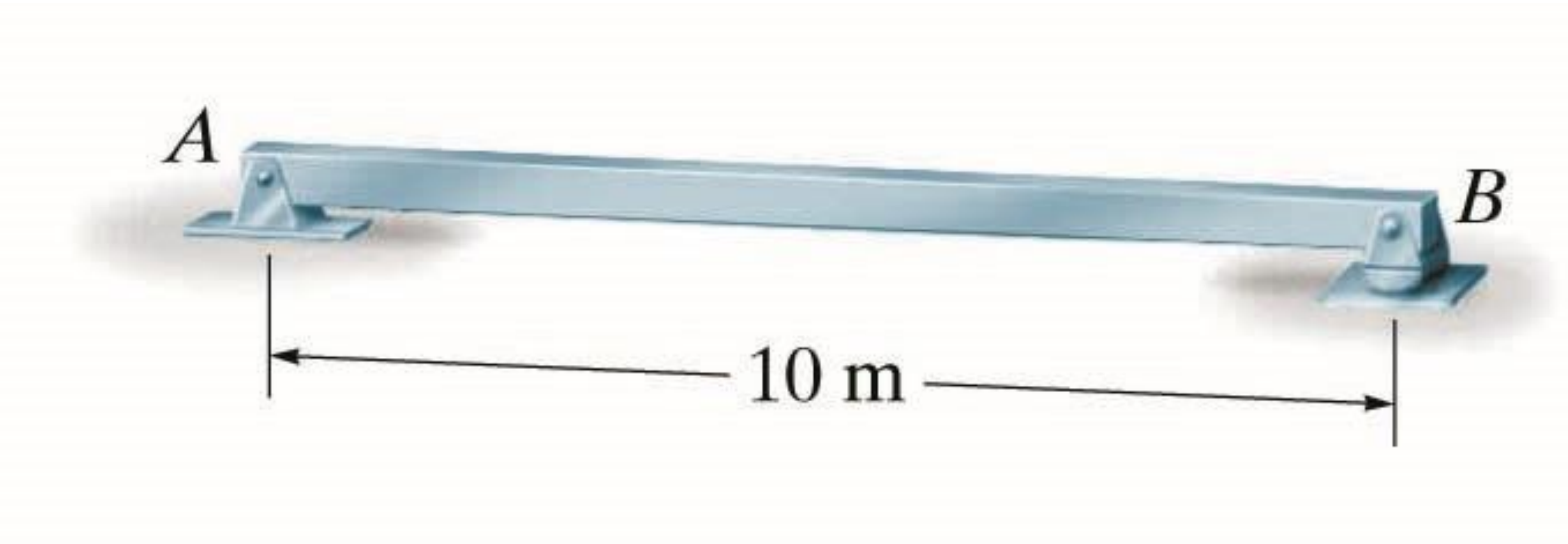
Influence Lines

- Influence-Line Eqns
  - The influence line can also be constructed by placing the unit load at a variable position, *x*, on the member & then computing the value of *R*, *V* or *M* at the point as a function of *x*
  - The eqns of the various line segments composing the influence line can be determined & plotted

Influence Lines

Example 6.1

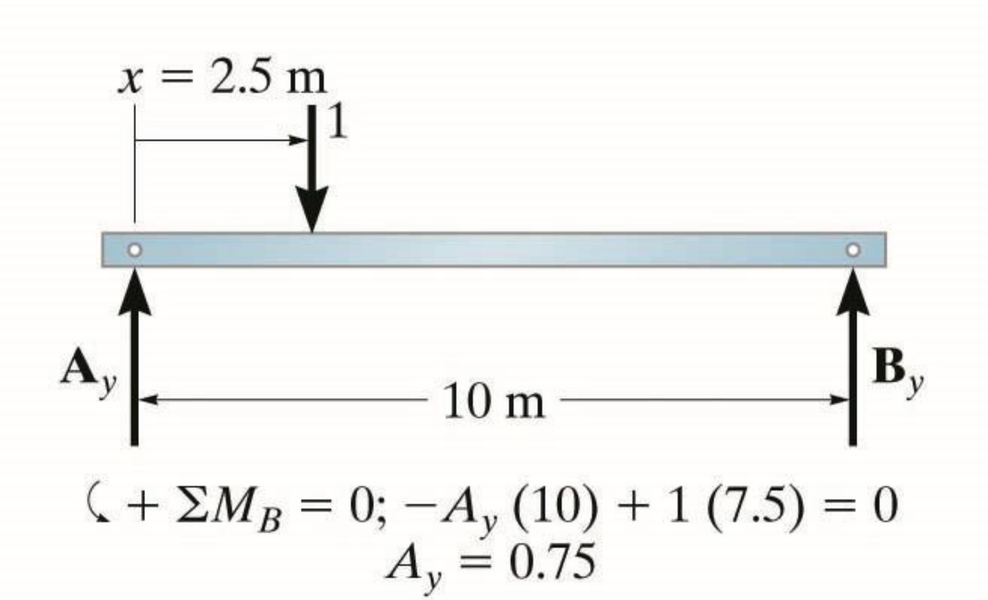
Construct the influence line for the vertical reaction at A of the beam.

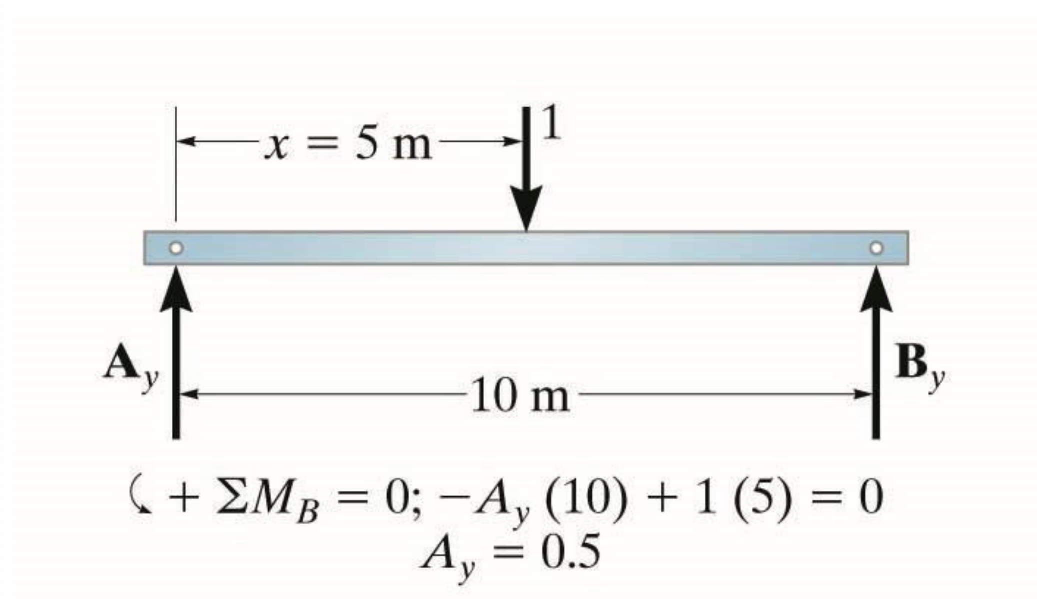


Influence Lines

Example 6.1 (Solution)

Tabulate Values
A unit load is placed on the beam at each selected point *x* & the value of A_y is calculated by summing moments about *B*.

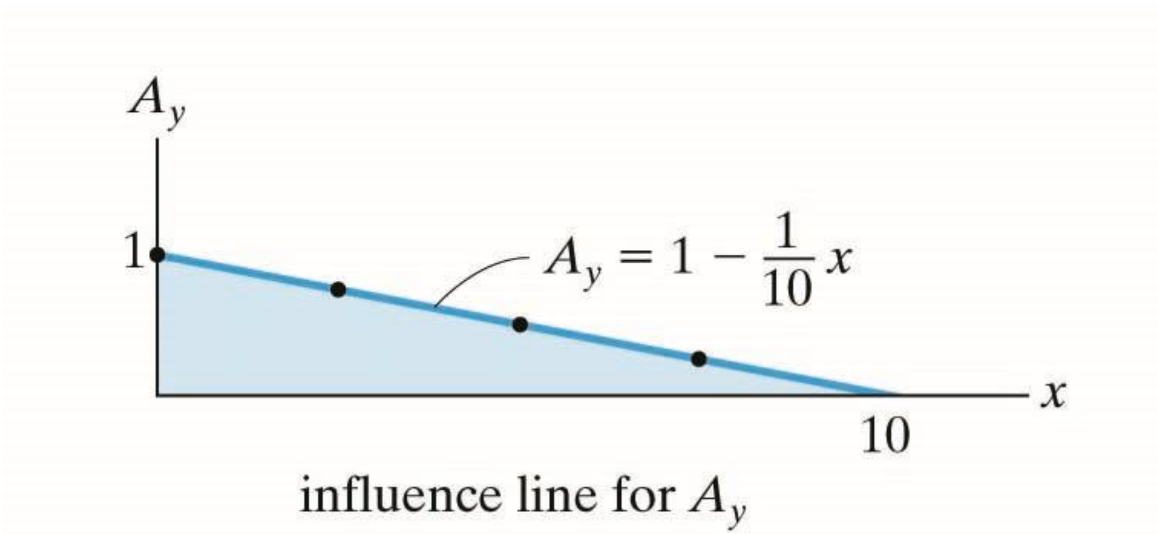




Influence Lines

Example 6.1 (Solution)

Tabulate Values



x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0

Influence Lines

Example 6.1 (Solution)

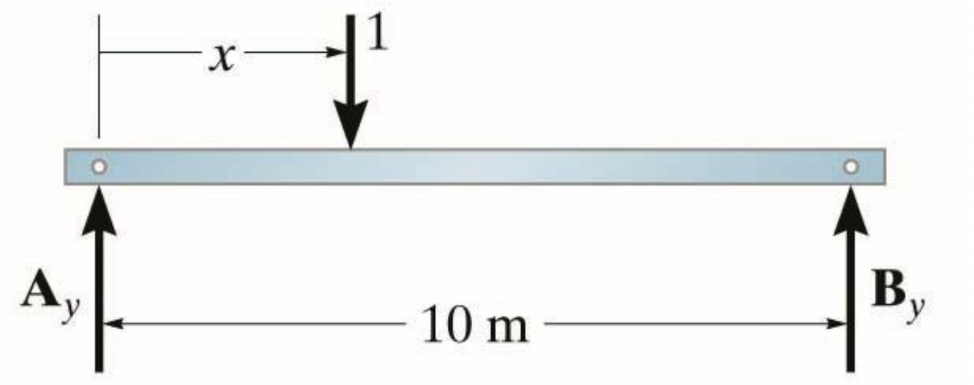
Influence-Line Equation

The reaction as a function of x can be determined from

$$\Sigma M_B = 0$$

$$-A_y(10) + (10 - x)(1) = 0$$

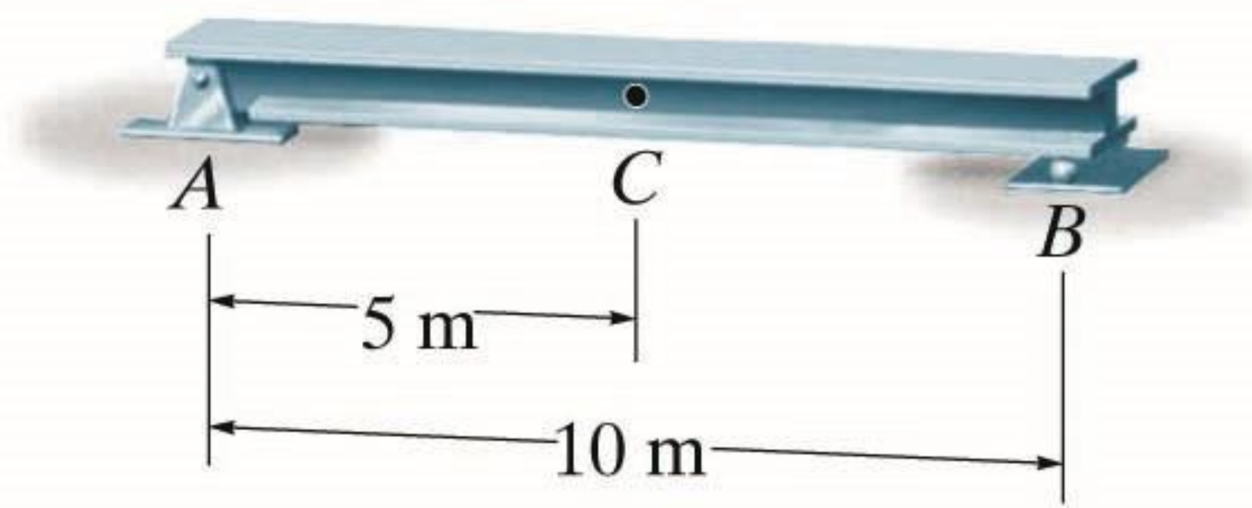
$$A_y = 1 - \frac{1}{10}x$$



Influence Lines

Example 6.5

Construct the influence line for the moment at *C* of the beam.

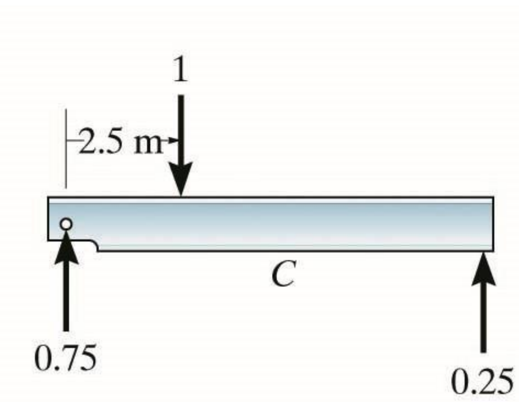


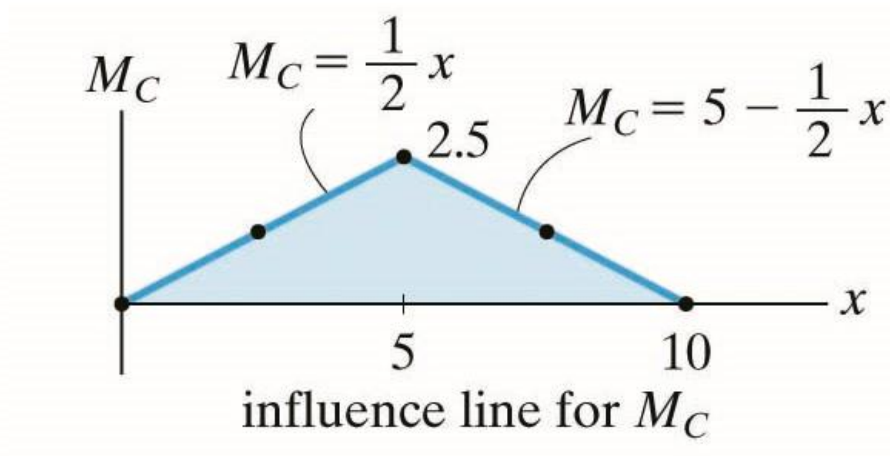
Influence Lines

Example 6.5 (Solution)

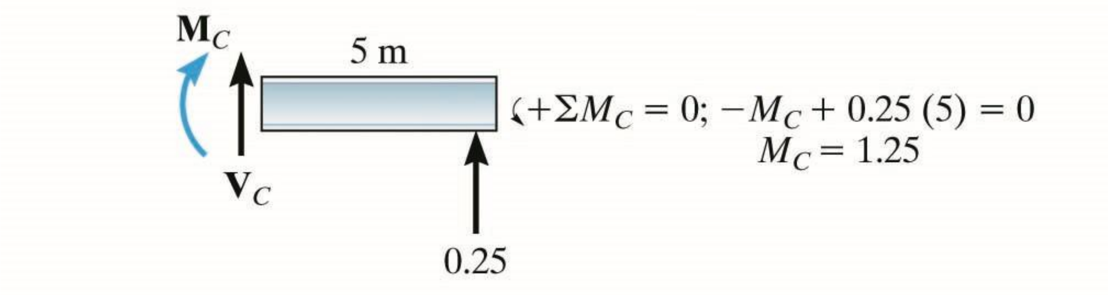
Tabulate Values

At each selected position of the unit load, the value of M_C is calculated using the method of sections.





x	M_C
0	0
2.5	1.25
5	2.5
7.5	1.25
10	0



Influence Lines

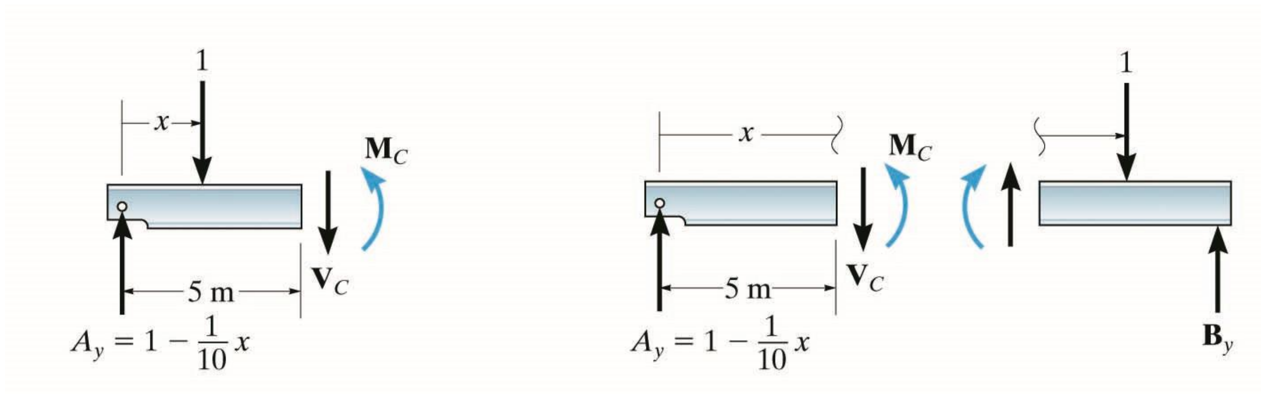
Example 6.5 (Solution)

Influence-Line Equations

$$\Sigma M_C = 0$$

$$M_C + 1(5 - x) - (1 - \frac{1}{10}x)5 = 0$$

$$M_C = \frac{1}{2}x \quad \text{for } 0 \le x < 5\,\text{m}$$



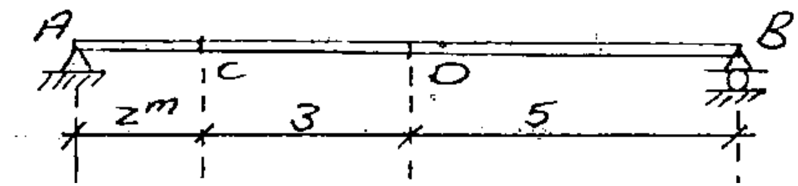
$$\Sigma M_C = 0$$

$$M_C - (1 - \frac{1}{10}x)5 = 0$$

$$M_C = 5 - \frac{1}{2}x \quad \text{for } 5\,\text{m} < x \le 10\,\text{m}$$

HW 6-1

Draw I.L for R_A, R_B, V_C, M_C, V_D, M_D



6.2
INFLUENCE LINES FOR BEAMS

6.2

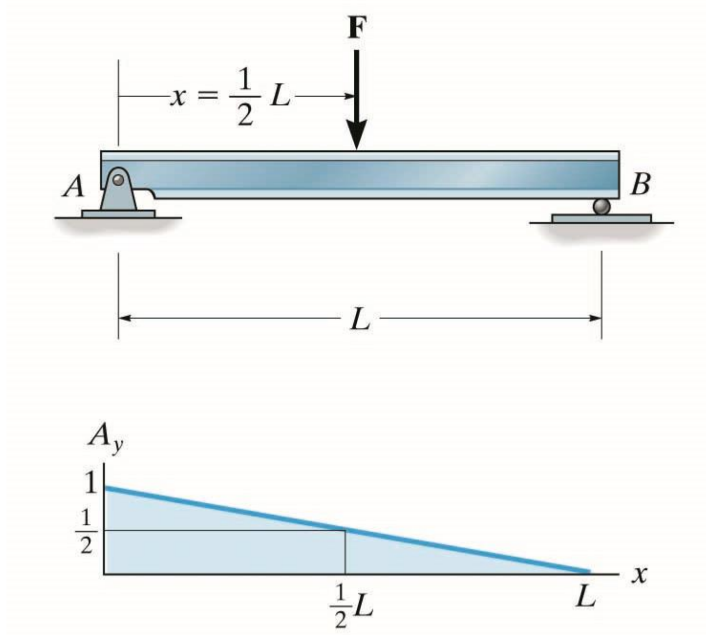
Influence Lines for Beams

- Once the influence line for a function has been constructed, it will be possible to position live loads on the beam which will produce the max value of the function

- 2 types of loadings will be considered:
  - Concentrated force
  - Uniform load

Influence Lines for Beams

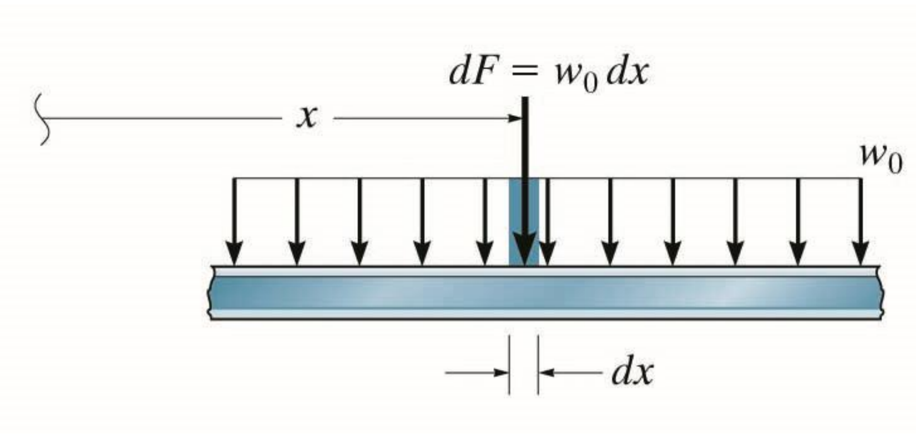
- Concentrated force
  - For any concentrated force, **F** acting on the beam, the value of the function can be found by multiplying the ordinate of the influence line at position *x* by magnitude of **F**
  - Consider Fig 6.7, influence line for A_y
  - For unit load, $A_y = ½$
  - For a force of F, $A_y = (½) F$

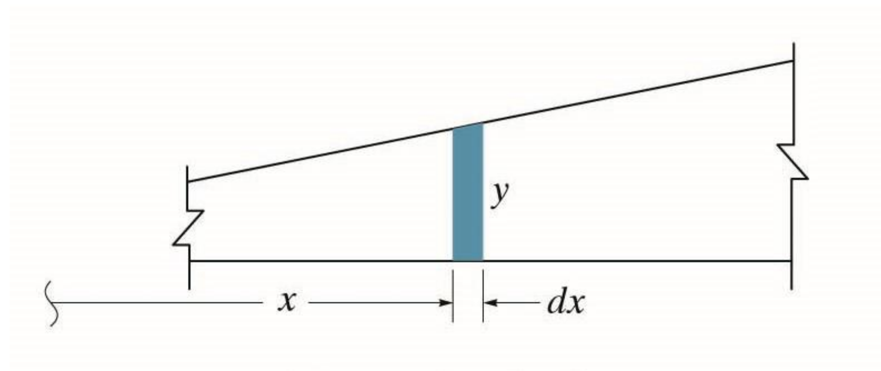


influence line for A_y

Influence Lines for Beams

- Uniform load
  - Each *dx* segment of this load creates a concentrated force of $dF = w_0 dx$
  - If $d\mathbf{F}$ is located at x, where the influence-line ordinate is y, the value of the function is $(dF)(y) = (w_0 dx)y$
  - The effect of all concentrated forces is determined by integrating over the entire length of the beam





influence line for function

Influence Lines for Beams

- Uniform load

$$\int w_o y dx = w_o \int y dx$$

- Since $\int y dx$ is equivalent to the area under the influence line, in general:

- value of the function caused by a uniform load = the area under the influence line *x* intensity of the uniform load

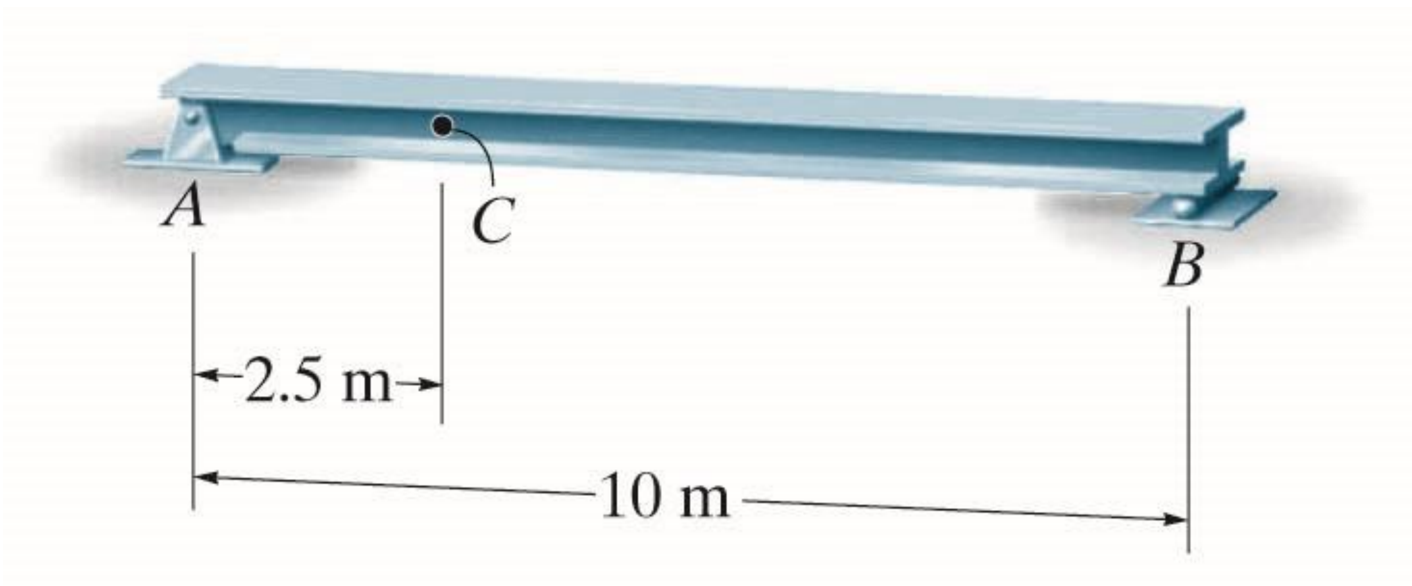
Influence Lines for Beams

Example 6.7

Determine the max +ve shear that can be developed at point *C* in the beam due to:

➔A concentrated moving load of 4 kN, and

➔A uniform moving load of 2 kN/m



Influence Lines for Beams

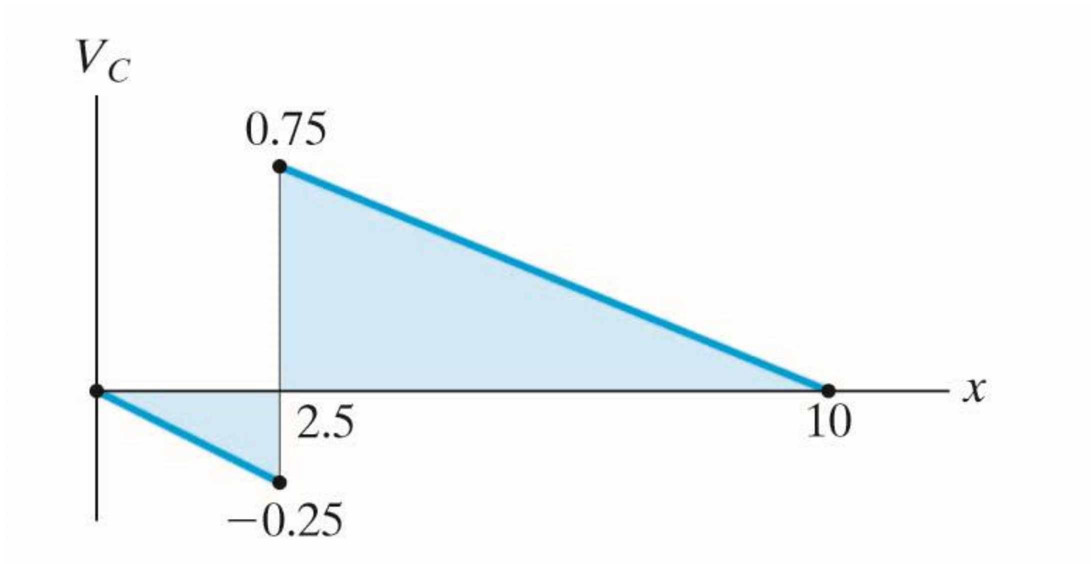
Example 6.7 (Solution)

Concentrated force

The max +ve positive shear at *C* will occur when the 4 kN force is located at *x* = 2.5 m.

The ordinate at this peak is +0.75, hence:

$$V_C = 0.75(4kN) = 3kN$$



Influence Lines for Beams

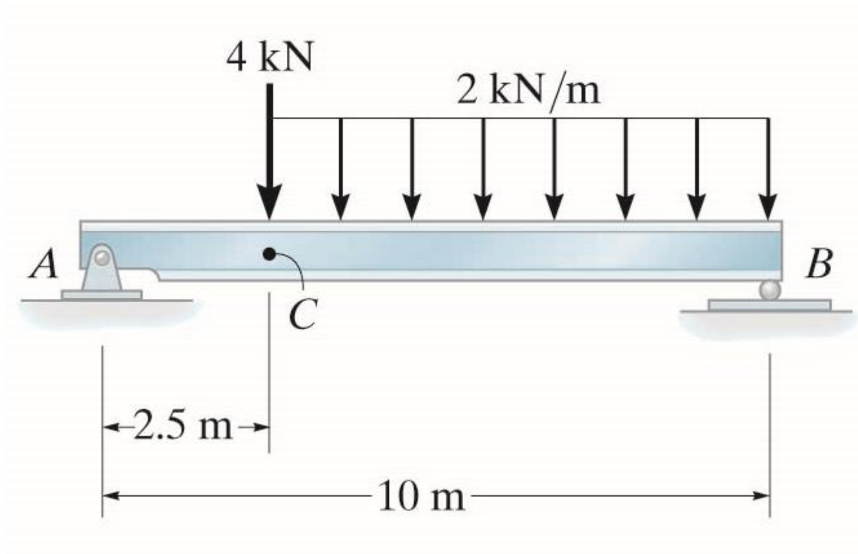
Example 6.7 (Solution)

Uniform load

The uniform moving load creates the max +ve influence for V_C when the load acts on the beam between x = 2.5 m and x = 10 m

The magnitude of $\mathbf{V}_C$ due to this loading is:

$$V_C = \left[\frac{1}{2}(10\,\text{m} - 2.5\,\text{m})(0.75)\right](2\,\text{kN/m})$$
$$= 5.625\,\text{kN}$$

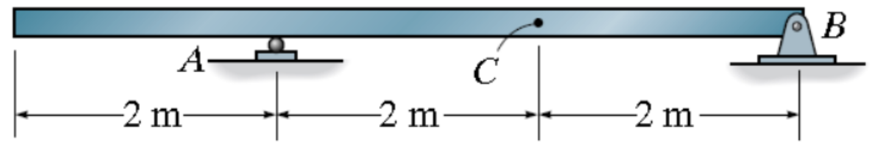


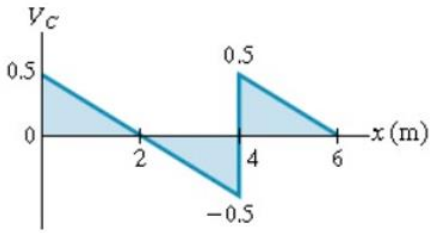
Total max shear at C:

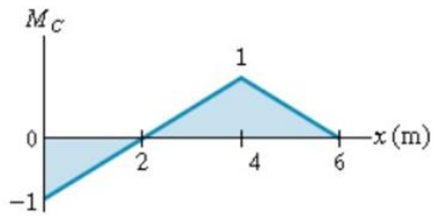
$$(V_C)_{\max} = 3\,\text{kN} + 5.625\,\text{kN} = 8.625\,\text{kN}$$

Example

The beam supports a distributed live load of 1.5 kN/m and single concentrated load of 8 kN. The dead load is 2 kN/m. Determine (a) the maximum positive moment at *C*, (b) the maximum positive shear at *C*.







$$(M_C)_{max(+)} = 8(1) + \left[\frac{1}{2}(6-2)(1)\right](1.5) + \left[\frac{1}{2}(2)(-1)\right](2) + \left[\frac{1}{2}(6-2)(1)\right](2)$$

$$= 13.0\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

$$(V_C)_{max(+)} = 8(0.5) + \left[\frac{1}{2}(2)(0.5)\right](1.5) + \left[\frac{1}{2}(6-4)(0.5)\right](1.5) + \left[\frac{1}{2}(2)(0.5)\right](2) + \left[\frac{1}{2}(4-2)(-0.5)\right](2) + \left[\frac{1}{2}(6-4)(0.5)\right](2)$$

$$= 6.50\ \text{kN} \qquad \textit{Ans.}$$

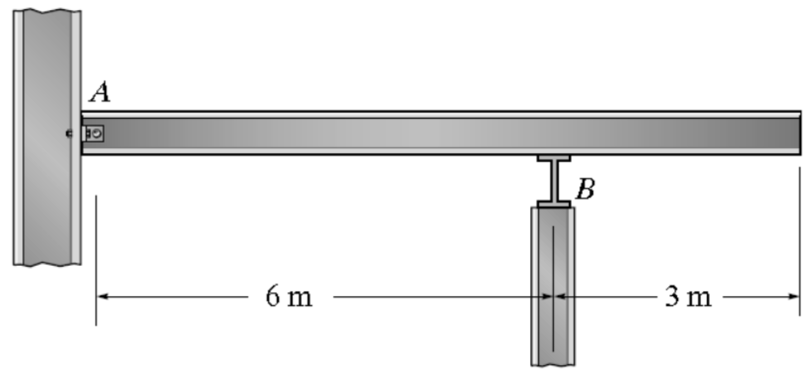
HW 6-2

A uniform live load of 4 kN/m and a single live concentrated force of 6 kN are to be placed on the beam. The beam has a weight of 2 kN/m. Determine

(a) the maximum vertical reaction at support B,

(b) The maximum negative moment at point B.

Assume the support at A is a pin and B is a roller.



Ans.
By +max=49.5 KN
MB -max=-45 KN.m

6.3

QUALITATIVE INFLUENCE LINES

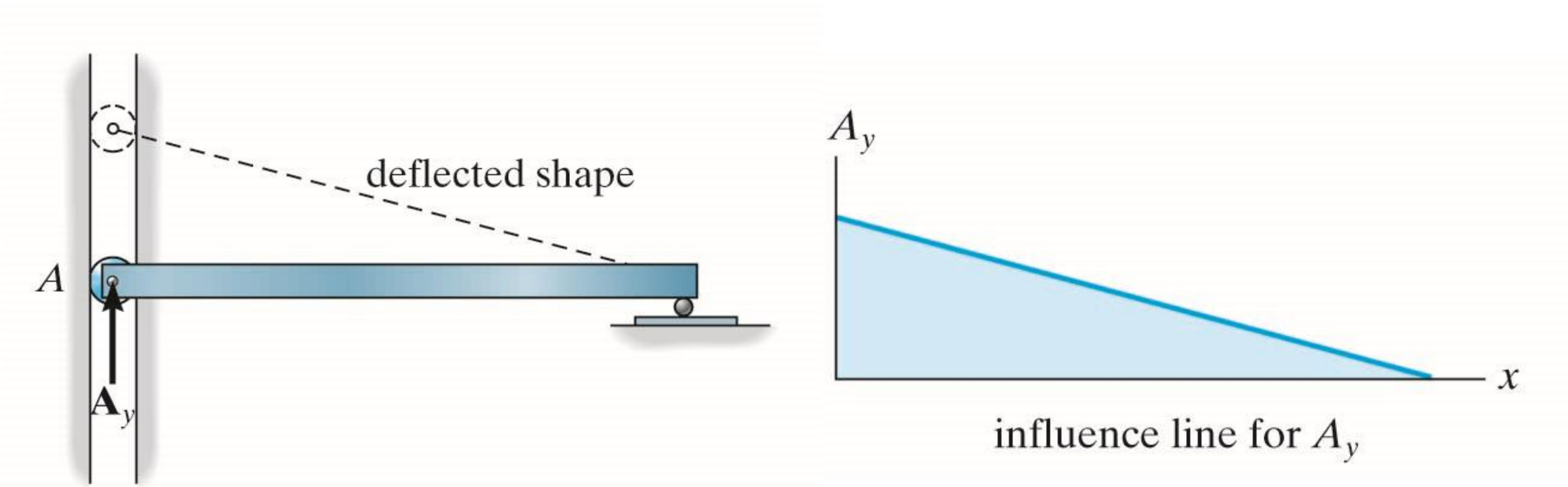
6.3

Qualitative Influence Lines

- The Müller-Breslau Principle states that the influence line for a function is to the same scale as the deflected shape of the beam when the beam is acted upon by the function
- If the shape of the influence line for the vertical reaction at *A* is to be determined, the pin is first replaced by a roller guide

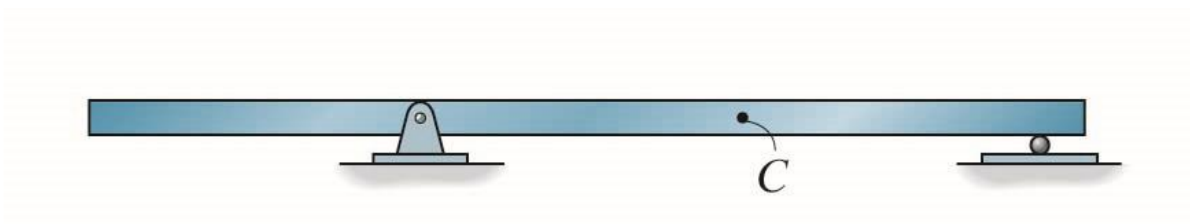
Qualitative Influence Lines

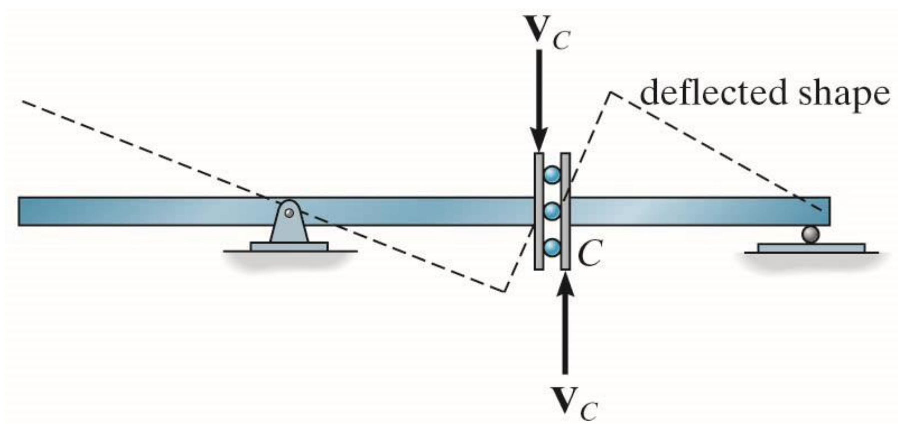
- When the +ve force $\mathbf{A}_y$ is applied at *A*, the beam deflects to the dashed position which rep the general shape of the influence line

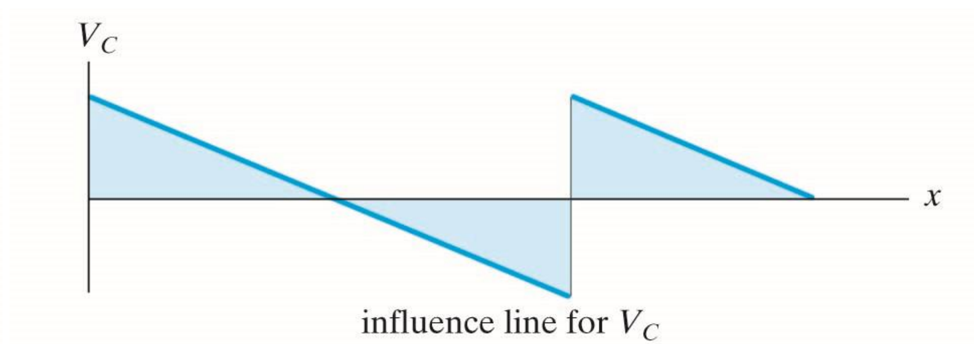


Qualitative Influence Lines

- If the shape of the influence line for shear at *C* is to be determined, the connection at *C* may be symbolized by a roller guide
- Applying a +ve shear force $\mathbf{V}_c$ to the beam at *C* & allowing the beam to deflect to the dashed position

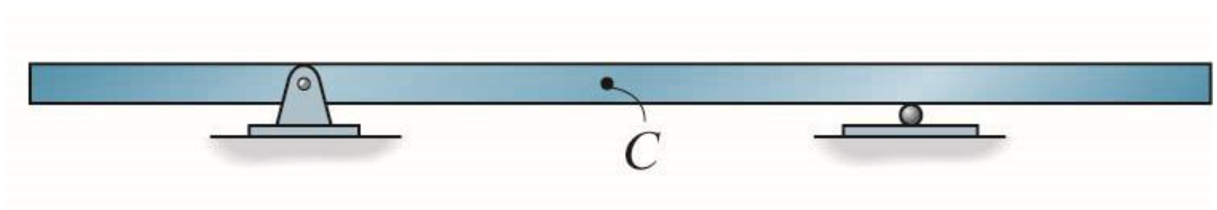


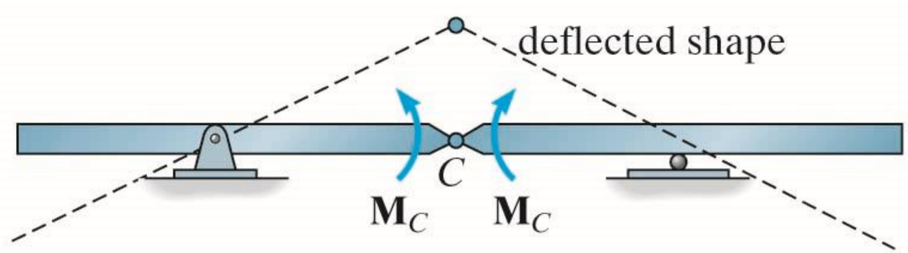


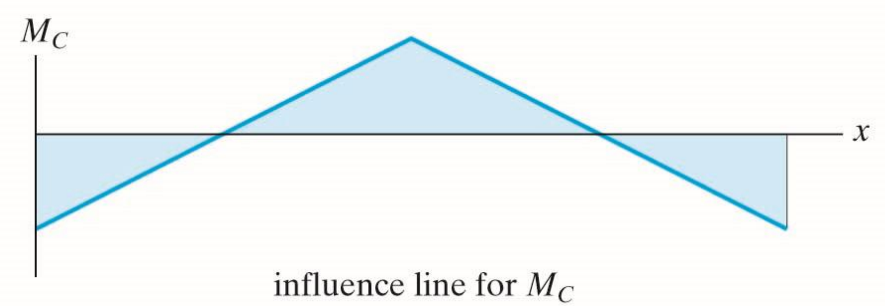


Qualitative Influence Lines

- If the shape of influence line for the moment at *C* is to be determined, an internal hinge or pin is placed at *C*
- Applying +ve moment $\mathbf{M}_c$ to the beam, the beam deflects to the dashed line



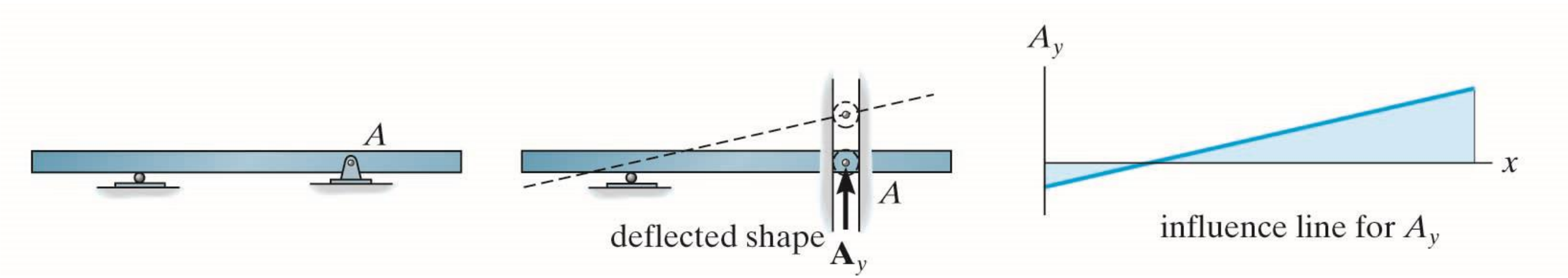


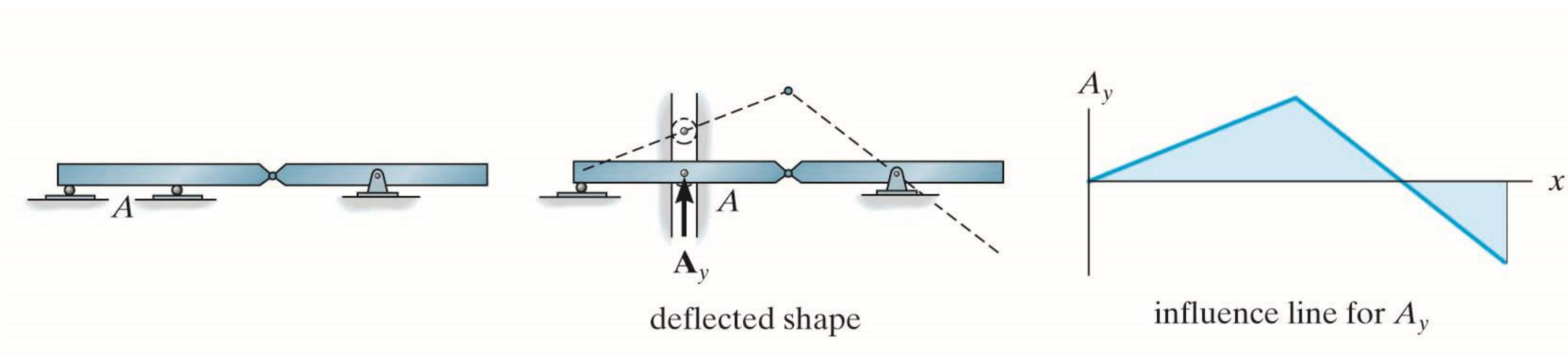


Qualitative Influence Lines

Example 6.9

For each beam sketch the influence line for the vertical reaction at *A*.

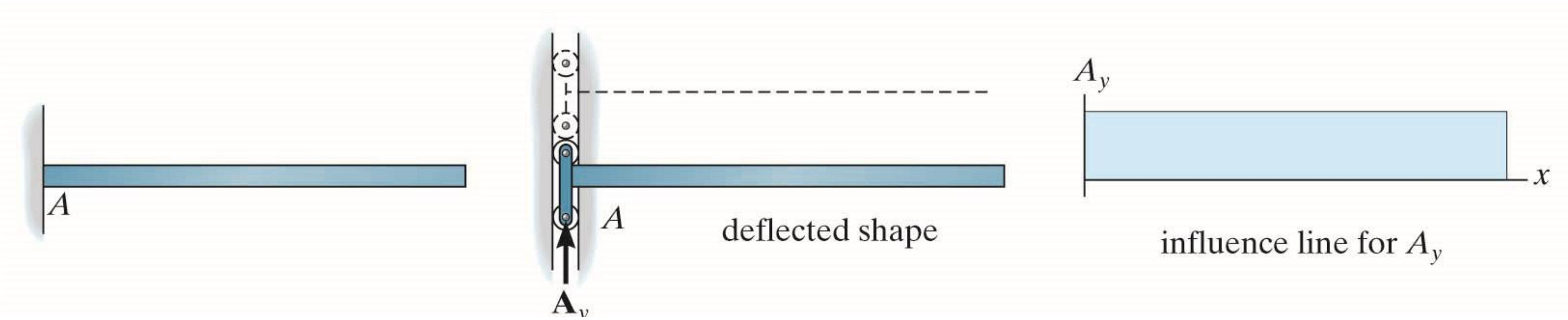




Qualitative Influence Lines

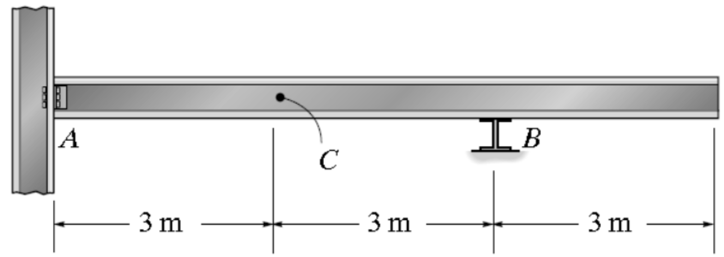
Example 6.9

For each beam sketch the influence line for the vertical reaction at *A*.



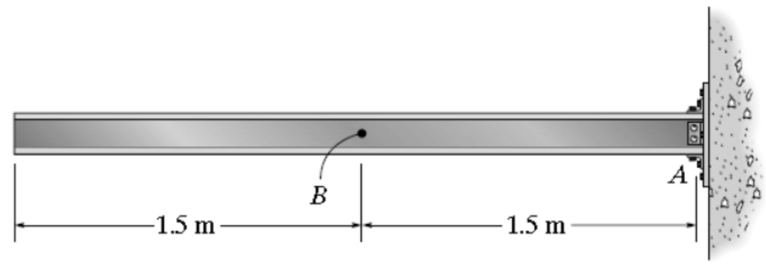
HW 6-3

Draw the influence lines for (a) the moment at *C*, (b) the reaction at *B*, and (c) the shear at *C*. Assume *A* is pinned and *B* is a roller.



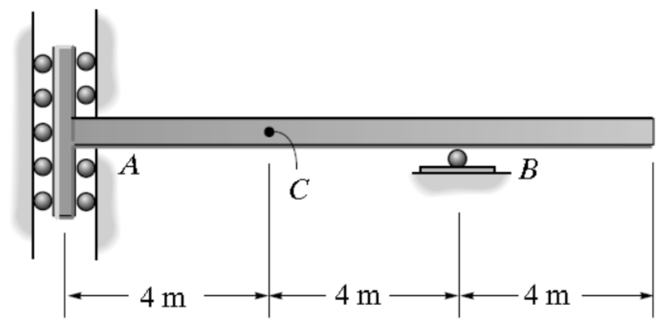
HW 6-4

Draw the influence lines for (a) the vertical reaction at A, (b) the moment at A, and (c) the shear at B. Assume the support at A is fixed.



HW 6-5

Draw the influence line for (a) the moment at B,
(b) the shear at C, and (c) the vertical reaction at B. The support at A resists only a horizontal force and a bending moment.



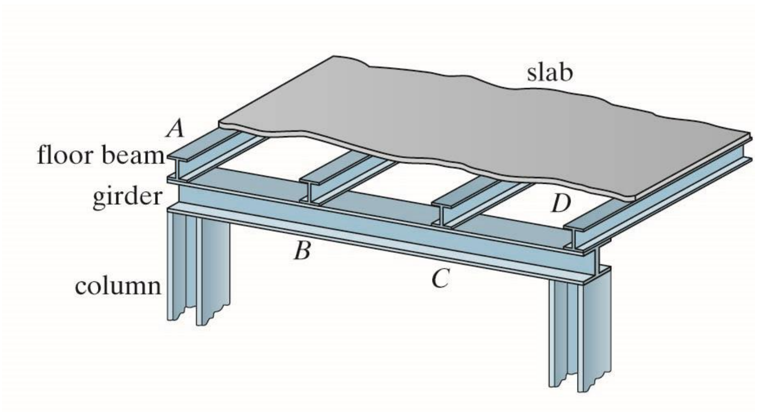
6.4

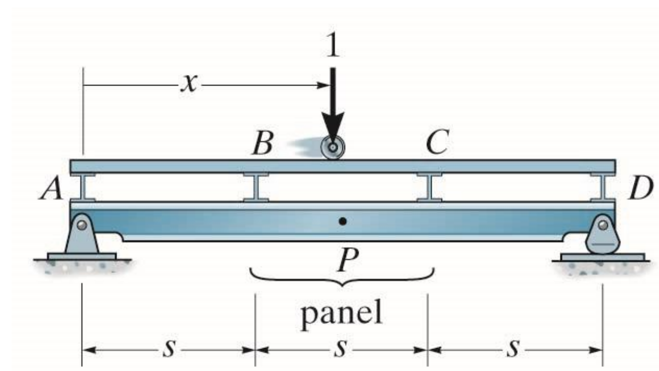
INFLUENCE LINES FOR FLOOR GIRDERS

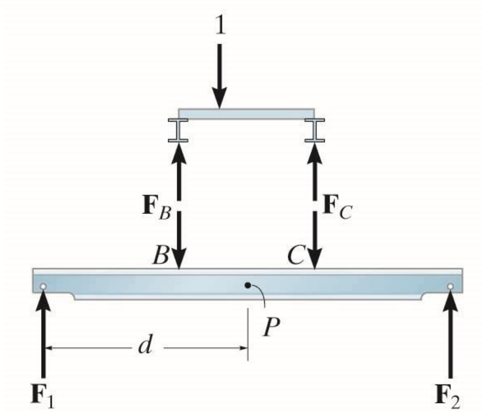
6.4

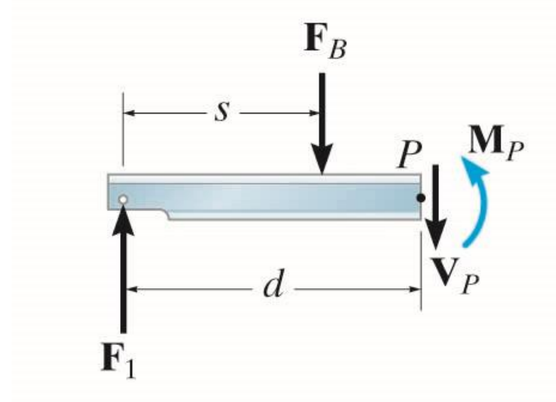
Influence Lines for Floor Girders

- Floor loads are transmitted from slabs to floor beams then to side girders & finally supporting columns









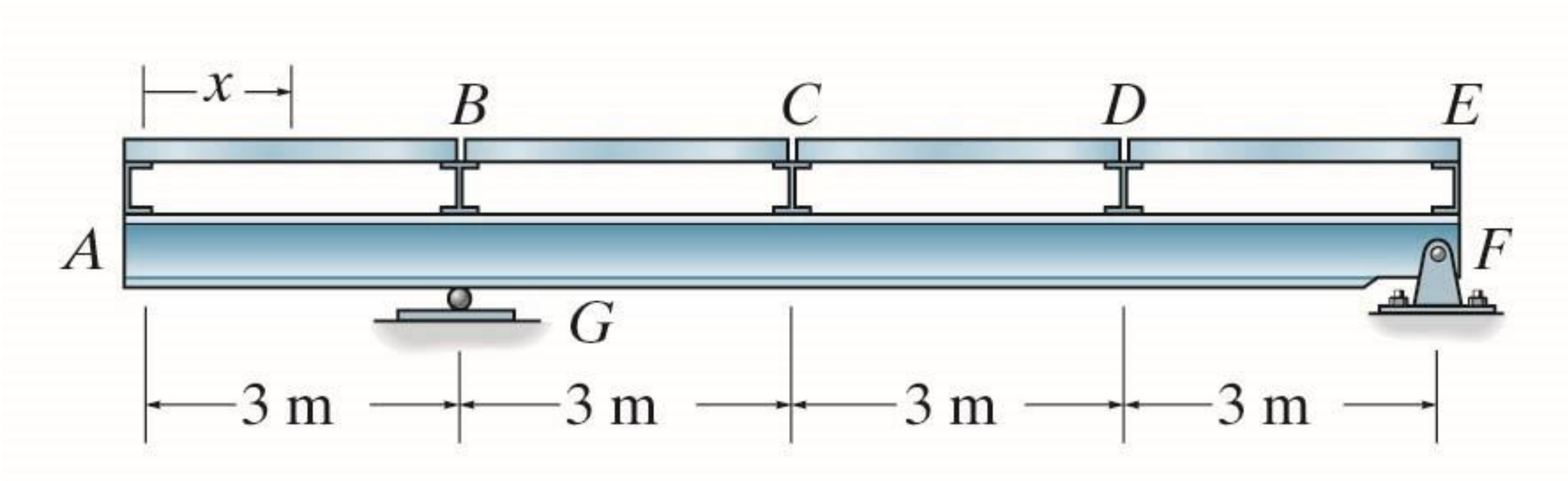
Influence Lines for Floor Girders

- The influence line for a specified point on the girder can be determined using the same statics procedure
- In particular, the value for the internal moment in a girder panel will depend upon where point *P* is chosen for the influence line
- Magnitude of $\mathbf{M}_P$ depends upon the point's location from end of the girder
- Influence lines for shear in floor girders are specified for panels in the girder and not specific points along the girder
- This shear is known as girder shear

Influence Lines for Floor Girders

Example 6.13

Draw the influence line for the shear in panel *CD* of the floor girder.

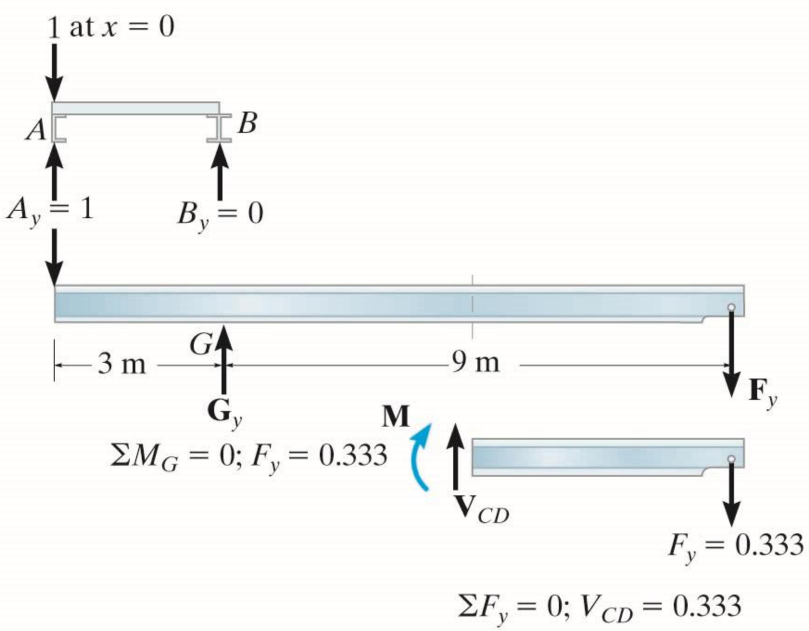


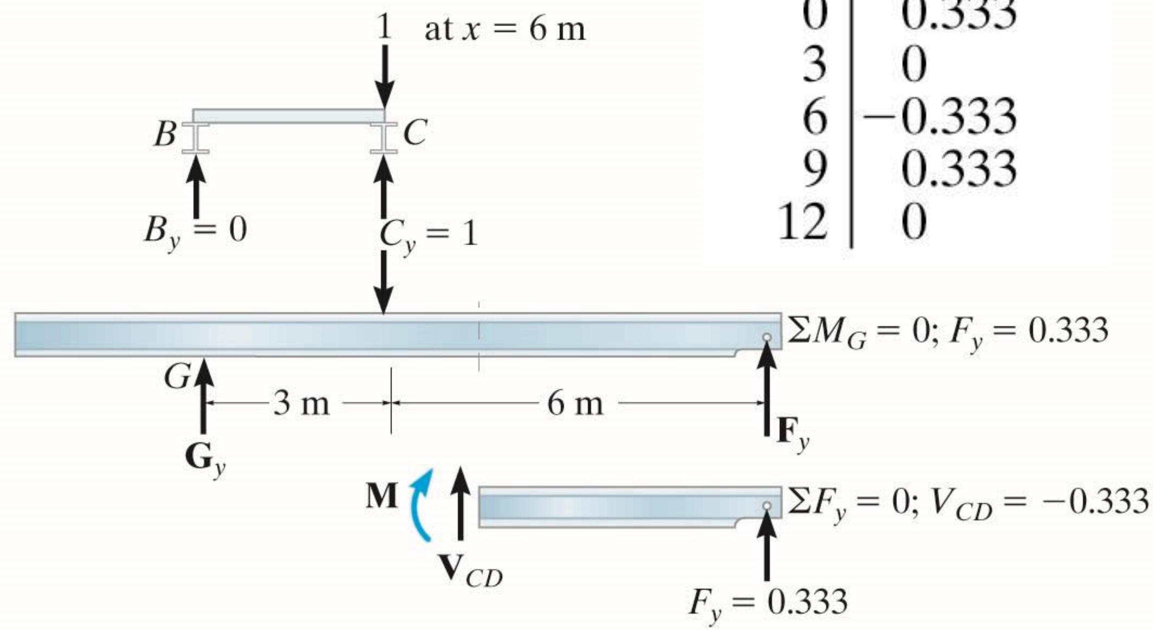
Influence Lines for Floor Girders

Example 6.13 (solution)

The unit load is placed at each floor beam location & the shear in panel *CD* is calculated. Finally a segment of the girder is considered & the internal panel shear V_{CD} is calculated.

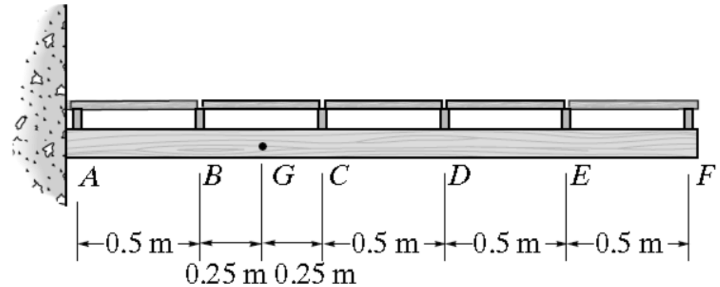
x	V_{CD}
0	0.333
3	0
6	−0.333
9	0.333
12	0





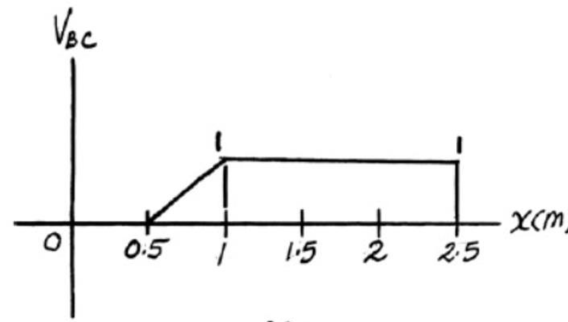
Example

A uniform live load of 1.8 kNm and a single concentrated live force of 4 kN are placed on the floor beams. Determine (a) the maximum positive shear in panel BC of the girder and (b) the maximum moment in the girder at *G*.



Referring to the influence line for the shear in panel *BC* shown in Fig. *a*, the maximum positive shear is

$$(V_{BC})_{\max(+)} = 1(4) + \left[\frac{1}{2}(1 - 0.5)(1)\right](1.8) + [(2.5 - 1)(1)](1.8) = 7.15 \text{ kN} \quad \textbf{Ans.}$$



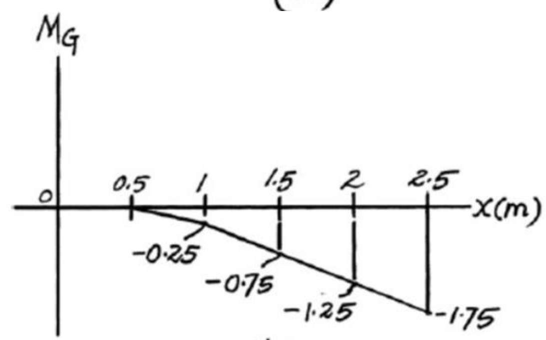
(a)

Referring to the influence line for the moment at *G* Fig. *b*, the maximum negative moment is

$$(M_G)_{\max(-)} = -1.75(4)\left[\frac{1}{2}(1 - 0.5)(-0.25)\right](1.8)$$

$$+ \left\{\frac{1}{2}(2.5 - 1)[-0.25 + (-1.75)]\right\}(1.8)$$

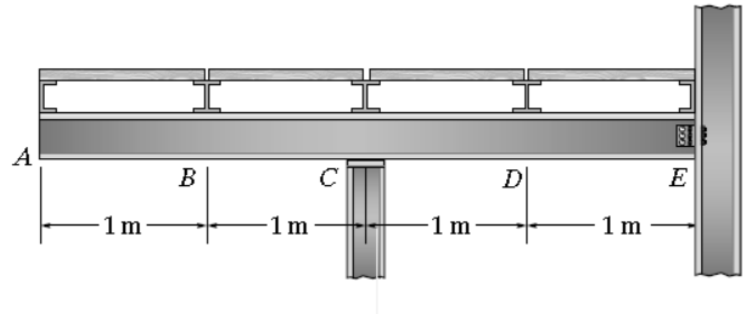
$$= -9.81 \text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$



(b)

HW 6-6

A uniform live load of 30 KN/m and a single concentrated live force of 30 KN are placed on the floor beams. If the beams also support a uniform dead load of 5.25 KN/m, determine (a) the maximum positive shear in panel CD of the girder and (b) the maximum negative moment in the girder at D. Assume the support at C is a roller and E is a pin.



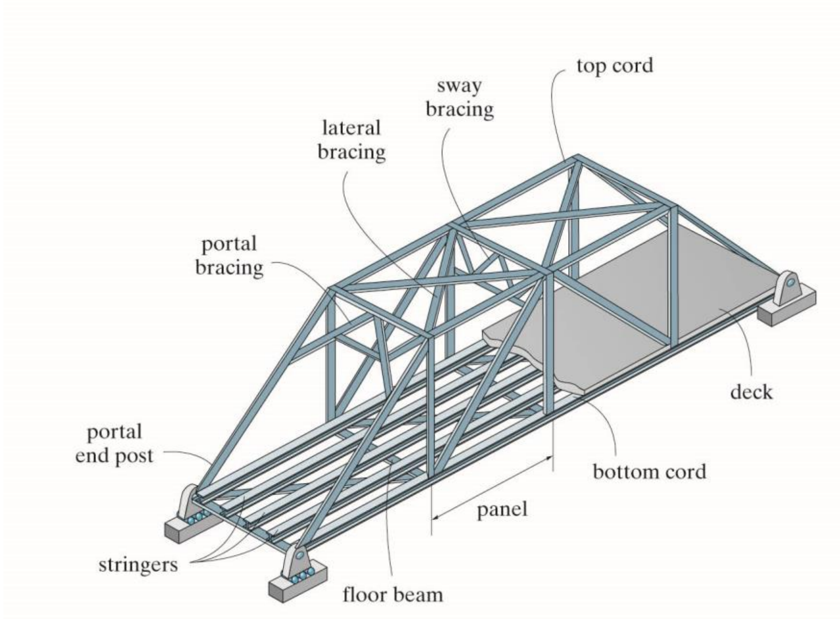
Ans.
V CD +max= 82.9 KN
MD max=-62.6 KN.m

6.5
INFLUENCE LINES FOR TRUSSES

6.5

Influence Lines for Trusses

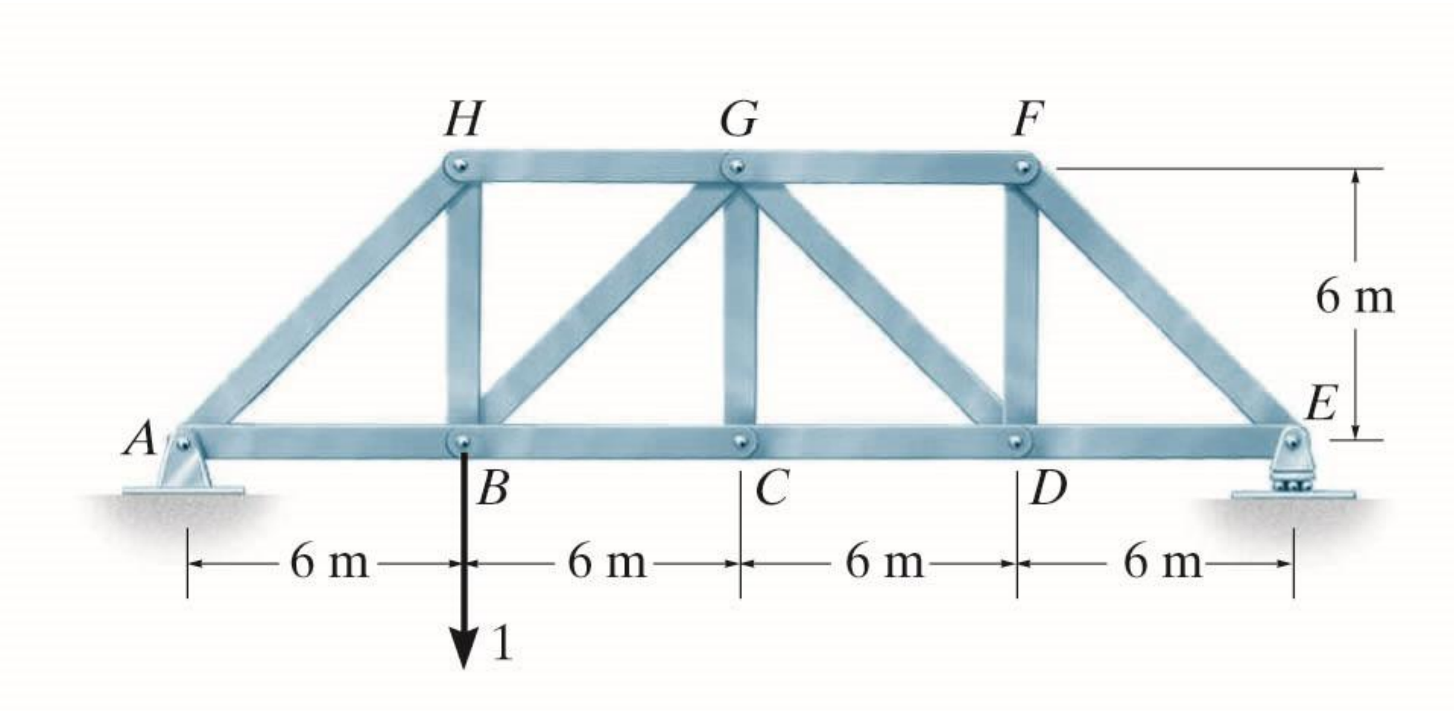
- The loading on the bridge deck is transmitted to stringers which in turn transmit the loading to floor beams and then to joints along the bottom cord
- We can obtain the ordinate values of the influence line for a member by loading each joint along the deck with a unit load and then use the method of joints or method of sections to calculate the force in the member



Influence Lines for Trusses

Example 6.15

Draw the influence line for the force in member *GB* of the bridge truss.



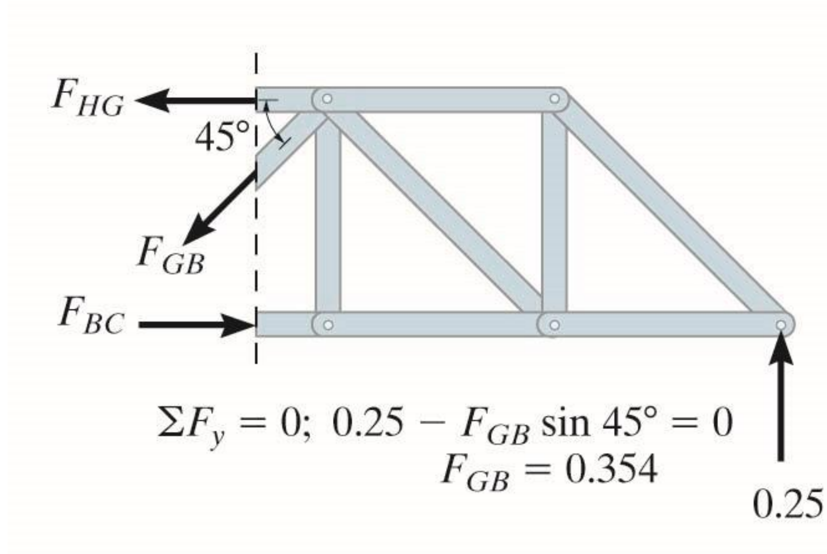
Influence Lines for Trusses

Example 6.15 (solution)

Each successive joint at the bottom cord is loaded with a unit load and the force in member *GB* is calculated using the method of sections.

Since the influence line extends over the entire span of truss, member *GB* is referred to as a primary member.

x	F_{GB}
0	0
6	0.354
12	−0.707
18	−0.354
24	0

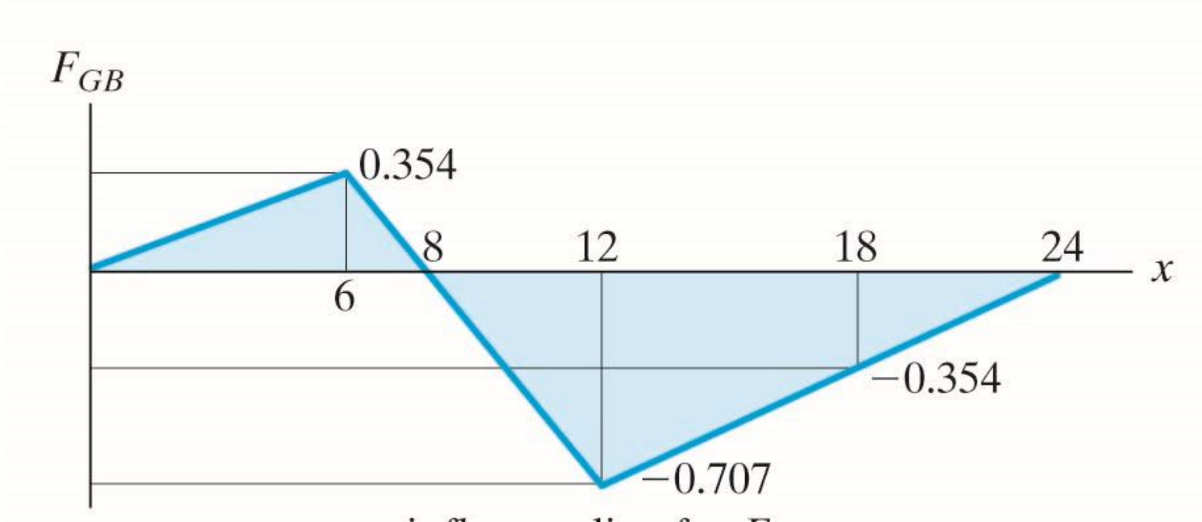


Influence Lines for Trusses

Example 6.15 (solution)

This means that *GB* is subjected to a force regardless of where the bridge deck is loaded.

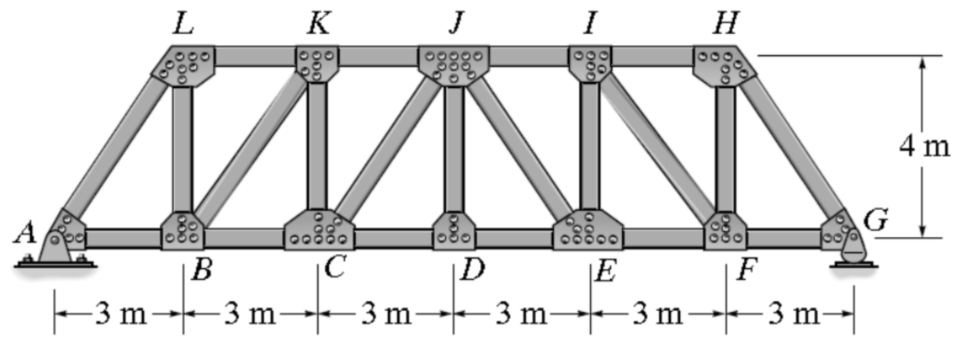
The point of zero force is determined by similar triangles.



influence line for F_{GB}

HW 6-7

Draw the influence line for the force in (a) member *KJ* and (b) member *CJ*.



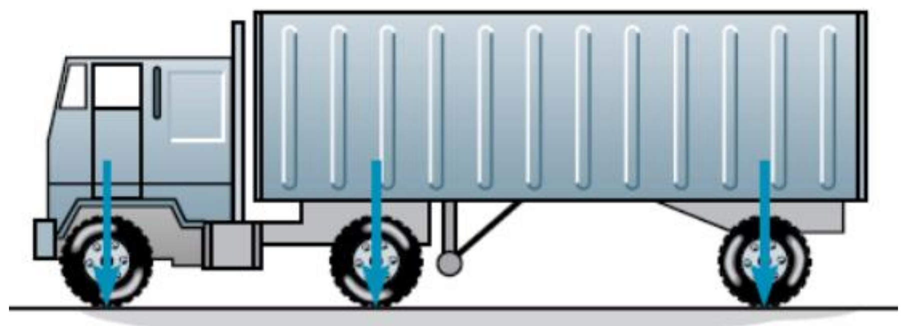
6.6

MAXIMUM INFLUENCE AT A POINT DUE TO A SERIES OF CONCENTRATED LOADS

6.6

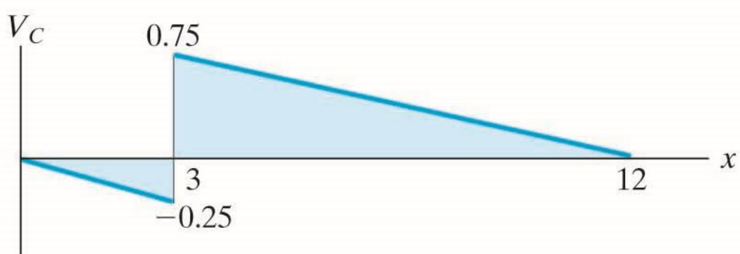
Maximum Influence at a Point due to a Series of Concentrated Loads

- The max effect caused by a live concentrated force is determined by multiplying the peak ordinate of the influence line by the magnitude of the force
- In some cases, e.g. wheel loadings, several concentrated loadings must be placed on structure
- Trial-and-error procedure can be used or a method that is based on the change in function that takes place as the load is moved

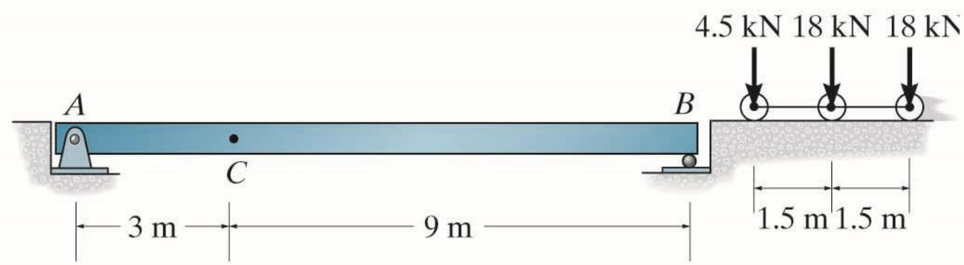


Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - Consider the simply supported beam with associated influence line for shear at point *C*
  - The max +ve shear at *C* is to be determined due to the series of concentrated loads moving from right to left
  - Critical loading occurs when one of the loads is placed just to the right of *C*



influence line for V_C



Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - By trial & error, each of three possible cases can therefore be investigated

$$\text{Case 1}: (V_C)_1 = 4.5(0.75) + 18(0.625) + 18(0.5) = 23.63kN$$

$$\text{Case 2}: (V_C)_2 = 4.5(-0.125) + 18(0.75) + 18(0.625) = 24.19kN$$

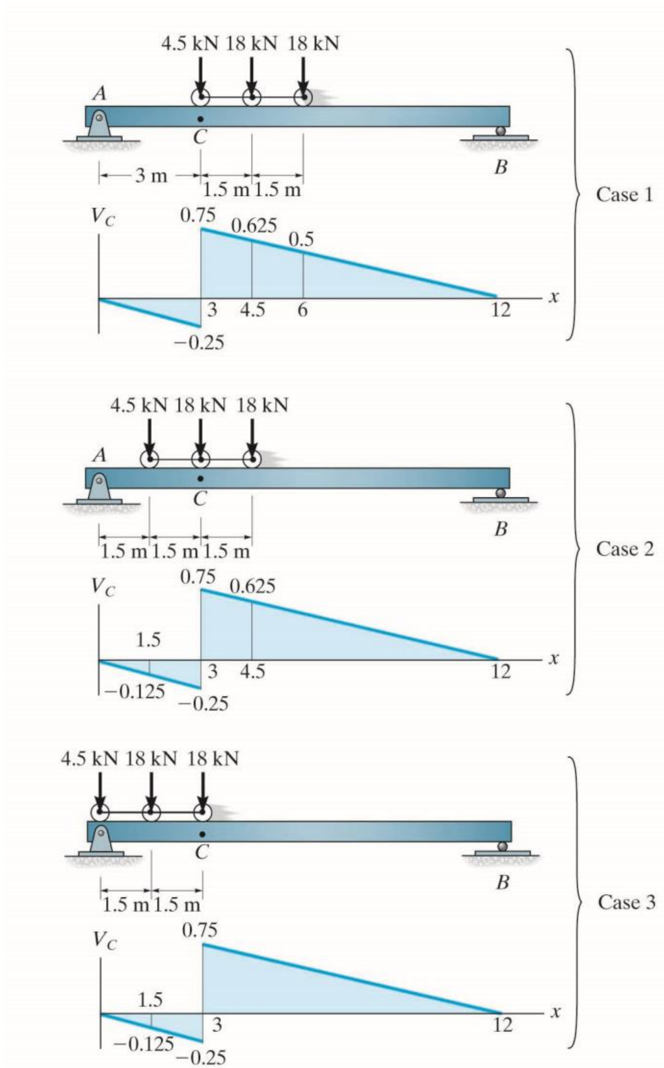
$$\text{Case 3}: (V_C)_3 = 4.5(0) + 18(-0.125) + 18(0.75) = 11.25kN$$

Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - Case 2 yields the largest value for V_C and therefore rep the critical loading
  - Investigation of Case 3 is unnecessary since by inspection such an arrangement of loads would yield $(V_C)_3 < (V_C)_2$
  - Trial-and-error can be tedious at times
  - The critical position of the loads can be determined in a more direct manner by finding ΔV which occurs when the loads are moved from Case 1 to 2, then from Case 2 to 3
  - As long as computed ΔV is +ve, the new position will yield a larger shear
  - Each movement is investigated until a –ve ΔV is computed

$$\Delta V = Ps(x_2 - x_1)$$

Maximum Influence at a Point due to a Series of Concentrated Loads

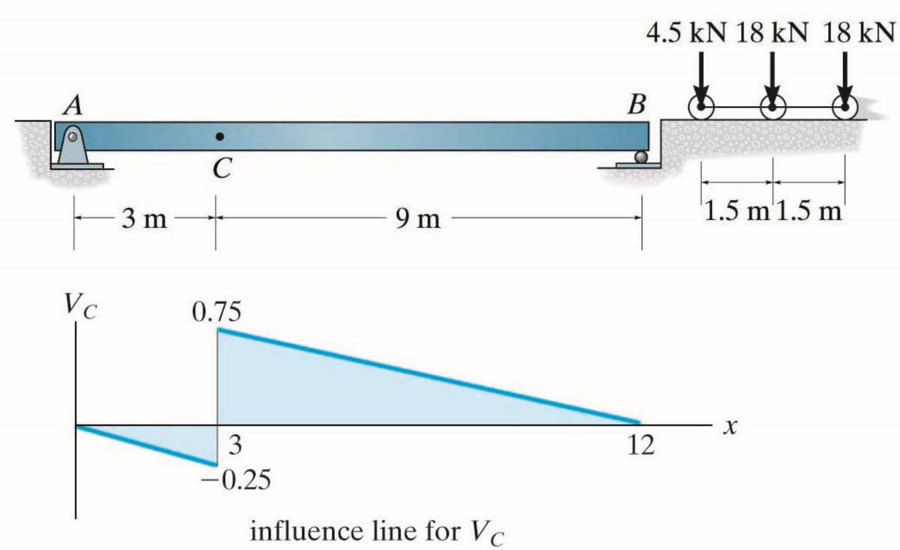


Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - If the load moves past a point where there is a discontinuity in the influence line, the change in shear is:

$$\Delta V = P(y_2 - y_1)$$

  - Use of above eqn will be illustrated with
  - reference to the beam, loading & influence line for V_c shown



$$\text{slope, } s = 0.75/(12-3) = 0.75/(9) = 0.0833$$

$$\text{jump at C} = 0.75 + 0.25 = 1$$

Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - Consider the loads moving 1.5 m
  - When this occurs, the 4.5 kN load jumps down (-1) & all the loads move up the slope of the influence line
  - This causes a change of shear

$$\Delta V_{1-2} = 4.5(-1) + [4.5 + 18 + 18](0.0833)(1.5) = +0.563\,\text{kN}$$

  - Since ΔV_{1-2} is +ve, Case 2 will yield a larger value for V_C than case 1

$$\Delta V_{2-3} = 18(-1) + [4.5 + 18 + 18](0.0833)(1.5) = -12.94\,\text{kN}$$

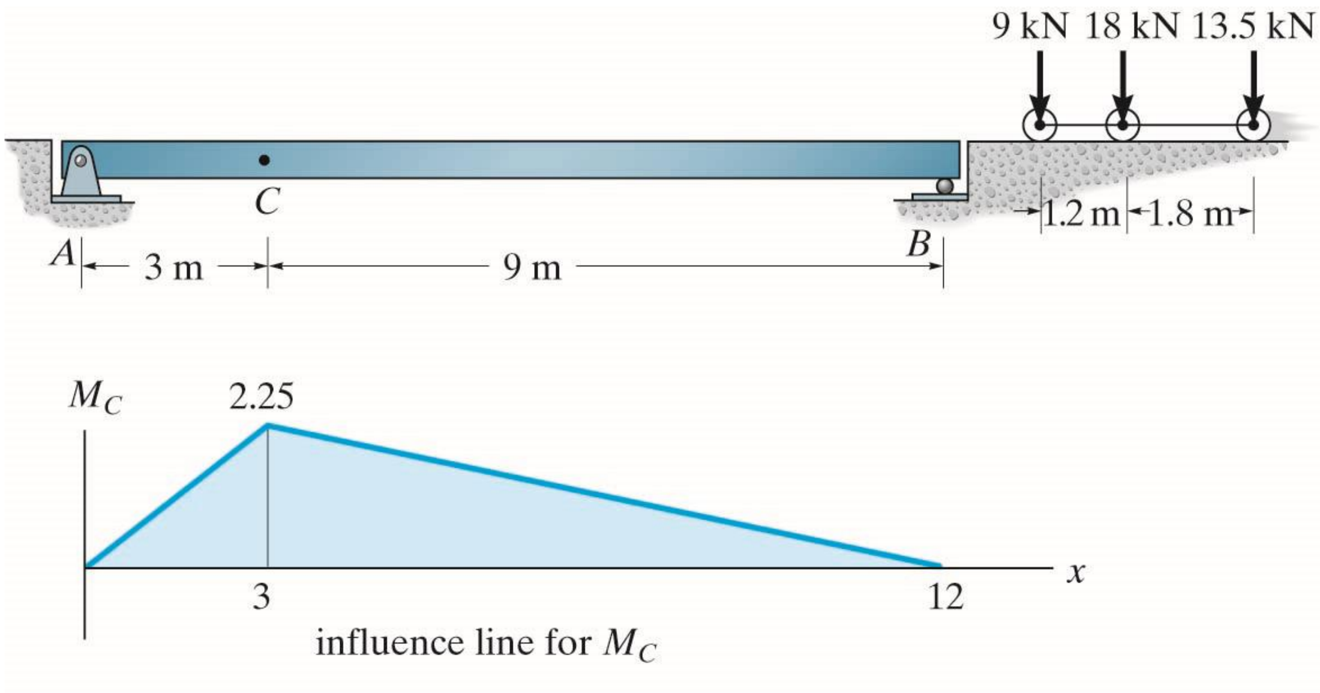
  - Since ΔV_{2-3} is -ve, Case 2 is the position of the critical loading

Maximum Influence at a Point due to a Series of Concentrated Loads

- Moment

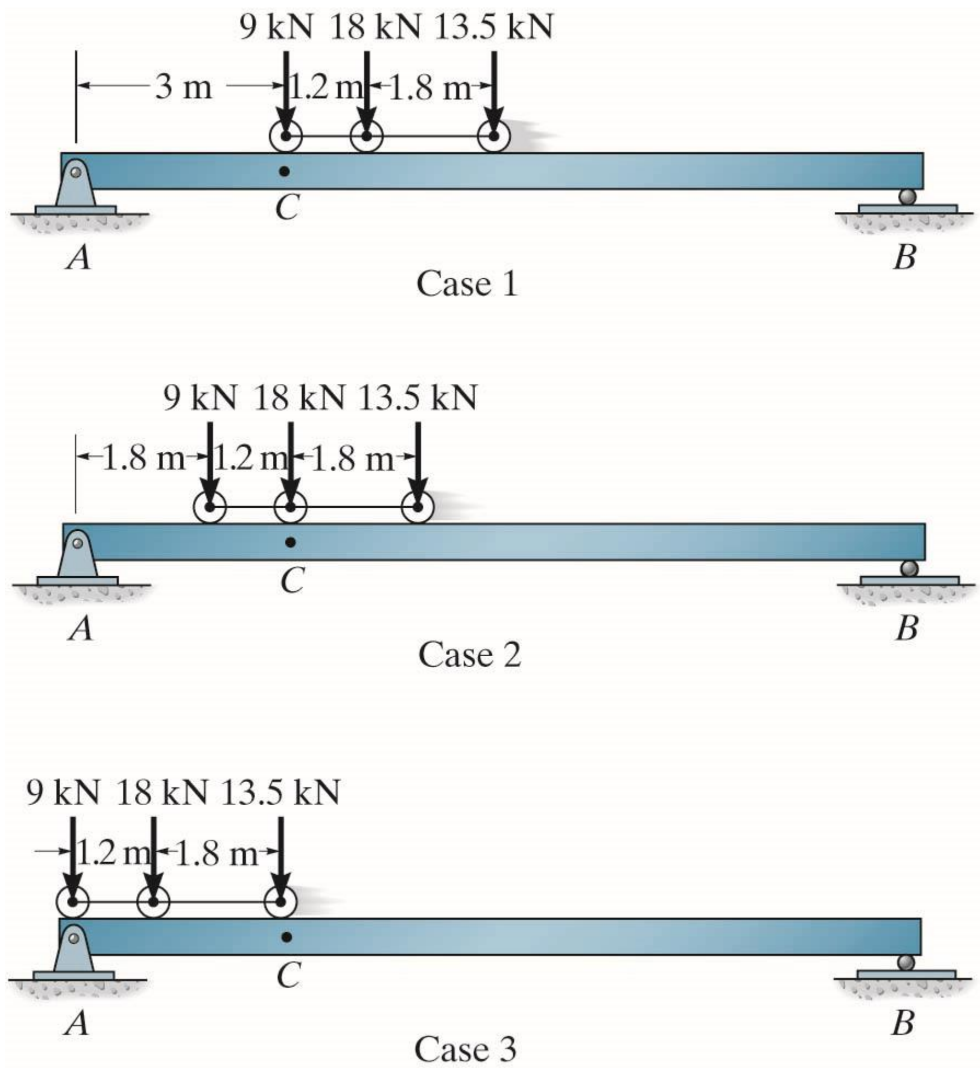
$$\Delta M = Ps(x_2 - x_1)$$

- Consider the beam, loading & influence line for the moment at point *C*



Maximum Influence at a Point due to a Series of Concentrated Loads

- Moment
  - When the loads of Case 1 are moved to Case 2, it is observed that the 9 kN load decreases $\Delta M_{1\text{-}2}$
  - Likewise, the 18 kN and 13.5 kN forces cause an increase of $\Delta M_{1\text{-}2}$



Maximum Influence at a Point due to a Series of Concentrated Loads

- Moment

$$\Delta M_{1-2} = 9\left(\frac{2.25}{3}\right)(1.2) + (18+13.5)\left(\frac{2.25}{12-3}\right)(1.2) = 1.35\,\text{kN}\bullet\text{m}$$

- Since Δ $M_{1\text{-}2}$ is +ve, we compute for loads moved from Cases 2 to 3

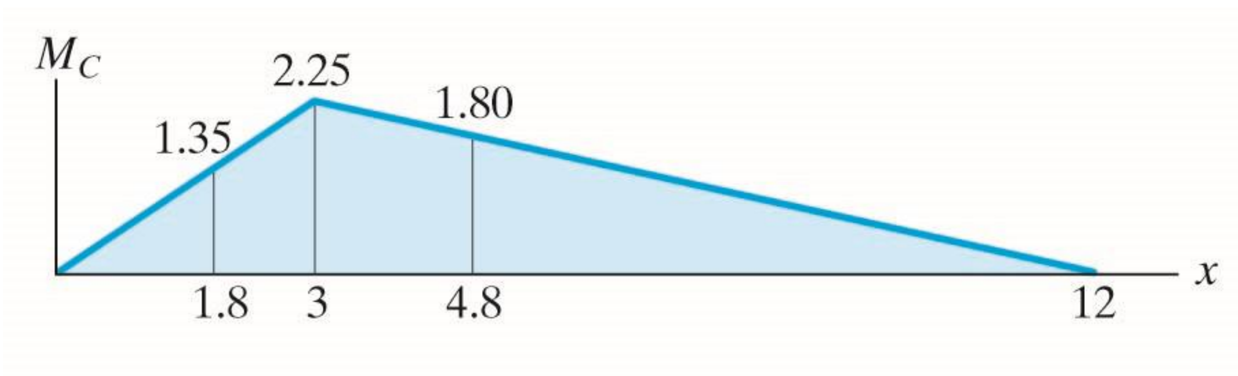
$$\Delta M_{2-3} = -(9+18)\left(\frac{2.25}{3}\right)(1.8) + 13.5\left(\frac{2.25}{12-3}\right)(1.8) = -30.38\,\text{kN}\bullet\text{m}$$

- Δ $M_{1\text{-}2}$ -ve, the greatest moment at *C* will occur when the beam is loaded as shown in Case 2

Maximum Influence at a Point due to a Series of Concentrated Loads

- Moment
  - The max moment at *C* is therefore,

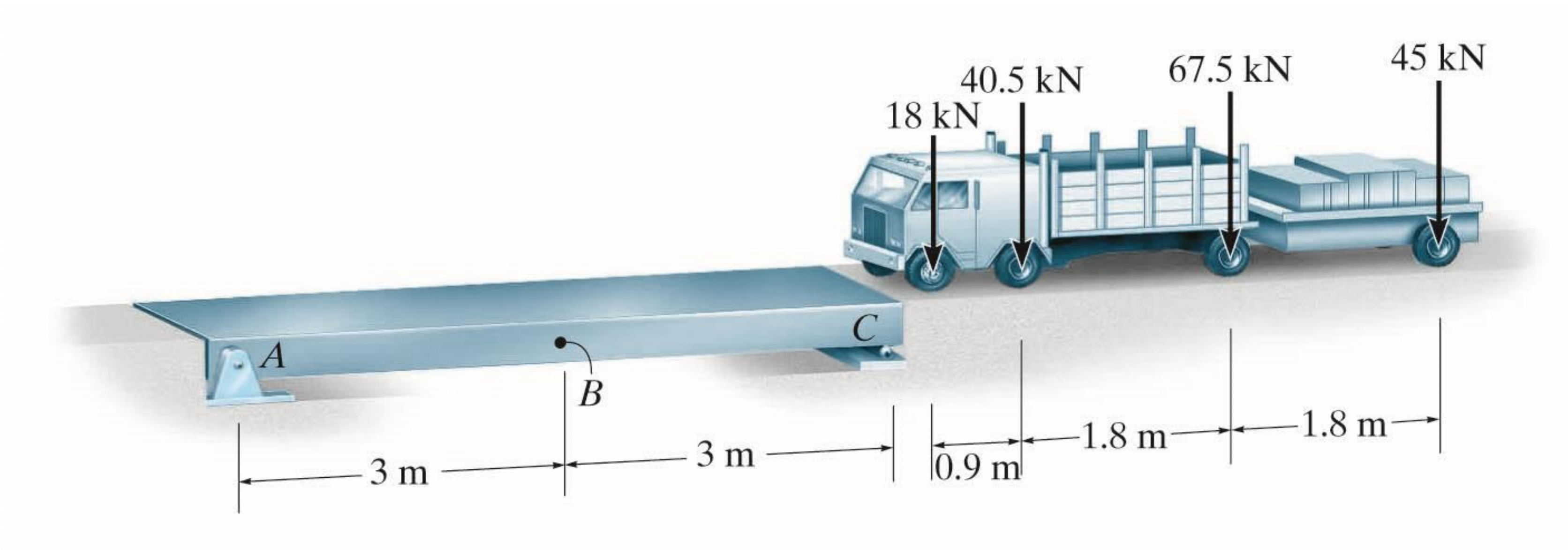
$$(M_C)_{\max} = 9(1.35) + 18(2.25) + 13.5(1.8) = 77.0\,\text{kN}\bullet\text{m}$$



Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18

Determine the maximum positive shear created at point *B* in the beam due to the wheel loads of the moving truck.



Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

➔ 0.9 m movement of the 18 kN load

Imagine that the 18 kN load acts just to the right of point *B* so that we obtain its max +ve influence.

Beam segment *BC* is 3 m long, the 45 kN load is not as yet on the beam.

When the truck moves 0.9 m to the left, the 18 kN load jumps downward on the influence line 1 unit.

Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

Although the 45 kN load also moves forward 0.9 m, it is still not on the beam. Thus,

$$\Delta V_B = 18(-1) + (18 + 40.5 + 67.5)\left(\frac{0.5}{3}\right)0.9 = +0.9\,\text{kN}$$

➔ 1.8 m movement of the 40.5 kN load

When the 40.5 kN load acts just to the right of *B* & the truck moves 1.8 m to the left, we have

$$\Delta V_B = 40.5(-1) + (18 + 40.5 + 67.5)\left(\frac{0.5}{3}\right)(1.8) + 45\left(\frac{0.5}{3}\right)(1.2)$$

$$= +6.3\,\text{kN}$$

Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

➔ 1.8 m movement of the 67.5 kN load

If the 67.5 kN load is positioned just to the right of *B* & then the truck moves 1.8 m to the left, the 18 kN load moves only 0.3 m until it is off the beam.

Likewise, the 40.5 kN load moves only 1.2 m until it is off the beam

$$\Delta V_B = 67.5(-1) + 18\left(\frac{0.5}{3}\right)(0.3) + 40.5\left(\frac{0.5}{3}\right)(1.2) + (67.5+45)\left(\frac{0.5}{3}\right)(1.8)$$

$$= -24.8\,\text{kN}$$

Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

➔ 1.8 m movement of the 67.5 kN load (cont'd)

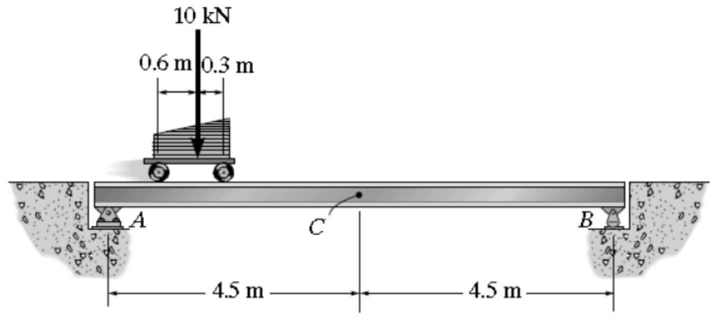
Since ΔV_B is -ve, the correct position of the loads occur when 67.5 kN is just to the right of *B*.

$$
\begin{aligned}
(V_B)_{\max} &= 18(-0.05) + 40.5(-0.2) + 67.5(0.5) + 45(0.2) \\
&= 33.8 \text{ kN}
\end{aligned}
$$

In practice, one also has to consider motion of the truck from left to right & then choose the max value between these 2 situations.

HW 6-8

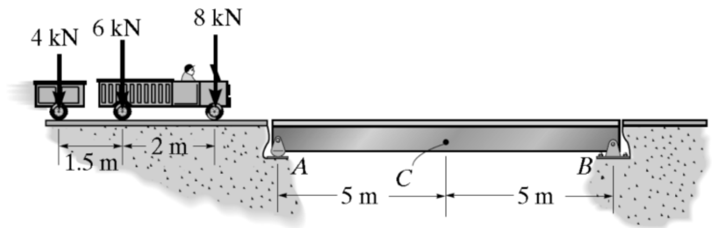
Determine the maximum moment at *C* caused by the moving load.



Ans.
Mc +max=21 KN.m

HW 6-9

Determine the maximum positive moment at point C on the single girder caused by the moving load.



Ans.
Mc +max=34 KN.m

6.7

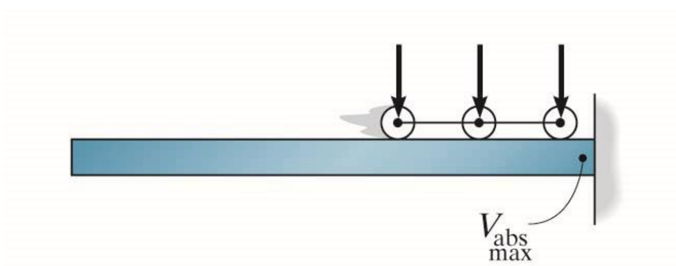
ABSOLUTE MAXIMUM SHEAR AND MOMENT

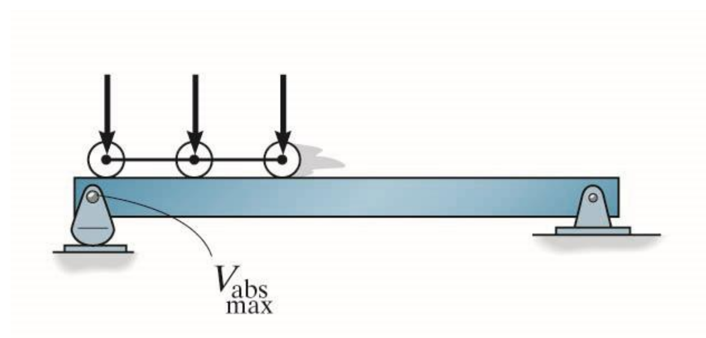
6.7

Absolute Maximum Shear and Moment

- A more general problem involves the determination of both the location of the point in beam & the position of the loading on the beam so that one can obtain the absolute max shear & moment caused by the loads

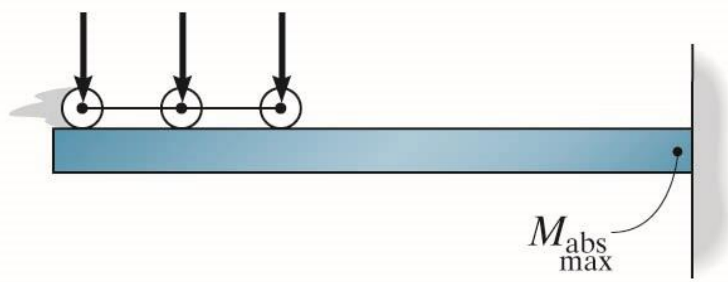
- Shear
  - For cantilevered beam, the absolute max shear will occur at a point just next to the fixed support
  - For simply supported beams the absolute max shear will occur just next to one of the supports





Absolute Maximum Shear and Moment

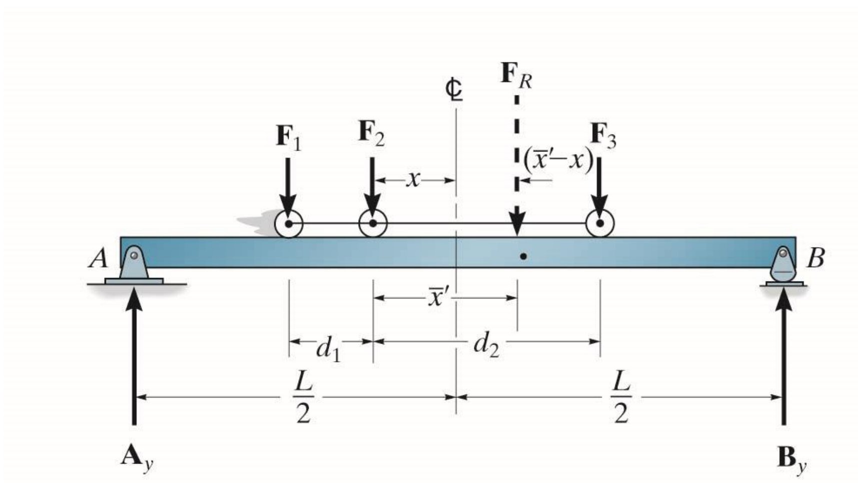
- Moment
  - The absolute max moment for a cantilevered beam occurs at a point where absolute max shear occurs
  - The concentrated loads should be positioned at the far end of the beam

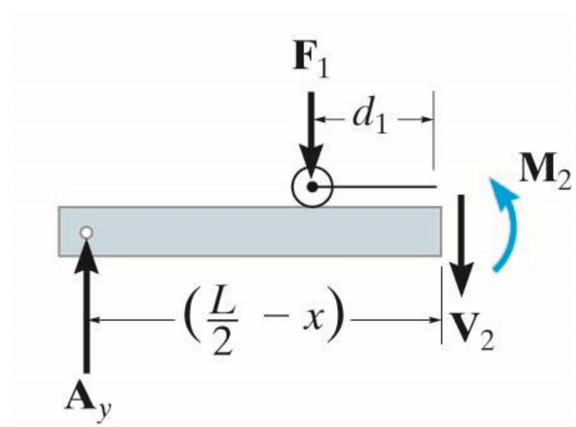


  - For a simply supported beam, the critical position of the loads & the associated absolute max moment cannot, in general, be determined by inspection
  - The position can be determined analytically

Absolute Maximum Shear and Moment

- Moment
  - Consider a beam subjected to forces, $\mathbf{F}_1$, $\mathbf{F}_2$ & $\mathbf{F}_3$
  - The moment diagram for a series of concentrated forces consists of straight line segments having peaks at each force
  - Assume the absolute max moment occurs under $\mathbf{F}_2$
  - The position of the 3 loads on the beam will be specified by the dist *x* measured from $\mathbf{F}_2$ to the beam's centerline





Absolute Maximum Shear and Moment

- Moment
  - To determine a specific value of *x*, first obtain the resultant force of the system $\mathbf{F}_R$ & its distance measured from $\mathbf{F}_2$
  - Moments are summed about *B*, yielding the beam's left reaction $\mathbf{A}_y$

$$\sum M_B = 0$$

$$A_y = \frac{1}{L}(F_R)\left[\frac{L}{2} - (\bar{x}' - x)\right]$$

Absolute Maximum Shear and Moment

- Moment
  - If the beam is sectioned just to the left of **F**$_2$, **M**$_2$ under **F**$_2$ is:

$$\sum M = 0$$

$$M_2 = (A_y)\left(\frac{L}{2} - x\right) - F_1 d_1$$

$$= \frac{1}{L}(F_R)\left[\frac{L}{2} - (\overline{x}' - x)\right]\left(\frac{L}{2} - x\right) - F_1 d_1$$

$$= \frac{F_R L}{4} - \frac{F_R \overline{x}'}{2} - \frac{F_R x^2}{L} + \frac{F_R x \overline{x}'}{L} - F_1 d_1$$

Absolute Maximum Shear and Moment

- Moment
  - For max M_2, we require:

$$\frac{dM_2}{dx} = \frac{-2F_R x}{L} + \frac{F_R \overline{x}'}{L} = 0 \quad \text{or} \quad x = \frac{\overline{x}'}{2}$$

Absolute Maximum Shear and Moment

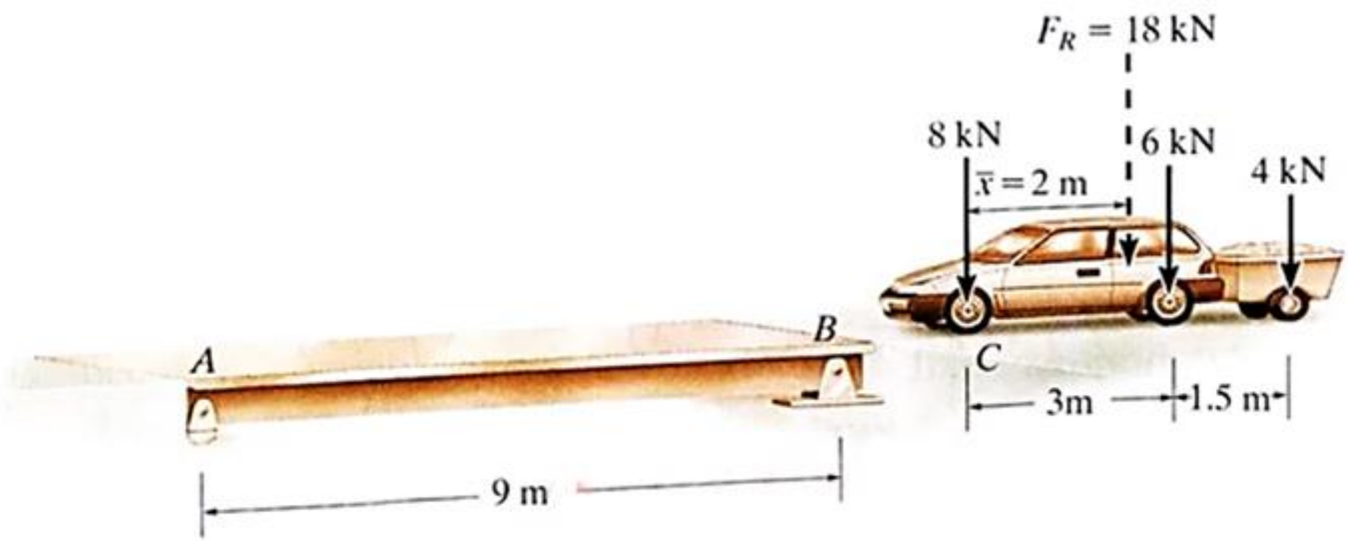
- Moment
  - Hence, we may conclude that the absolute max moment in a simply supported beam occurs under one of the concentrated forces such that this force is positioned on the beam so that it & the resultant force of the system are equidistant from the beam's centerline

- Envelope of Max influence-line values
  - An elementary way to proceed requires constructing influence lines for the shear or moment at selected points along the entire length of the beam & then computing the max shear or moment in the beam for each point
  - These values when plotted yield an "envelope of maximums", from which both the absolute maximum value of shear or moment and its location can be found.

Absolute Maximum Shear and Moment

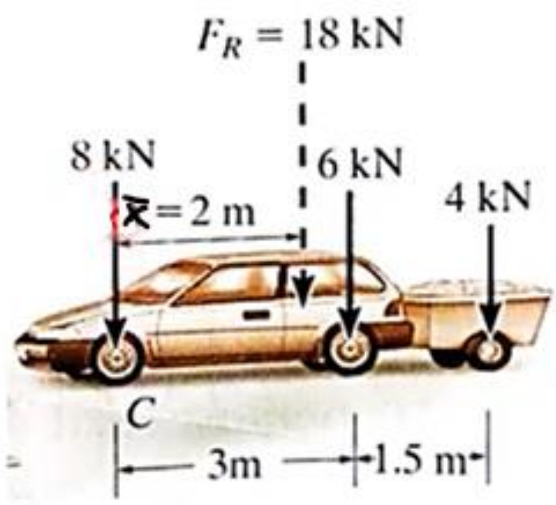
Example 6.21

Determine the absolute maximum moment in the simply-supported bridge deck shown below.



Absolute Maximum Shear and Moment

Example 6.21 (solution)

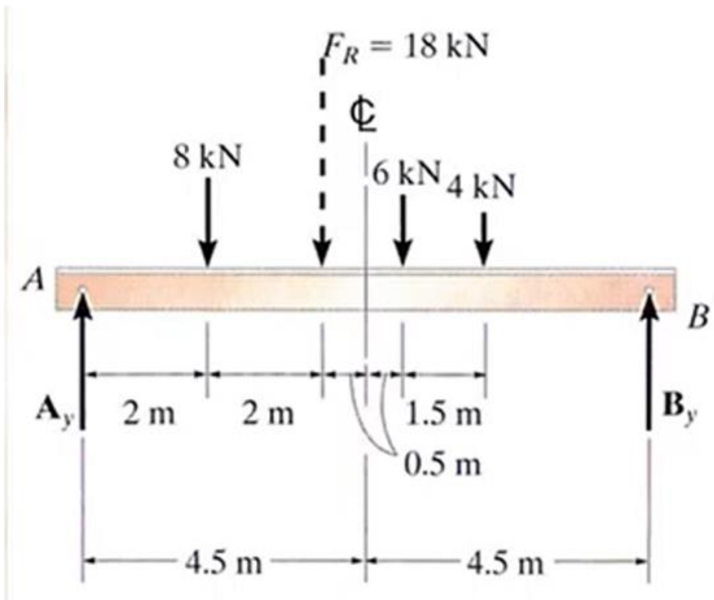


$$F_R = 8 + 6 + 4 = 18\text{ kN}$$

$$18\bar{x} = 6(3) + 4(4.5)$$

$$\bar{x} = 2\text{ m}$$

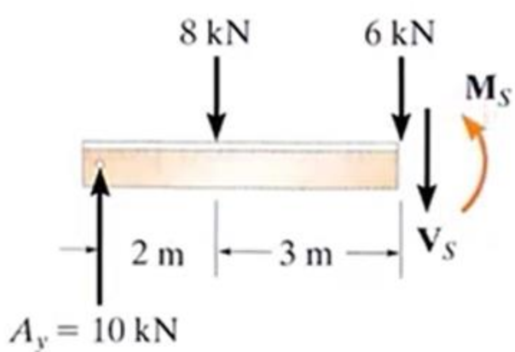
Assume the absolute max moment occurs under the 6-kN load



$$\circlearrowleft + \sum M_B = 0; \quad -A_y(9) + 18(5) = 0$$

$$A_y = 10\text{ kN}$$

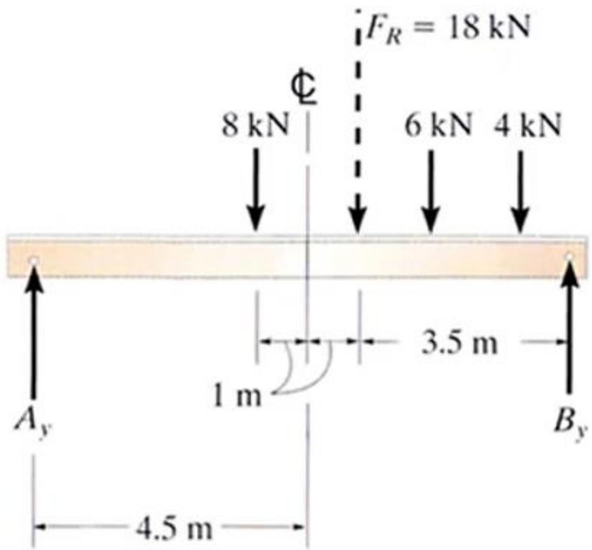
Now using the left section of the beam



$$\circlearrowleft + \sum M_S = 0; \quad -10(5) + 8(3) + M_S = 0$$

$$M_S = 26.0\text{ kNm}$$

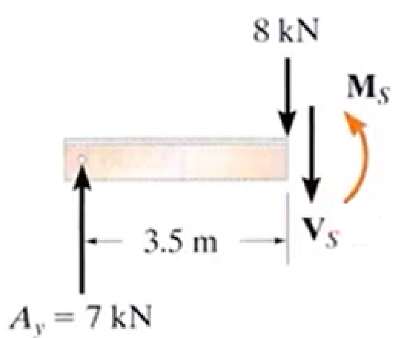
There is a possibility that the absolute max moment may occur under the 8-kN load, since 8 kN > 6 kN and $\mathbf{F}_R$ is between both 8 kN and 6 kN



$$\circlearrowleft + \sum M_B = 0; \quad -A_y(9) + 18(3.5) = 0$$

$$A_y = 7\ \text{kN}$$

Now using the left section of the beam



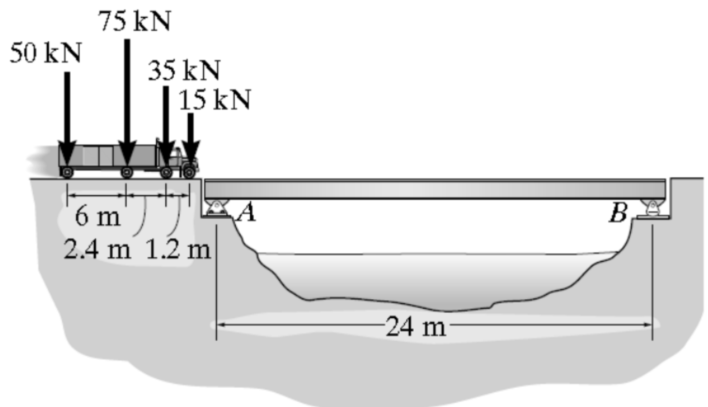
$$\circlearrowleft + \sum M_S = 0; \quad -7(3.5) + M_S = 0$$

$$M_S = 24.5\ \text{kNm}$$

By comparison the absolute max moment is $M_S = 26.0\ \text{kNm}$

HW 6-10

Determine the absolute maximum moment in the girder bridge due to the truck loading shown. The load is applied directly to the girder.



Ans.
M max=832.6 KN.m

CHAPTER 7:

APPROXIMATE ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES

7

Chapter Outline

7.1

USE OF APPROXIMATE METHODS

7.1

Use of Approximate Methods

- The analysis when using a model must satisfy both the conditions of:
  - Equilibrium
  - Compatibility of displacements at joints
- For an initial design, member sizes are not known & statically indeterminate analysis cannot be done
- A simpler model, i.e. statically determinate analysis, must be developed

Use of Approximate Methods

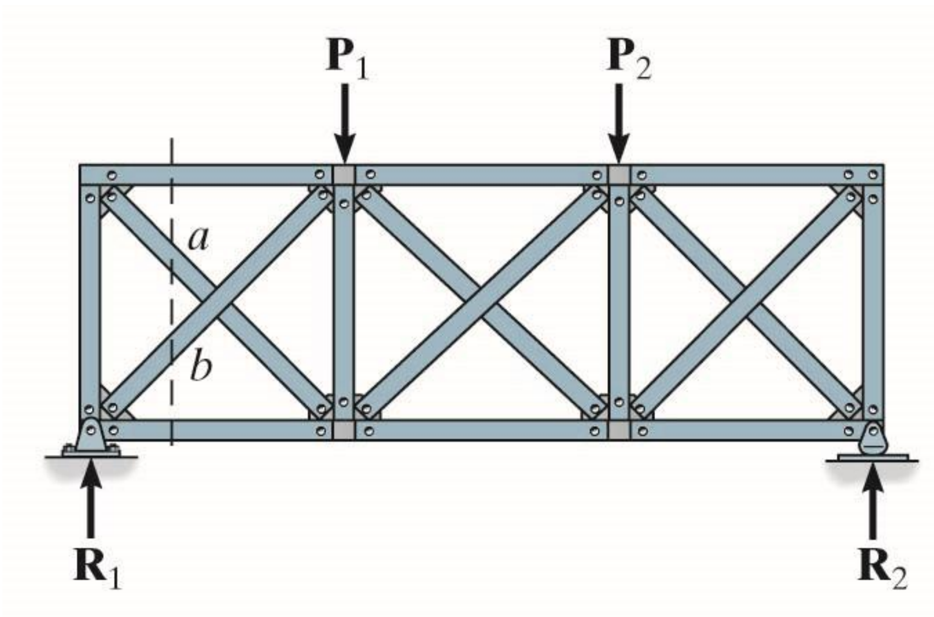
- The analysis of this model is known as an approximate analysis
- The preliminary design of the members can be made
- After which, the more exact indeterminate analysis can be performed & the design refined

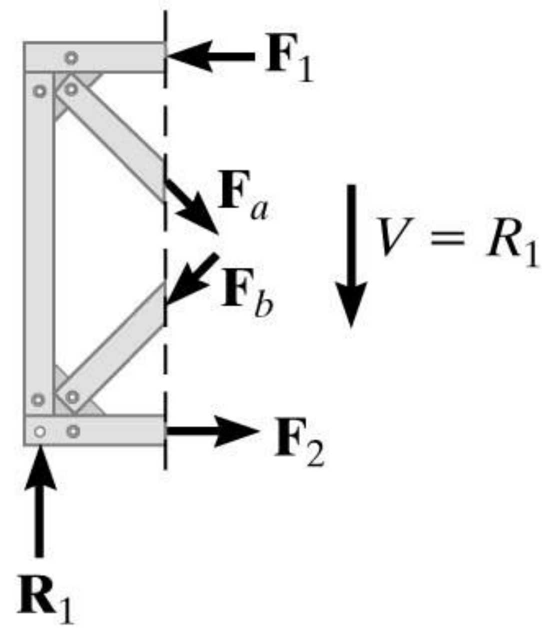
7.2
TRUSSES

7.2

Trusses

- The truss used for lateral bracing of a building is not considered a primary element
- It will therefore be analyzed using approximate methods
- In the case shown, the truss is indeterminate to the third degree





Trusses

- 3 assumptions must be made in order to reduce the truss to one that is statically determinate
- Assumptions may be made with regards to the following:
  - When 1 diagonal in the panel is in tension, the corresponding cross diagonal will be in compression
- Two methods of analysis are generally acceptable:
- Method 1
  - If the diagonals are intentionally designed to be long & slender, it is reasonable to assume they cannot support compression force
  - Otherwise, they may easily buckle
  - Hence, the compressive diagonal is assumed to be a zero-force member

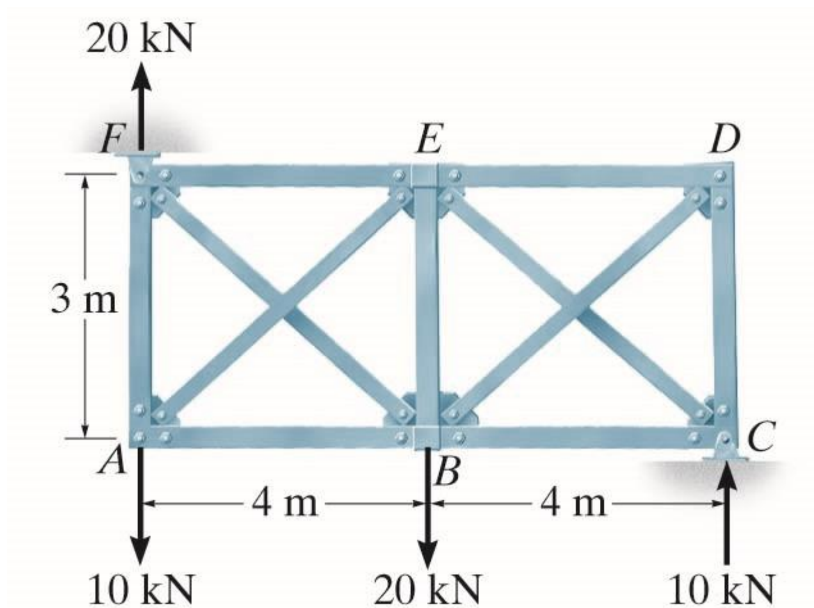
Trusses

- Method 2
  - If the diagonals are intended to be constructed from large rolled sections such as angles or channels, they may be equally capable of supporting a tensile & compressive force
  - We will assume that tension & compression diagonals each carry half the panel shear

Trusses

Example 7.1

Determine (approximately) the forces in the members of the truss. The diagonals are to be designed to support both tensile and compressive forces, and therefore each is assumed to carry half the panel shear. The support reactions have been computed.



Trusses

Example 7.1 (Solution)

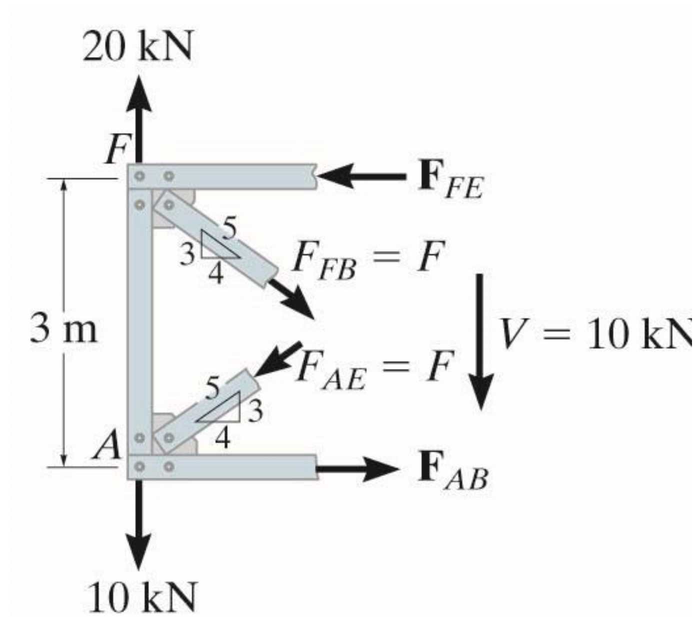
By inspection, the truss is statically indeterminate to the second degree. The 2 assumptions require the tensile & compressive diagonals to carry equal forces.

For a vertical section through the left panel, we have:

$$+\uparrow \sum F_y = 0$$

$$20 - 10 - 2\left(\frac{3}{5}\right)F = 0 \Rightarrow F = 8.33\,\text{kN}$$

$$F_{FB} = 8.33\,\text{kN(T)} \;\&\; F_{AE} = 8.33\,\text{kN(C)}$$



Trusses

Example 7.1 (Solution)

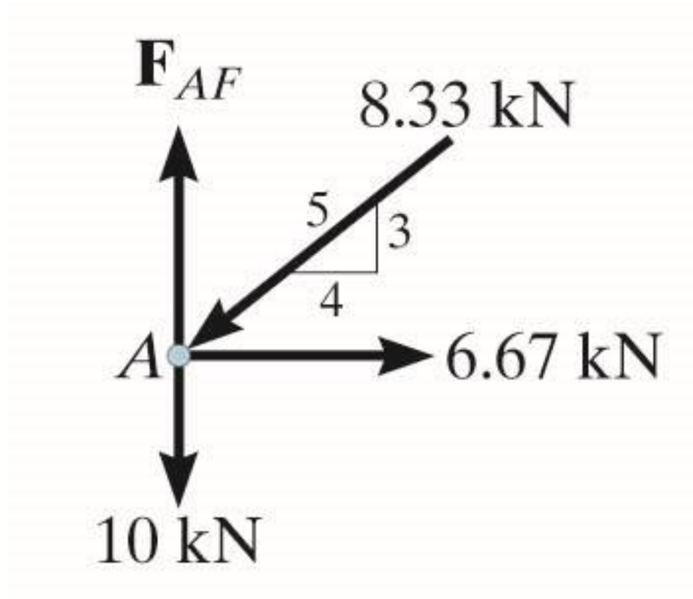
With anti-clockwise moments as +ve:

$$\sum M_A = 0$$

$$-8.33\left(\frac{4}{5}\right)(3) + F_{FE}(3) = 0 \Rightarrow F_{FE} = 6.67\,\text{kN(C)}$$

$$\sum M_F = 0$$

$$-8.33\left(\frac{4}{5}\right)(3) + F_{AB}(3) = 0 \Rightarrow F_{AB} = 6.67\,\text{kN(T)}$$



From Joint A, Fig 7.2(c),

$$+\uparrow \sum F_y = 0$$

$$F_{AF} - 8.33\left(\frac{3}{5}\right) - 10 = 0 \Rightarrow F_{AF} = 15\,\text{kN(T)}$$

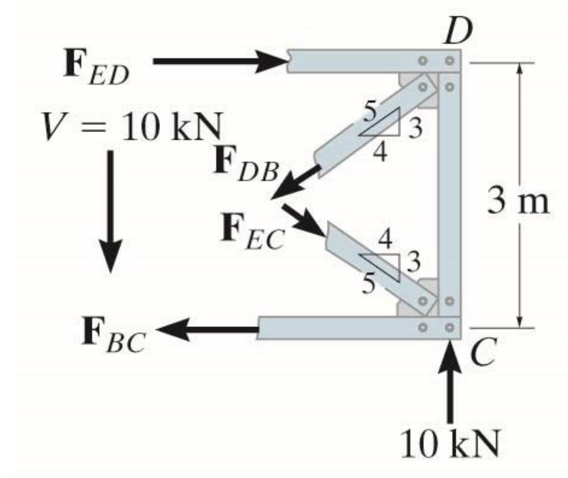
Trusses

Example 7.1 (Solution)

A vertical section through the right panel is shown in Fig 7.2(d).

$$F_{DB} = 8.33\,\text{kN(T)},\ \ F_{ED} = 6.67\,\text{kN(C)}$$

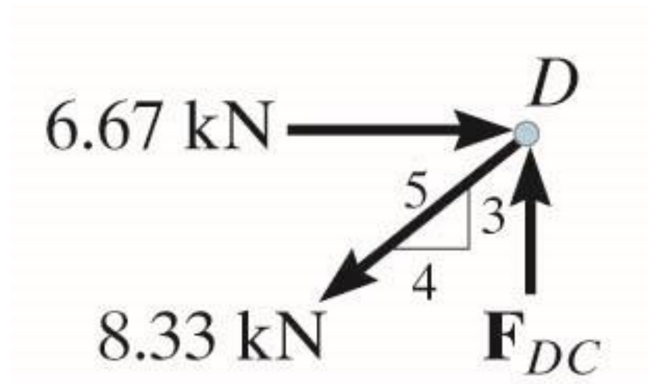
$$F_{EC} = 8.33\,\text{kN(C)},\ \ F_{BC} = 6.67\,\text{kN(T)}$$



Furthermore, using the free body diagrams of joints *D* & *E*, Fig 7.2(e) & 7.2(f), show that

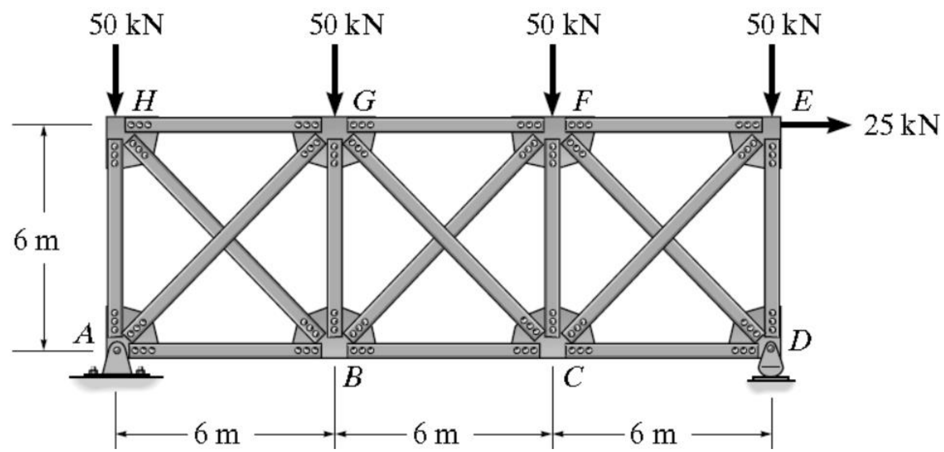
$$F_{DC} = 5\,\text{kN(C)}$$

$$F_{EB} = 10\,\text{kN(T)}$$



HW 7-1

Determine (approximately) the force in each member of the truss. Assume the diagonals can support either a tensile or a compressive force.



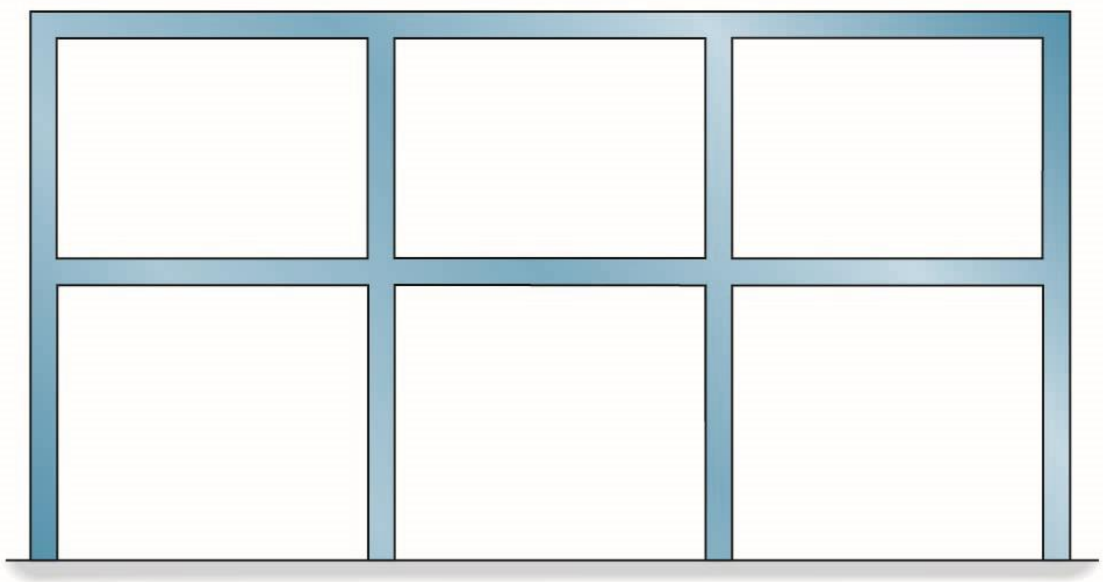
7.3

VERTICAL LOADS ON BUILDING FRAMES

7.3

Vertical Loads on Building Frames

- Building frames often consist of girders that are rigidly connected to columns
- This is to allow the structure to better able to resist the effects of lateral forces



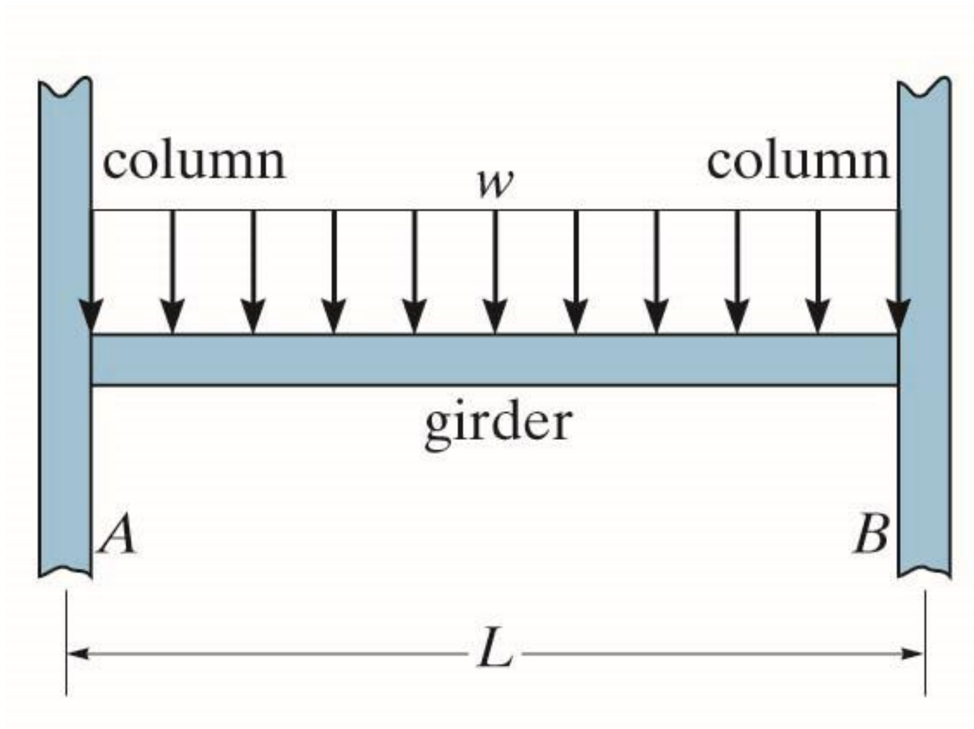
typical building frame

Vertical Loads on Building Frames

- One technique would be to consider only the members within a localised region of the structure
- This is possible if the deflections of the members within the region caused little disturbance to the members outside the structure
- The approximate location of the points of inflection can be specified
- These points are subjected to zero moments

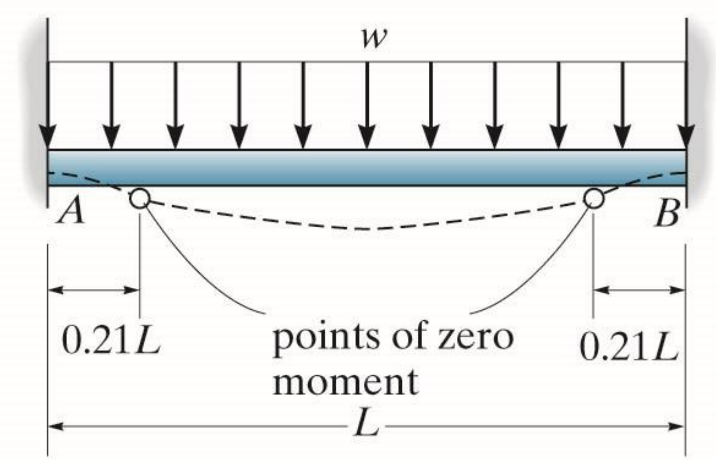
Vertical Loads on Building Frames

- Assumptions for approximate analysis
  - The column supports at *A* & *B* will each exert 3 reactions on the girder
  - The girder will be statically indeterminate to the third degree
  - 3 assumptions would be needed to perform an approximate analysis

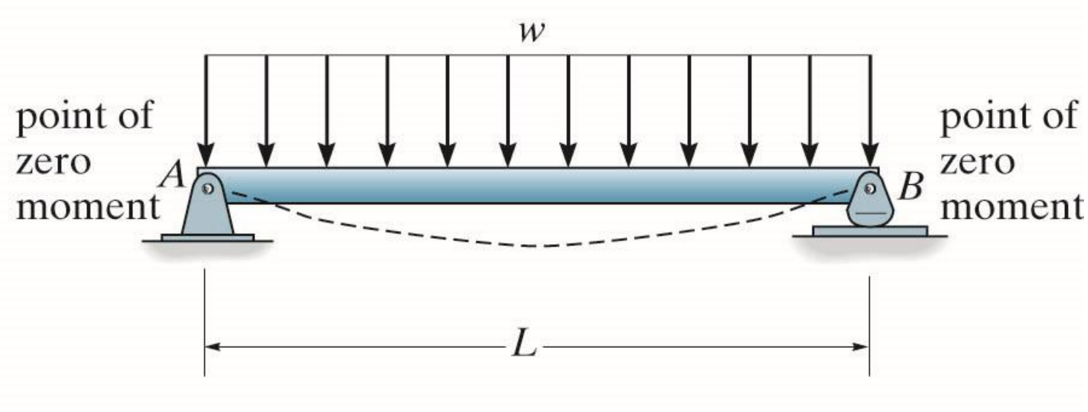


Vertical Loads on Building Frames

- Assumptions for approximate analysis
  - If the columns are stiff, no rotation at *A* & *B* will occur
  - However, if the column connections at *A* & *B* are very flexible, then zero moments will occur at the supports



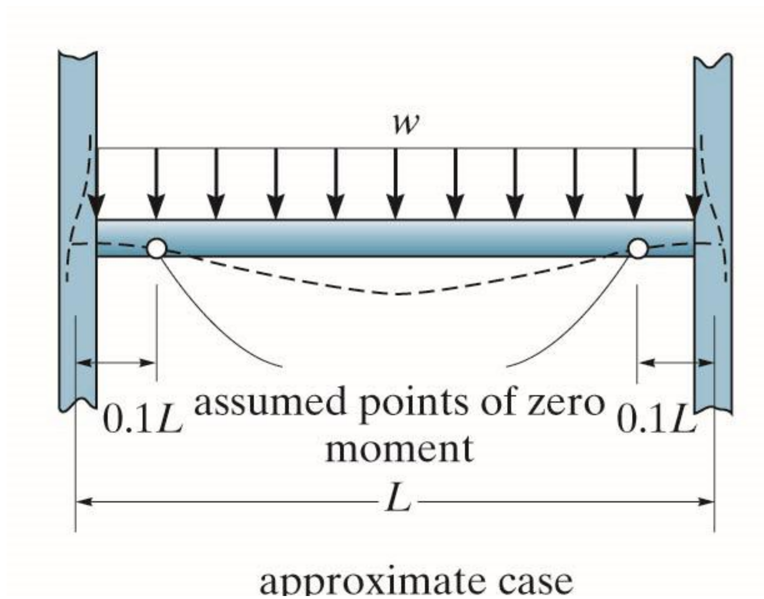
fixed supported



simply supported

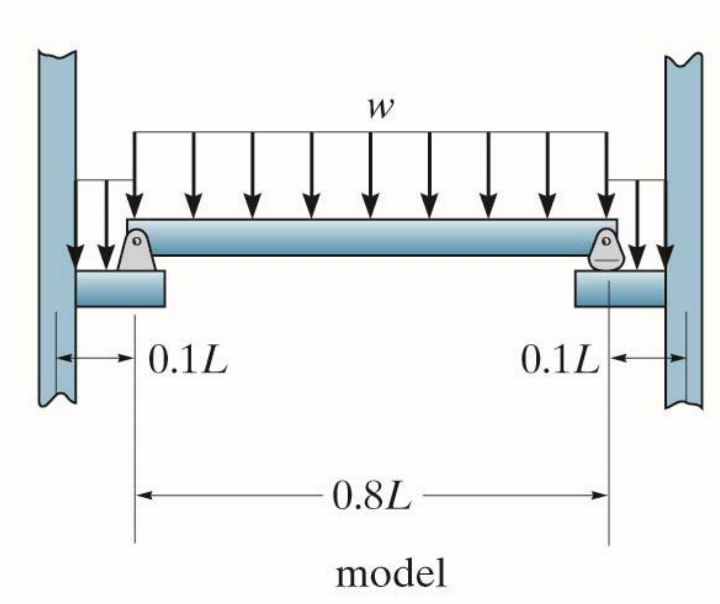
Vertical Loads on Building Frames

- Assumptions for approximate analysis
  - In reality, the columns will provide some flexibility at the supports
  - Therefore, point of zero moment occurs at the average point between the two extremes ➔ $(0.21L+0) / 2 \approx 0.1L$ from each support



Vertical Loads on Building Frames

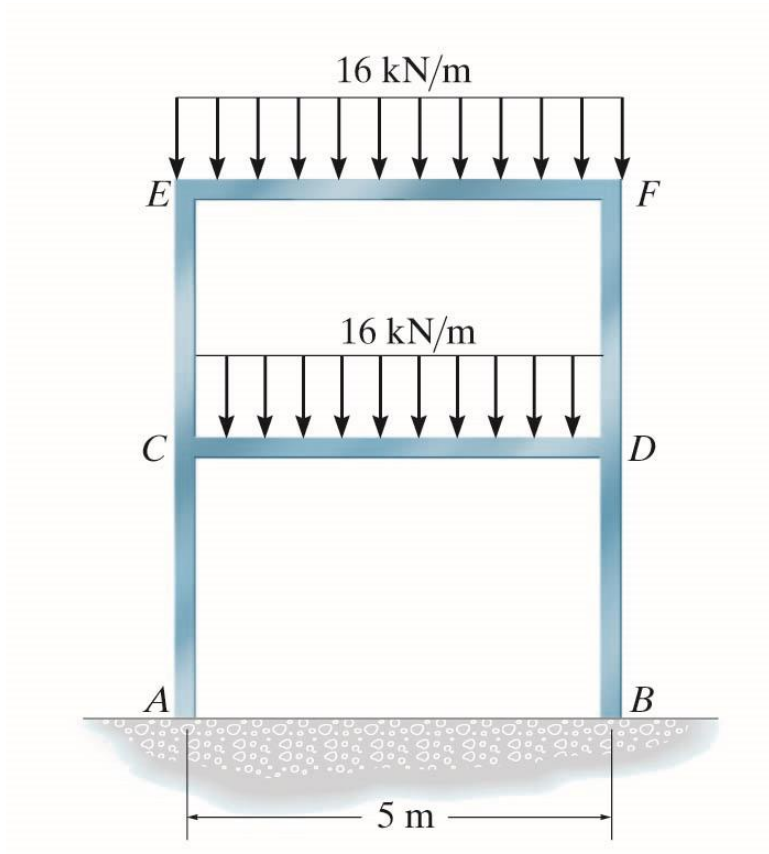
- Assumptions for approximate analysis
  - In summary the 3 assumptions are incorporated:
    - There is zero moment in the girder, 0.1L from the left support
    - There is zero moment in the girder, 0.1L from the right support
    - The girder does not support an axial force



Vertical Loads on Building Frames

Example 7.3

Determine (approximately) the moment at the joints *E* and *C* caused by members *EF* and *CD* of the building bent.



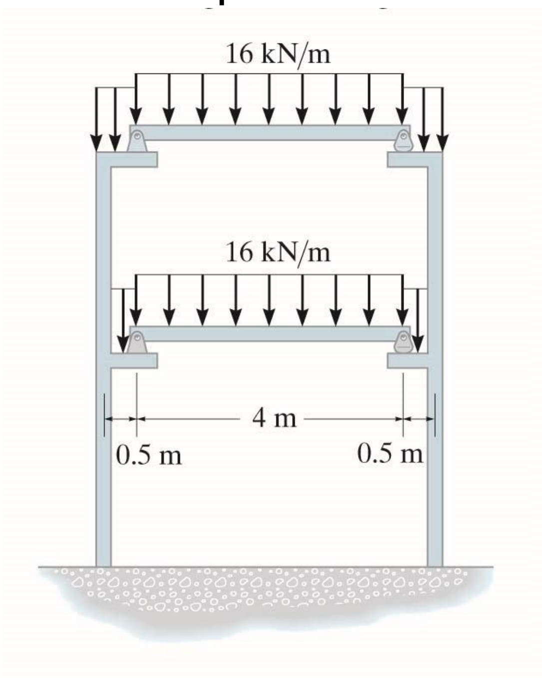
Vertical Loads on Building Frames

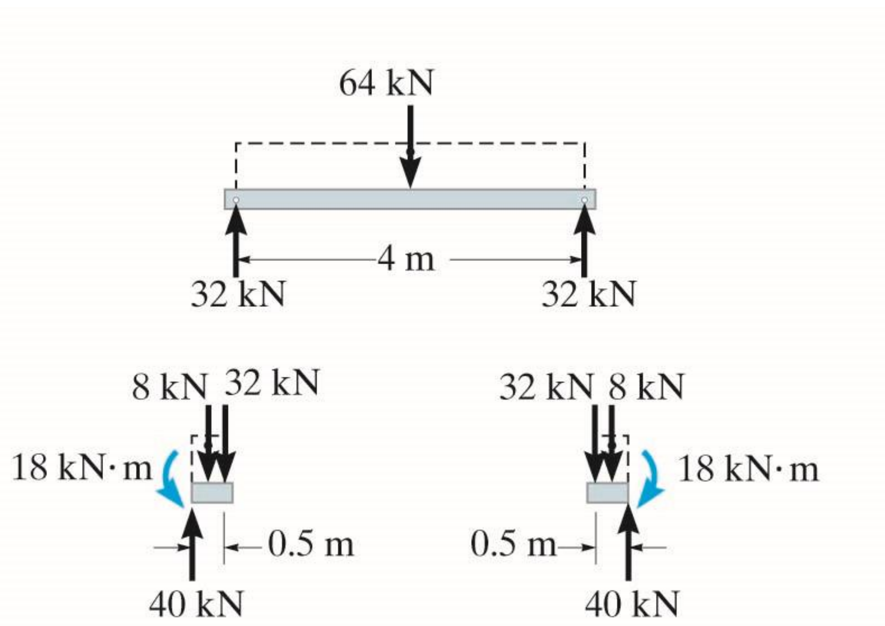
Example 7.3 (Solution)

For an approximate analysis, the frame is modeled as shown.

Note that the cantilevered spans supporting the center portion of the girder have a length of $0.1L = 0.5$ m

Equilibrium requires end reactions of center portion = 32 kN





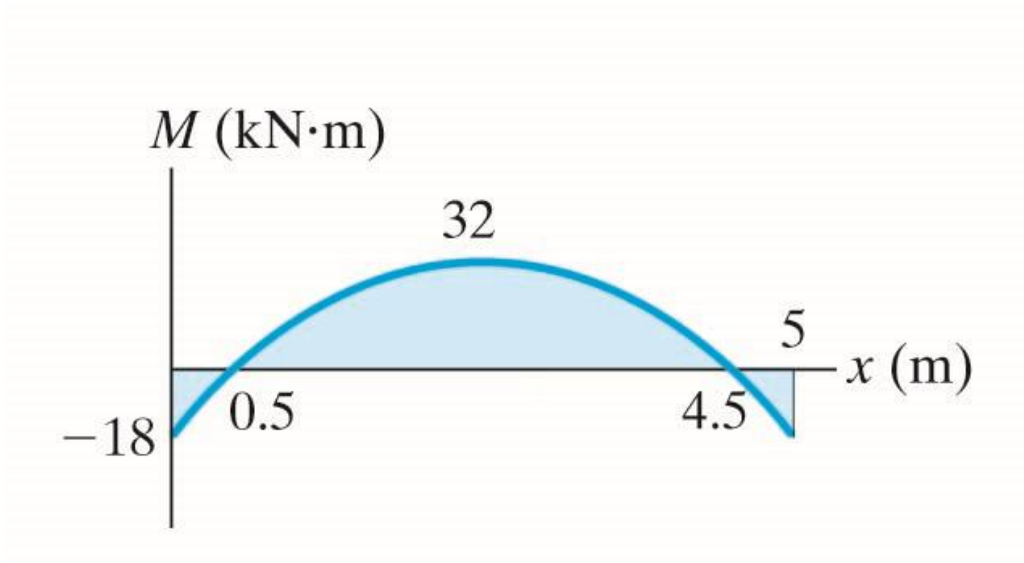
Vertical Loads on Building Frames

Example 7.3 (Solution)

Cantilevered spans are subjected to moment of:

$$M = 8(0.25) + 32(0.5) = 18\,\text{kN}\bullet\text{m}$$

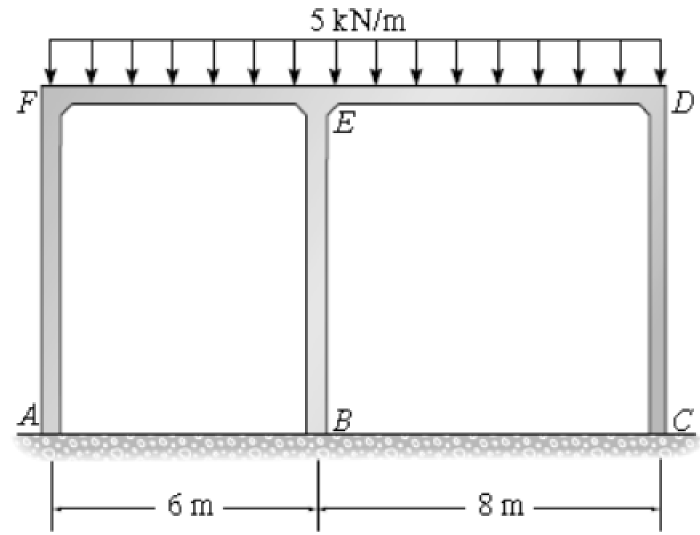
This approximate moment with opposite direction acts on the joints at *E* & *C*.



Vertical Loads on Building Frames

Example

Determine (approximately) the internal moments at joints *F* and *D* of the frame.

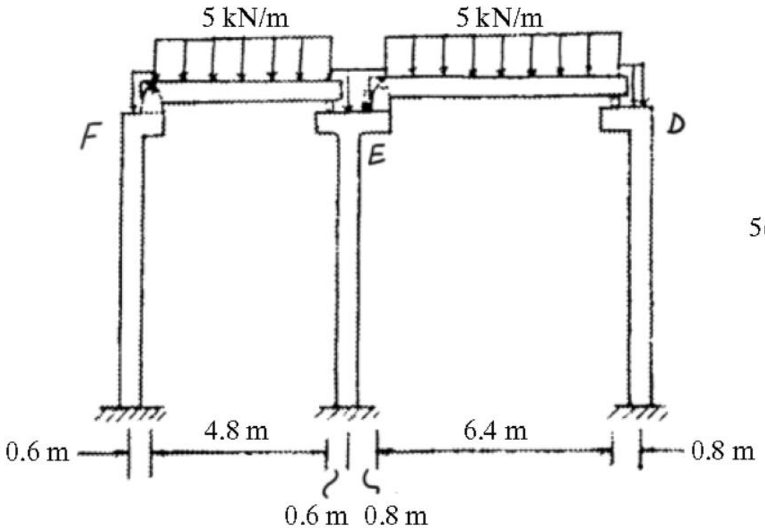


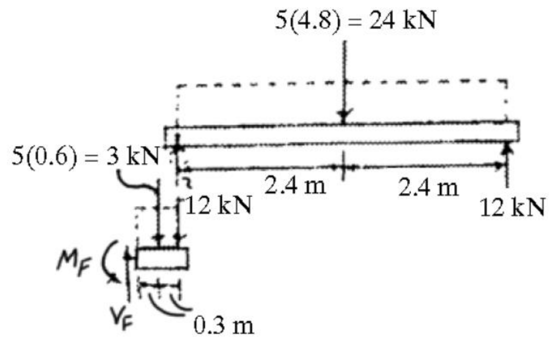
$$\circlearrowleft + \sum M_F = 0; \quad M_F - 3(0.3) - 12(0.6) = 0$$

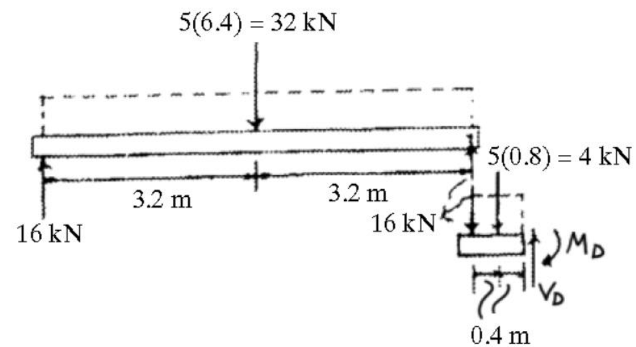
$$M_F = 8.1\ \mathrm{kN \cdot m}$$

$$\circlearrowleft + \sum M_D = 0; \quad -M_D + 4(0.4) + 16(0.8) = 0$$

$$M_D = 14.4\ \mathrm{kN \cdot m}$$

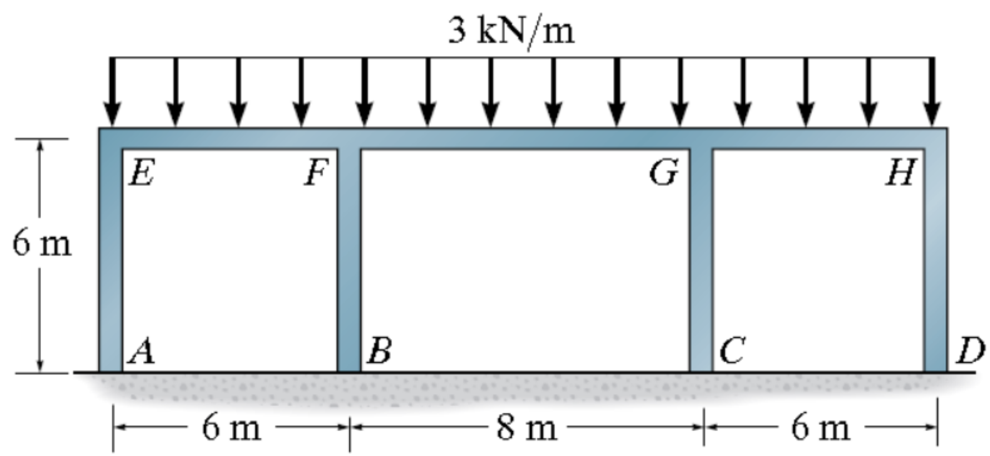






HW 7-2

Determine (approximately) the internal moments at joints *A* and *B* of the frame



Ans.
MA = 4.86 kN.m
MB = 3.78 kN.m

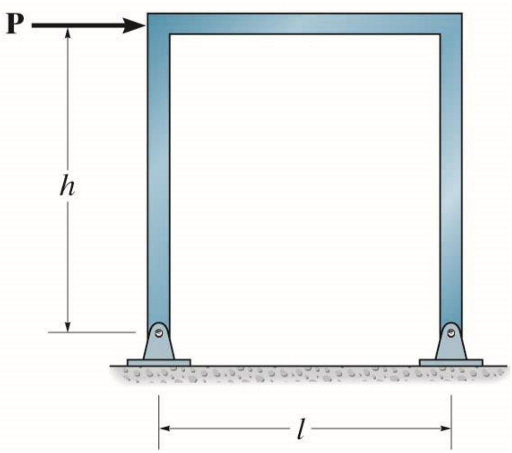
7.4

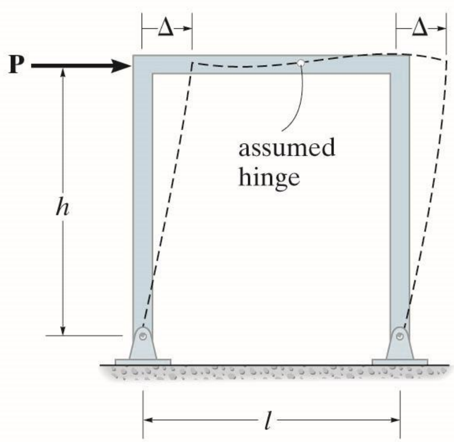
PORTAL FRAMES AND TRUSSES

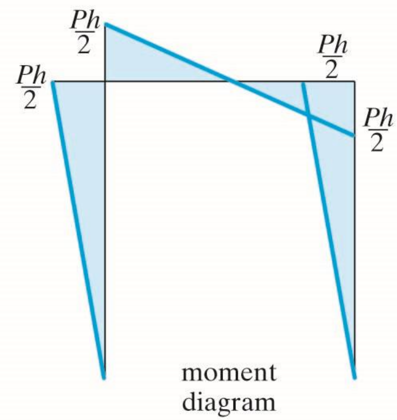
7.4

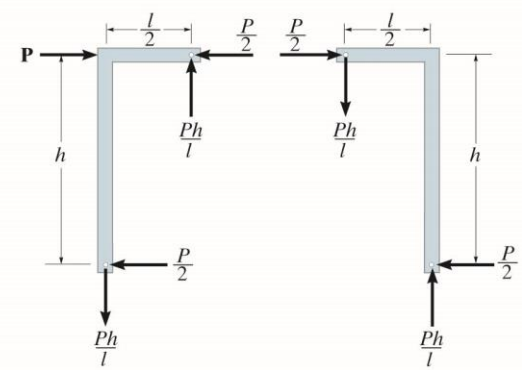
Portal Frames and Trusses

- Portal frames are used to transfer horizontal forces applied at the top of frame to the foundation
- Portals can be pin supported, fixed supported or supported by partial fixity









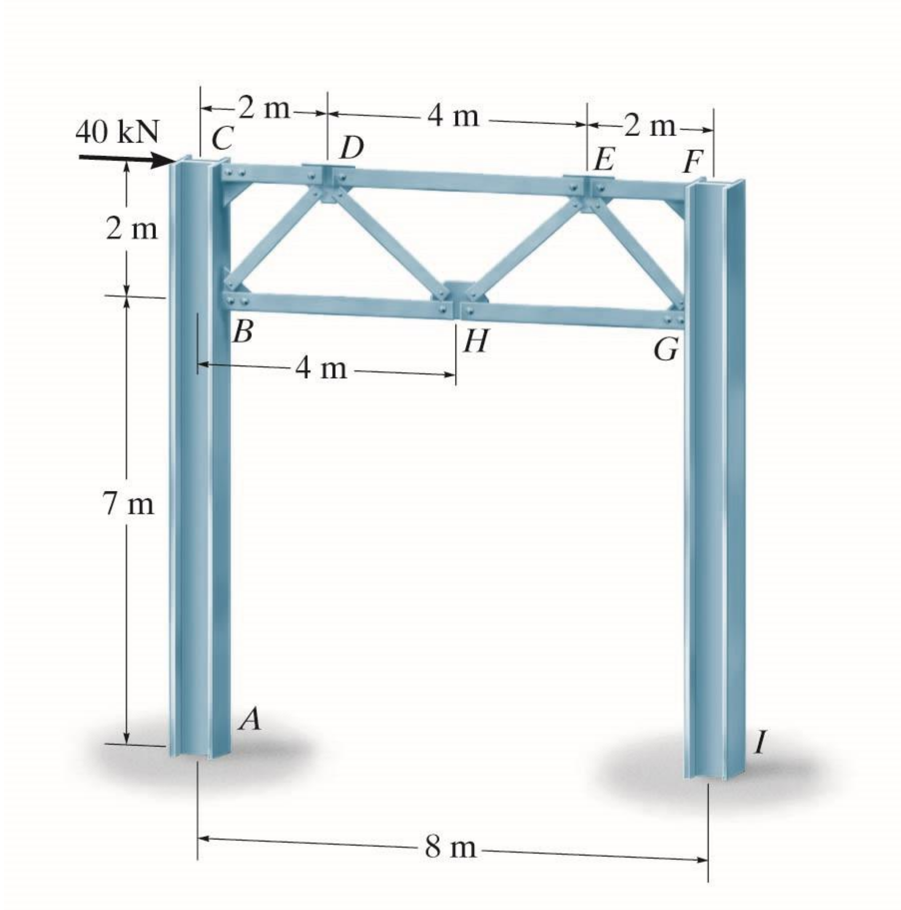
Portal Frames and Trusses

- We can analyse trussed portals using the same assumptions as those for simple portal frames
- For pin-supported columns, assume horizontal shear are equal
- For fixed-supported columns, assume horizontal reactions are equal and an point of inflection occurs on each column, midway between base of column & the lowest point of truss member connection to column

Portal Frames and Trusses

Example 7.4

Determine by approximate methods the forces acting in the members of the Warren portal.



Portal Frames and Trusses

Example 7.4 (Solution)

The truss portion *B*, *C*, *F*, *G* acts as a rigid unit
A point of inflection is assumed to exist at 7 m/2 = 3.5 m above *A* & *I*
Equal horizontal reactions act at the base of the column
Determine the reactions at the columns as follows:

➔Lower half of column

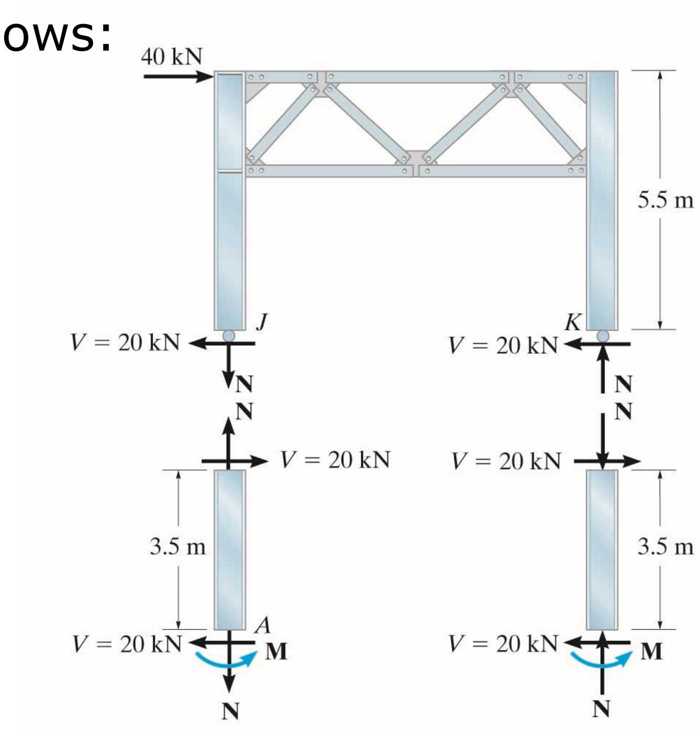
With anti-clockwise moments as +ve:

$$\sum M_A = 0; \quad M\text{-}3.5(20) = 0 \Rightarrow M = 70\,\text{kN} \bullet \text{m}$$

➔Upper half of column

With anti-clockwise moments as +ve:

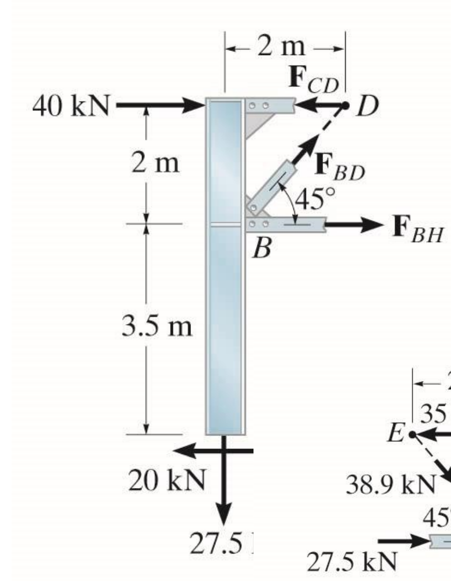
$$\sum M_J = 0; \quad -40(5.5) + N(8) = 0 \Rightarrow N = 27.5\,\text{kN}$$

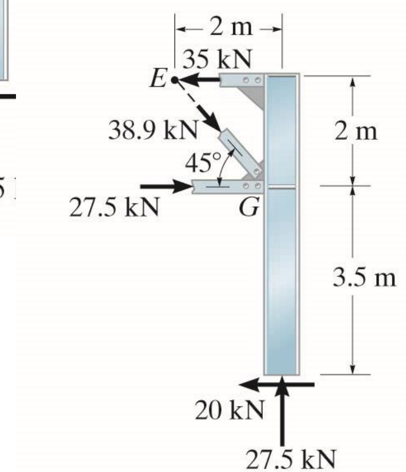


Portal Frames and Trusses

Example 7.4 (Solution)

Using the method of sections, we can proceed to obtain the forces in members *CD*, *BD* & *BH*





$$+\uparrow \sum F_Y = 0; \quad -27.5 + F_{BD}\sin 45^o = 0 \Rightarrow F_{BD} = 38.9\,\text{kN(T)}$$

With anti-clockwise moments as +ve:

$$\sum M_B = 0; \quad -20(3.5) - 40(2) + F_{CD}(2) = 0 \Rightarrow F_{CD} = 75\,\text{kN(C)}$$

$$\sum M_D = 0; \quad F_{BH}(2) - 20(5.5) + 27.5(2) = 0 \Rightarrow F_{BH} = 27.5\,\text{kN(T)}$$

Portal Frames and Trusses

Example 7.4 (Solution)

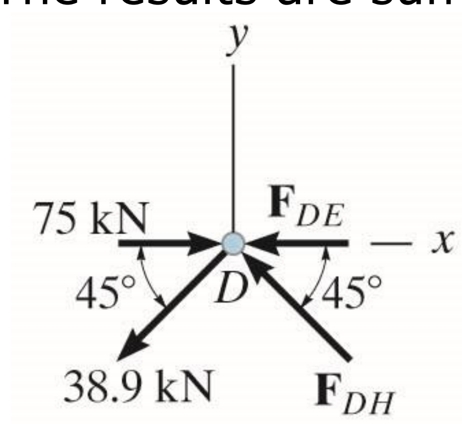
Using these results we can find the force in each of the other truss members using method of joints

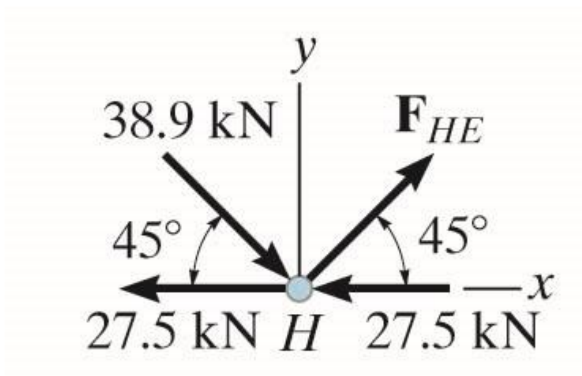
$$+\uparrow \sum F_Y = 0; \quad F_{DH}\sin 45^o - 38.9\sin 45^o = 0 \Rightarrow F_{DH} = 38.9\,\mathrm{kN(C)}$$

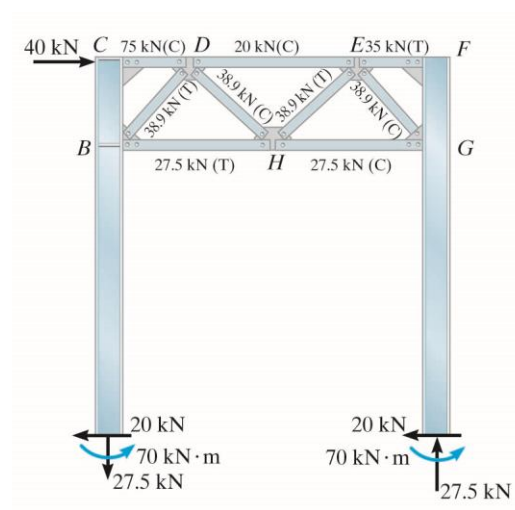
$$\underset{\rightarrow}{+} \sum F_X = 0; \quad 75 - 2(38.9\cos 45^o) - F_{DE} = 0 \Rightarrow F_{DE} = 20\,\mathrm{kN(C)}$$

$$+\uparrow \sum F_Y = 0; \quad F_{HE}\sin 45^o - 38.9\sin 45^o = 0 \Rightarrow F_{HE} = 38.9\,\mathrm{kN(T)}$$

The results are summarized as

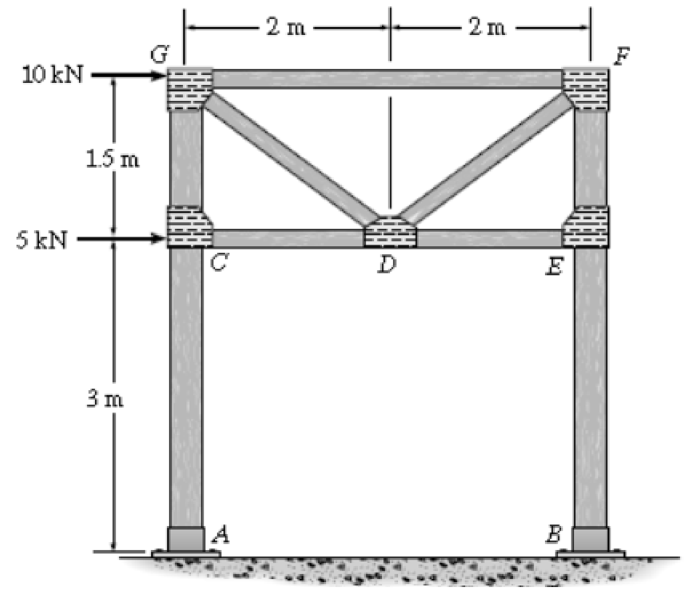






HW 7-3

Determine (approximately) the force in each truss member of the portal frame. Also find the reactions at the fixed column supports *A* and *B*. Assume all members of the truss to be pin connected at their ends.



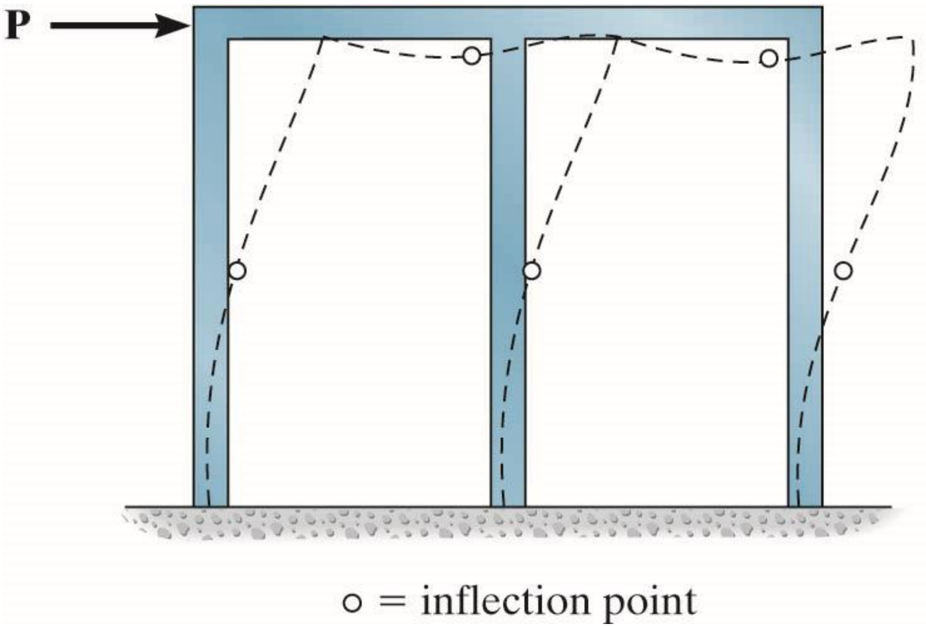
7.5

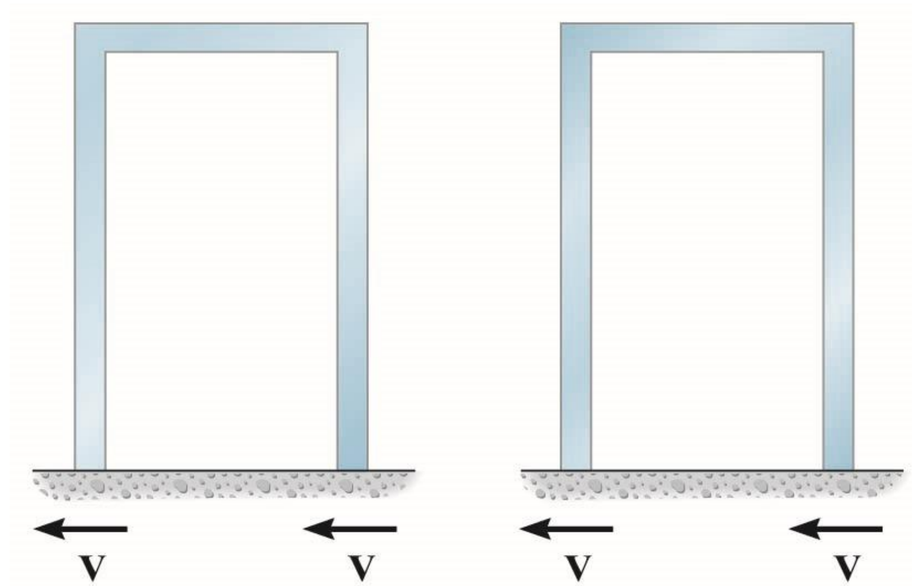
LATERAL LOADS ON BUILDING FRAMES: PORTAL METHOD

7.5

Lateral Loads on Building Frames: Portal Method

- A building bent deflects in the same way as a portal frame
- Each bent of the frame can be considered as a series of portals
- The interior columns would represent the effect of 2 portal columns & would carry 2x the shear *V* as the exterior columns





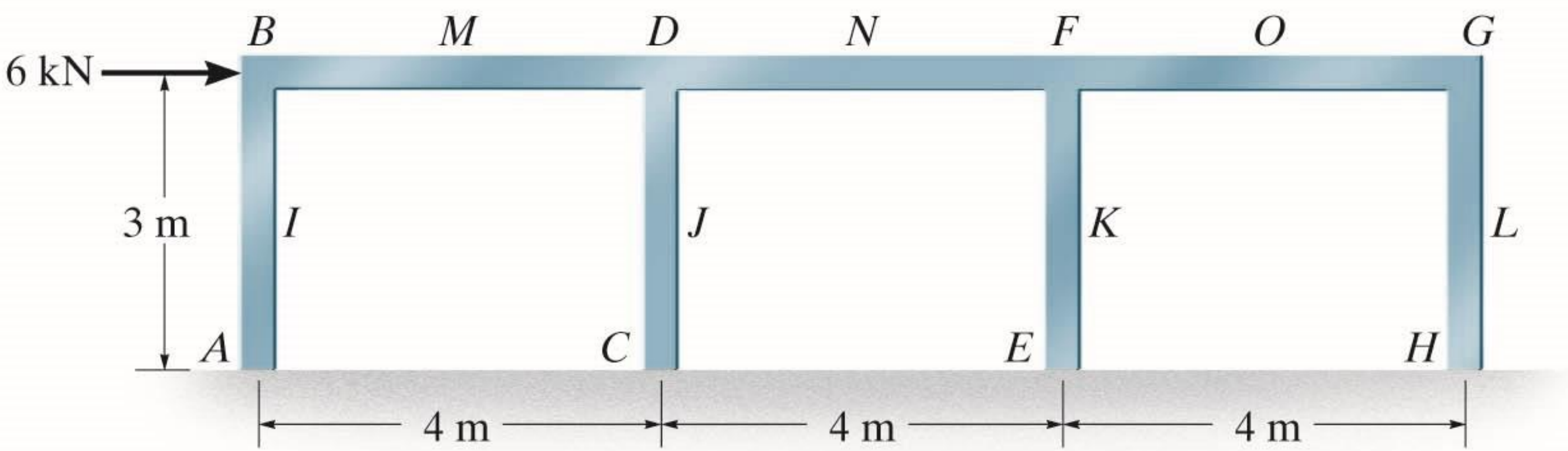
Lateral Loads on Building Frames: Portal Method

- The portal method for analyzing fixed supported building frames requires the following assumptions:
  - A hinge is placed at the center of each girder
  - A hinge is placed at the center of each column
  - At a given floor level, the shear at the int column hinges is 2x that at the ext column hinges
- These assumptions provide an adequate reduction of the frame to one that is statically determinate and yet stable under loading
- This method is more suitable for buildings having low elevation and uniform framing

Lateral Loads on Building Frames: Portal Method

Example 7.5

Determine (approximately) the reactions at the base of the columns of the frame. Use the portal method of analysis.

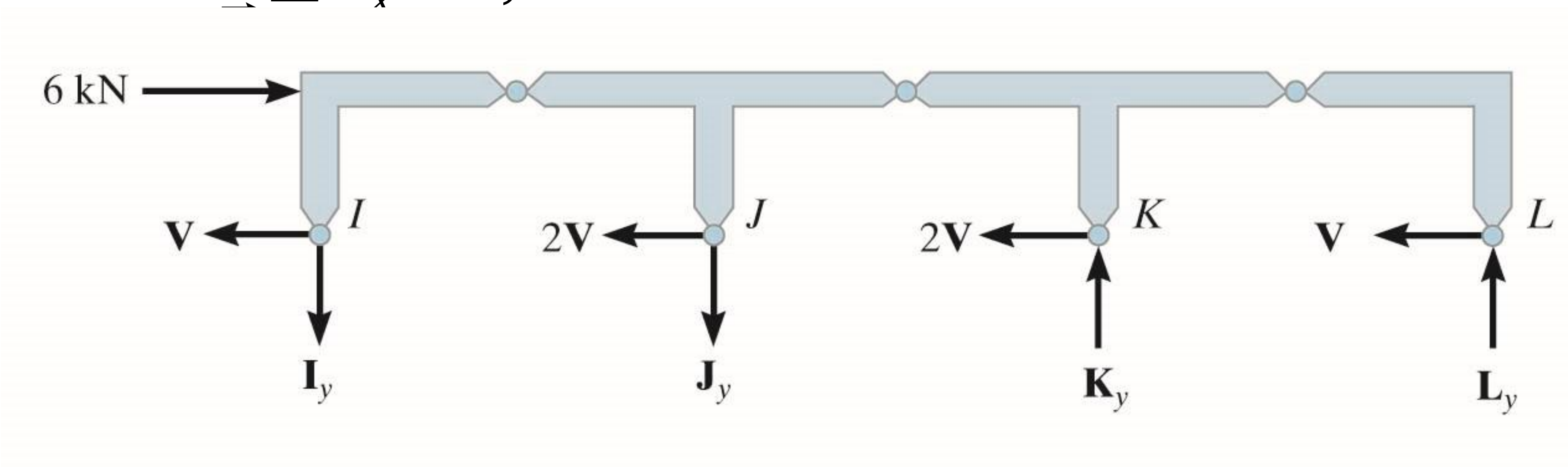


Lateral Loads on Building Frames: Portal Method

Example 7.5 (Solution)

Applying the first 2 assumptions of the portal method, we place hinges at the centers of the girders & columns of the frame.
A section through the column hinges at *I*, *J*, *K* & *L* yields the free body diagram. The third assumption regarding the column shear applies.

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad 6 - 6V = 0 \Rightarrow V = 1\text{kN}$$



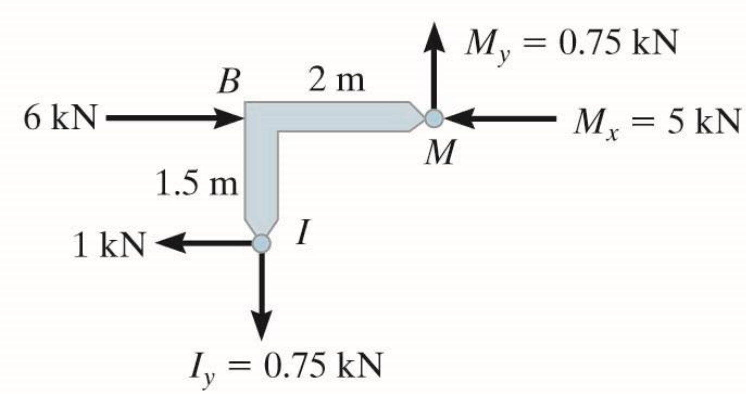
Lateral Loads on Building Frames: Portal Method

Example 7.5 (Solution)

Using this result, we proceed to dismember the frame at the hinges & determine their reactions.
As a general rule, always start analysis at the corner or joint where the horizontal load is applied.
The free-body diagram of segment *IBM* is shown.

The 3 reactions components at the hinges are determined by applying

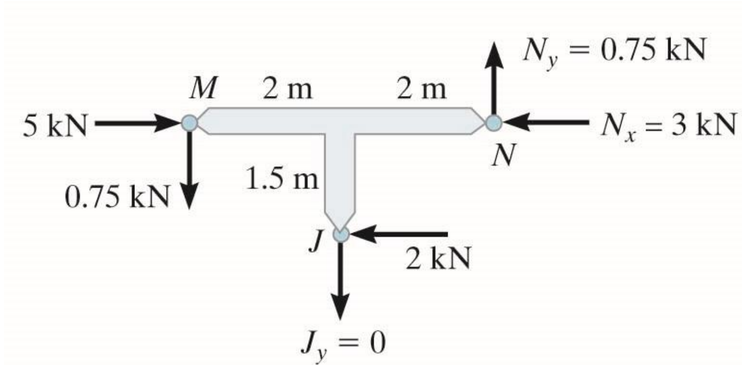
$$\sum M_M = 0; \;\; \sum F_x = 0; \;\; \sum F_y = 0$$



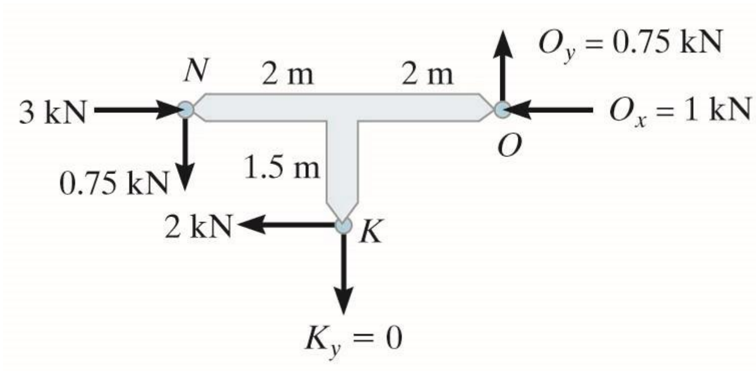
Lateral Loads on Building Frames: Portal Method

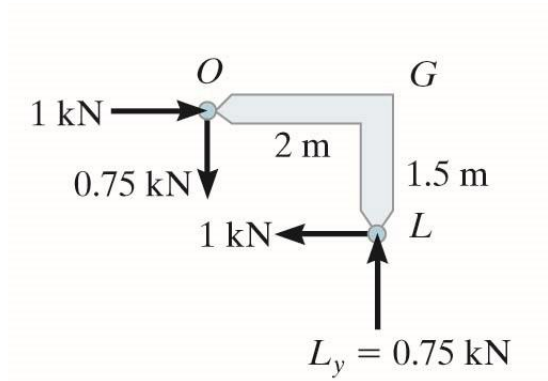
Example 7.5 (Solution)

The adjacent segment *MJN* is analyzed next.



This is followed by segment *NKO* and *OGL*.

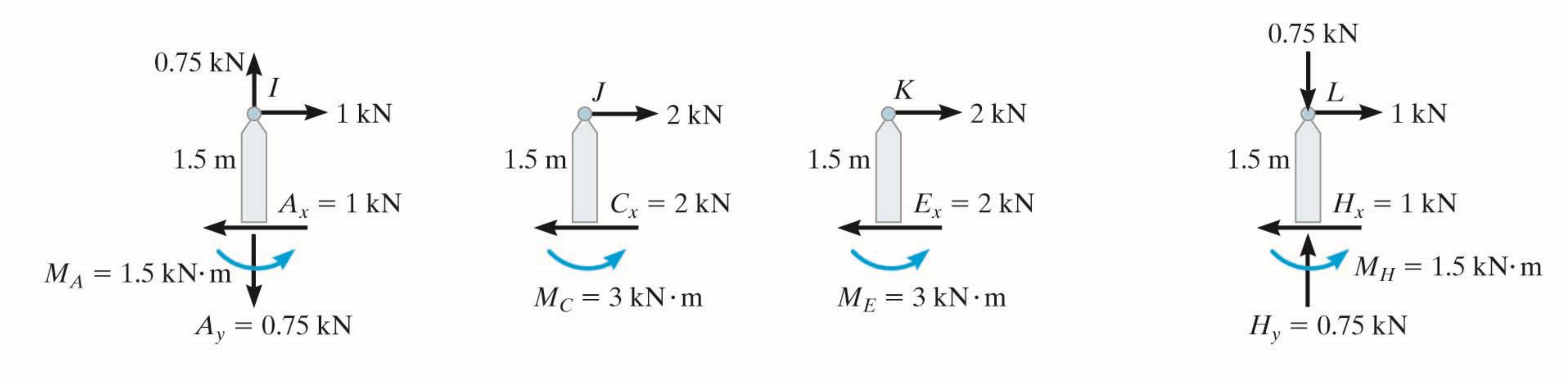




Lateral Loads on Building Frames: Portal Method

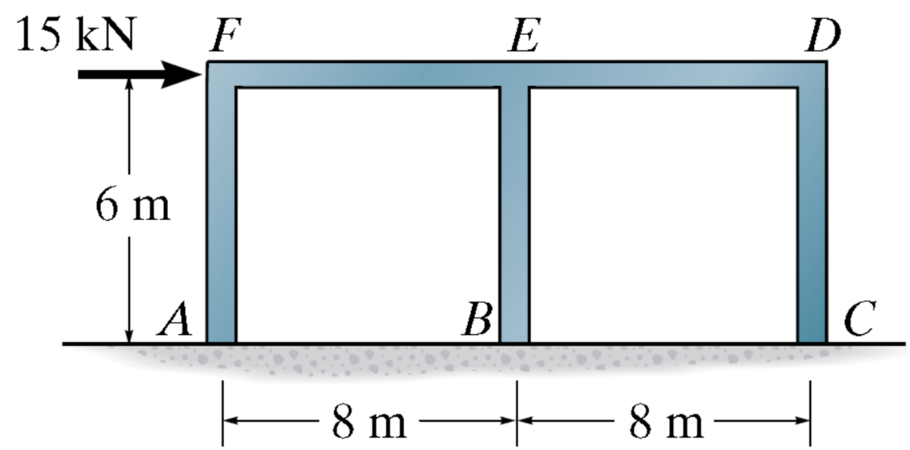
Example 7.5 (Solution)

Using these results, the free body diagram of the columns with their support reactions are shown.

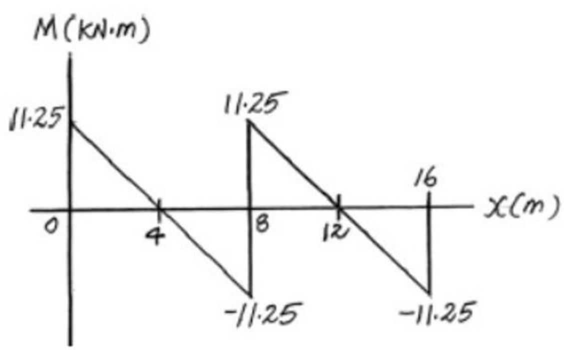


HW 7-4

Use the portal method of analysis and draw the moment diagram for girder *FED*.



Ans.



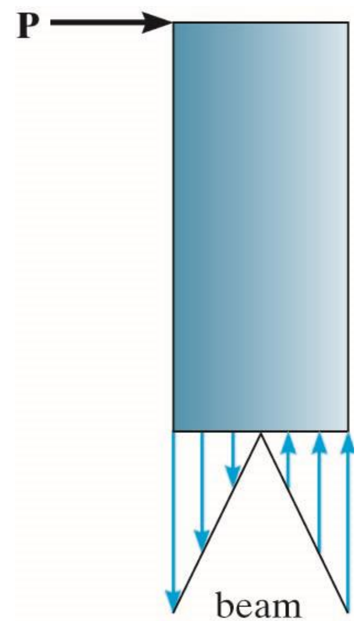
7.6

LATERAL LOADS ON BUILDING FRAMES: CANTILEVER METHOD

7.6

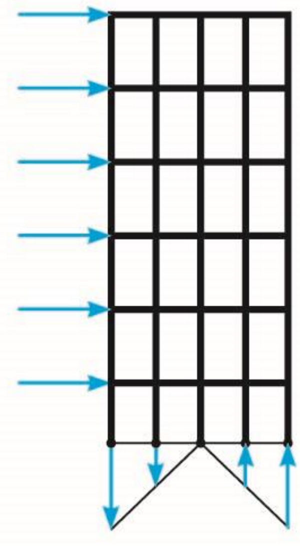
Lateral Loads on Building Frames: Cantilever Method

- This method is based on the same action as a long cantilevered beam subjected to a transverse load
- This causes a bending stress that varies linearly from the beam's neutral axis
- In a similar manner, the lateral loads on a frame tends to tip the frame over or cause a rotation about a neutral axis lying in the horizontal plane that passes through the columns at each floor level



Lateral Loads on Building Frames: Cantilever Method

- To counter this, the axial forces in the columns will be tensile on one side of the neutral axis & compressive on the other side
- It is reasonable to assume this axial stress has a linear variation from the centroid of column areas or the neutral axis
- This method is appropriate if the frame is tall & slender or has columns with different cross-sectional areas



building frame

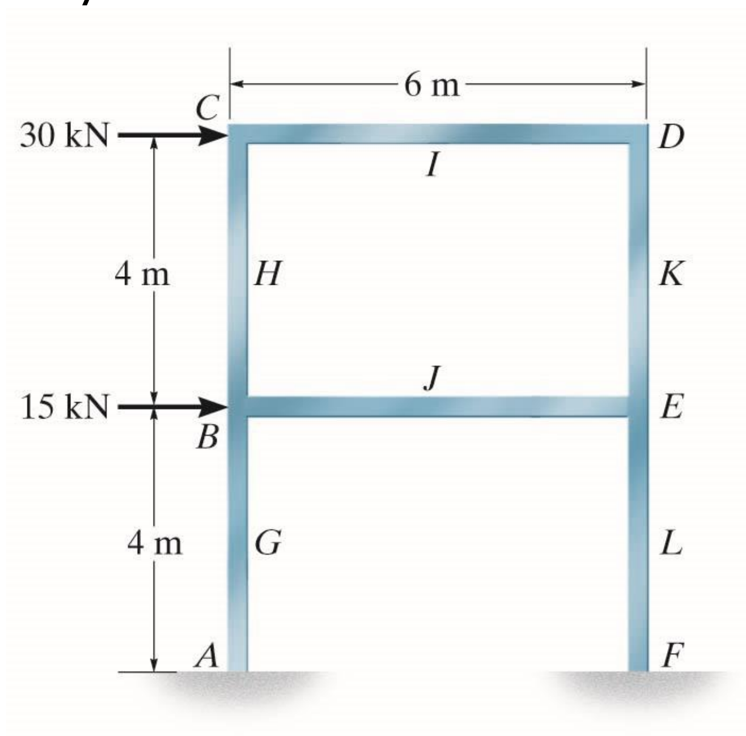
Lateral Loads on Building Frames: Cantilever Method

- In summary, the following assumptions apply for a fixed support frame
  - A hinge is placed at the center of each girder
  - A hinge is placed at the center of each column
  - The axial stress in a column is proportional to its distance from the centroid of the cross-sectional areas of the columns at a given floor level
  - Since stress = force per area, then in the case of equal cross-sectional areas, the force in a column is proportional to its distance from the centroid of column areas
- These assumptions reduce the frame to one that is both stable & statically determinate

Lateral Loads on Building Frames: Cantilever Method

Example 7.7

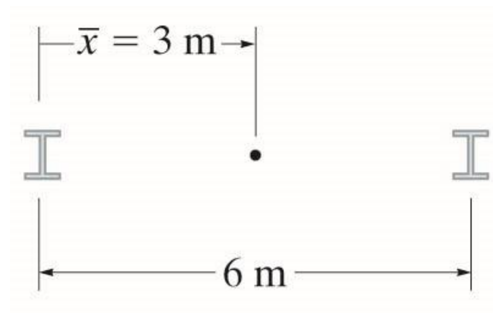
Determine (approximately) the reactions at the base of the columns of the frame. The columns are assumed to have equal cross-sectional areas. Use the cantilever method of analysis.



Lateral Loads on Building Frames: Cantilever Method

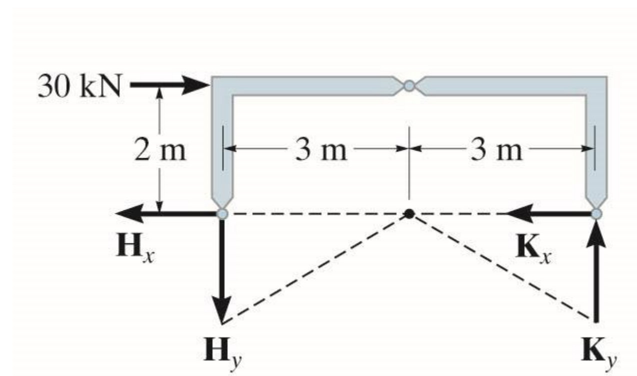
Example 7.7 (Solution)

Hinges are placed at midpoints of the columns & girders. The locations of these points are indicated by the letters *G* through *L*.
The axial force in each column is approximate distance from this point.
A section through the hinges *H* and *K* at the top floor yields the free body diagram as shown.



With anticlockwise moments + ve,

$$\sum M = 0 \quad -30(2) + 3H_y + 3K_y = 0$$



By proportional triangles

$$\frac{H_y}{3} = \frac{K_y}{3} \Rightarrow H_y = K_y = 10\,\text{kN}$$

Lateral Loads on Building Frames: Cantilever Method

Example 7.7 (Solution)

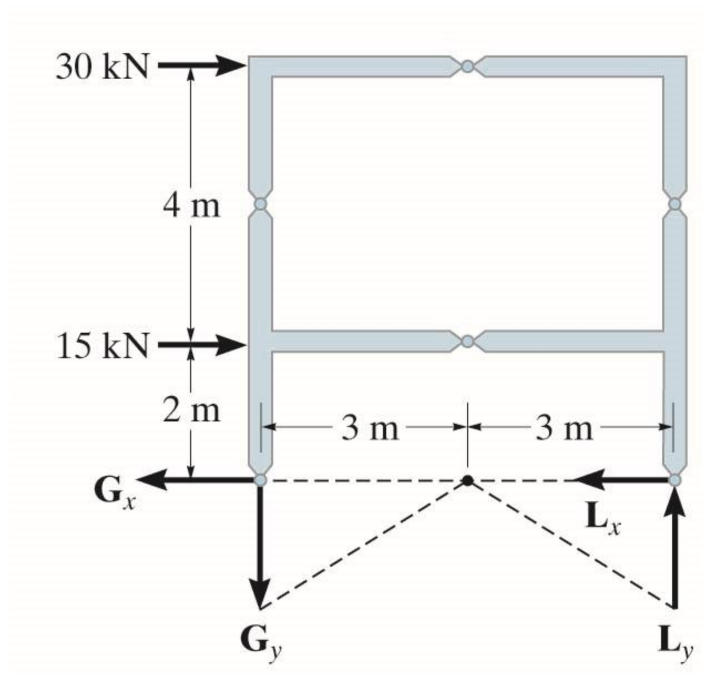
In a similar manner, using a section of the frame through the hinges at *G* & *L*, we have:

With anticlockwise moments + ve,

$$\sum M = 0 \quad -30(6) - 15(2) + 3G_y + 3L_y = 0$$

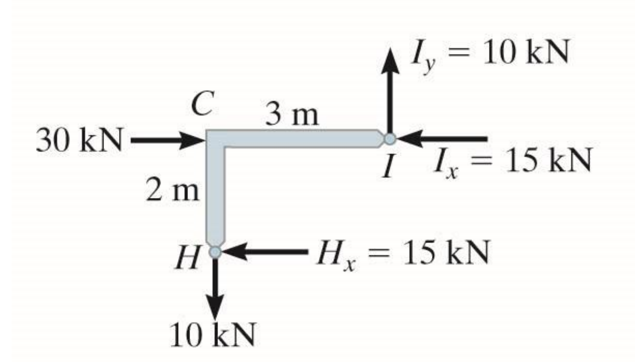
By proportional triangles

$$\frac{G_y}{3} = \frac{L_y}{3} \Rightarrow G_y = L_y \Rightarrow 35\,\text{kN}$$

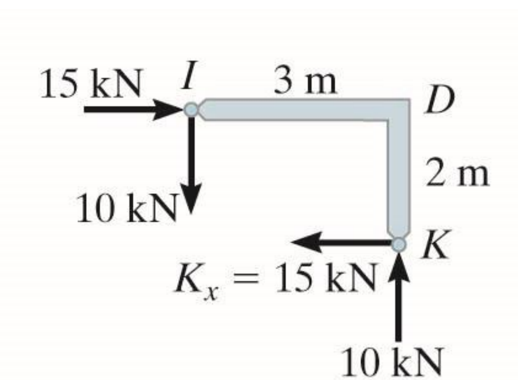


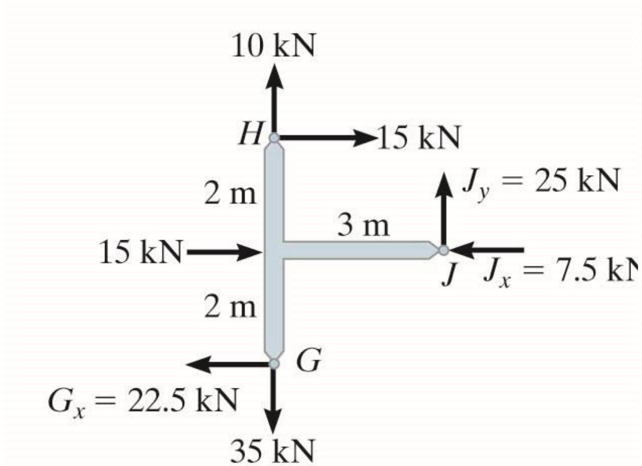
Lateral Loads on Building Frames: Cantilever Method

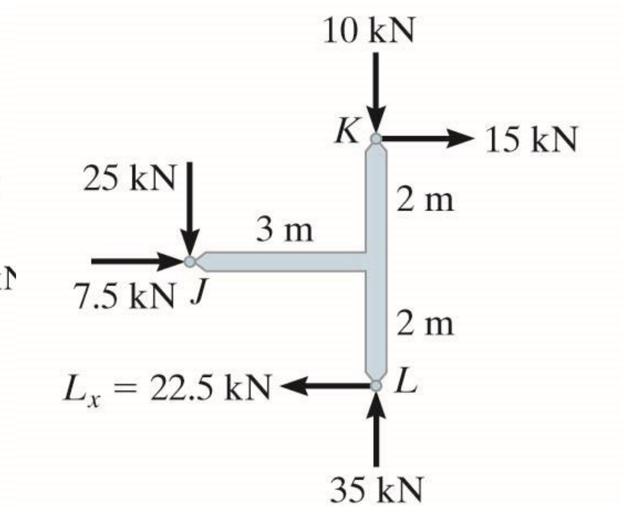
Example 7.7 (Solution)



Each part of the frame can be analyzed using the above results.
Beginning with the upper corner where the applied loading occurs, segment *HCI*.
Applying eqn of equilibrium yields the results for H_x, I_x and I_y.
Using these results, segment *IDK* is analyzed, next followed by *HJG* & *KJL*.



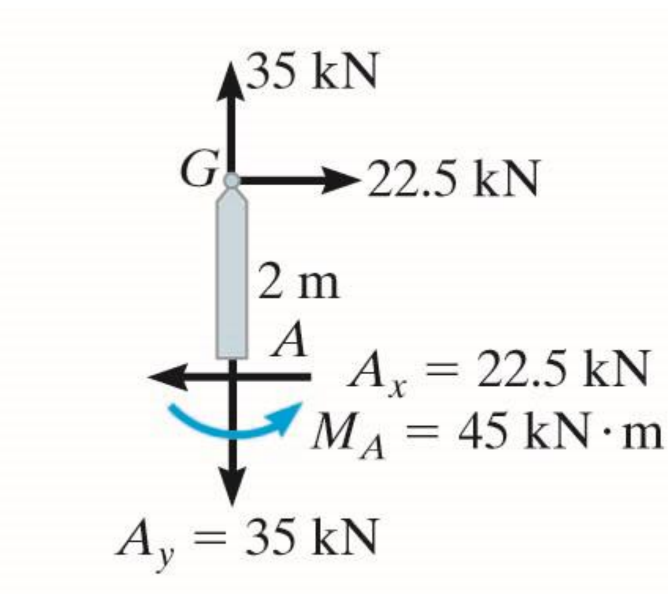


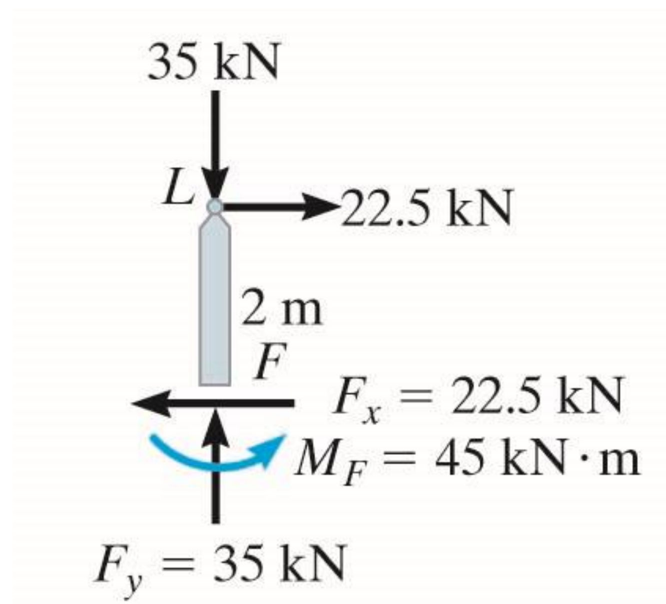


Lateral Loads on Building Frames: Cantilever Method

Example 7.7 (Solution)

Finally, the bottom portions of the columns.





HW 7-5

Use the cantilever method and determine (approximately) the reactions at *A*. All of the columns have the same cross-sectional area

Ans.

$A_x = 8.14\ \text{kN}$

$M_A = 12.2\ \text{kN}\cdot\text{m}$

$A_y = 12.59\ \text{kN}$